Bramwell *et al.* Reply: Zheng and Trimper [1] confirm the conjecture given in our Letter [2] that the probability distribution for order parameter fluctuations in the 2D and 3D Ising models at a temperature $T^*(L)$ slightly below $T_C(L \to \infty)$ approximates the universal functional form of the 2D-XY model in its low temperature phase [3]. They show quantitatively that $T_C - T^*(L)$ scales as $L^{-1/\nu}$. The XY-type scaling is, of course, only one locus in the L^{-1}, T plane and for general L^{-1}, T the probability distribution function (PDF) is not of the XY form [2,4]. The point of departure between our interpretation of this result and that of Zheng and Trimper is that we attribute the PDF to critical fluctuations and they do not. We are pleased to take the opportunity to discuss this point in detail.

The 2D-XY model is critical throughout the low temperature regime with diverging longitudinal fluctuations. It is therefore incorrect to think that critical "fluctuations are mainly rotational." The physics of a phase transition in a spin system with continuous symmetry is controlled by the divergence of the longitudinal susceptibility, the transverse susceptibility being infinite at all temperatures. The lengthening magnetization vector as order develops drives the diffusion constant around the circle (in the XY model) to zero in the thermodynamic limit, and consequently rotational symmetry is broken. The scalar magnetization is therefore a critical quantity, as can be seen through any finite-size scaling criterion. However, it is rather a special limit for critical fluctuations: despite the susceptibility diverging as $\sim L^{d-\eta}$ and $\sigma/\langle m \rangle$ remaining independent of system size, the latter ratio is small, $\approx \eta/4$ [3]. The result, paradoxically, is that the divergent fluctuations never bring the order parameter near the limits $m = 0$ and $m = 1$. The critical fluctuations therefore occur without ever changing the fixed topology (or symmetry) imposed by the corrections to the thermodynamic limit [3]. That is, there is a barrier to jump to arrive at $m = 0$, but $\langle m \rangle \sim L^{-\eta/2}$ is not an intensive variable and the free energy barrier is not extensive; it is a correction to the thermodynamic limit and a pure effect of criticality.

Zheng and Trimper find that, for the Ising model, the measured correlation length at $T^*(L)$ is small compared with L. This property should ensure that fluctuations in the Ising systems studied can be described in a similar way to those of the XY model. Indeed the authors point out that at $T^*(L)$, the order parameter remains far from the minimum of probability: $m = 0$. However, they are wrong to conclude that the fluctuations at this temperature are "not a characteristic property at the critical point": the observation of the universal fluctuations at constant $s = L\tau$ means that the "small" correlation length is fixed by the system size; it does diverge in the thermodynamic limit and the observed property is a critical property. If the correlation length remained finite (for example, if the transition were first order), the central limit theorem would apply and the limit distribution would be Gaussian, or a closely related function [5].

The critical point is a singular point in the thermodynamic limit and admits many definitions in a corresponding finite system. The existence of the family of loci $[T(L), L^{-1}]$, all collapsing onto T_C in the thermodynamic limit has been addressed in [4–6]. However, in view of the fact that many complex systems approximate the XY functional form, it seems that the important question for the Ising model is as follows: Does the locus of points identified here have any special properties, or is it just one of many curves?

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[1] B. Zheng and S. Trimper, preceding Comment, Phys. Rev. Lett. **87**, 188901 (2001).
[2] S. T. Bramwell, K. Christensen, J.-Y. Fortin, P. C. W. Holdsworth, H. J. Jensen, S. Lise, J. M. López, M. Nicodemi, J.-F. Pinton, and M. Sellitto, Phys. Rev. Lett. **84**, 3744 (2000).
[3] P. Archambault, S. T. Bramwell, and P. C. W. Holdsworth, J. Phys. A **30**, 8363 (1997).
[4] S. T. Bramwell, J.-Y. Fortin, P. C. W. Holdsworth, S. Peysson, B. Portelli, J.-F. Pinton, and M. Sellitto, Phys. Rev. E **63**, 041106 (2001).
[5] For a full discussion, see the appendix to Ref. [4].
[6] B. Portelli, P. C. W. Holdsworth, M. Sellitto, and S. T. Bramwell, Phys. Rev. E **64**, 036111 (2001).

