

Dissipationless Spin Transport in Thin Film Ferromagnets

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Metallic thin film ferromagnets generically possess spiral states that carry dissipationless spin currents. We relate the critical values of these supercurrents to micromagnetic material parameters, identify the circumstances under which the supercurrents will be most robust, and propose experiments which could reveal this new collective transport behavior.

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In ferromagnetic metals and semiconductors, quasiparticle states can be manipulated by external magnetic fields that couple to the spin-magnetization-density collective coordinate. This property is responsible for related robust magnetoresistance effects that occur in various geometries such as anisotropic magnetoresistance in bulk samples, giant magnetoresistance [1,2] in metallic multilayers, and tunnel magnetoresistance [3,4] in tunnel junctions. In this paper we propose a distinctly different type of spin-dependent transport effect, in which spin current is carried collectively rather than by quasiparticles. Because the spin current is nonzero when its quasiparticles are in equilibrium, it is carried without dissipation. This spin-supercurrent state occurs only in easy-plane ferromagnets and will be robust only when anisotropy within the easy plane is weak. We propose an experiment to observe this effect in thin films of ferromagnetic metals.

The key observation that motivates this proposal arises by considering the class of excited states obtained from the ferromagnetic ground state by following its adiabatic evolution as equal and opposite constant vector potentials are introduced for up and down spins, with the spin-quantization axis perpendicular to the ferromagnet's easy plane. The many-particle Hamiltonian is

$$\mathcal{H} = \sum_i \left[\frac{\hbar^2}{2m} \left(\mathbf{k}_i + \frac{\mathbf{Q}\sigma_z}{2} \right)^2 + v(\mathbf{r}_i) \right] + \mathcal{H}_{\text{el-el}}, \quad (1)$$

with $\mathcal{H}_{\text{el-el}} = \sum_{i < j} e^2 / |\mathbf{r}_i - \mathbf{r}_j|$. The vector potentials for spin σ are $\mathbf{A}_\sigma = \mathbf{Q}_\sigma (\hbar c / e)$ with $\mathbf{Q}_\uparrow = -\mathbf{Q}_\downarrow = \mathbf{Q} / 2$. They can be removed by gauge transformations, multiplying the many-particle wave function by $\exp(i\mathbf{Q}_\sigma \cdot \mathbf{r})$ for each electron with spin σ . In a paramagnet, the ground-state wave function would, therefore, evolve trivially with \mathbf{Q} and the ground-state energy would be independent of \mathbf{Q} . In a ferromagnet, however, a change in the phase relationship between up spins and down spins alters the magnetic order and will change the energy.

We start with a ground state that has a spontaneous spin density along the $\hat{\mathbf{x}}$ direction. Its magnitude is $m(\mathbf{r}) =$

$\langle \Psi_\uparrow^\dagger(\mathbf{r}) \Psi_\downarrow(\mathbf{r}) \rangle_0$, where $\Psi_\sigma(\mathbf{r})$ is an electron field operator for an electron with spin $\sigma = \uparrow, \downarrow$, and $\langle \cdots \rangle_0$ denotes a ground-state expectation value. For small \mathbf{Q} , $\Psi_\sigma(\mathbf{r}) \rightarrow \exp(i\mathbf{Q}_\sigma \cdot \mathbf{r}) \Psi_\sigma(\mathbf{r})$ and the order parameter rotates in the $\hat{\mathbf{x}}\text{-}\hat{\mathbf{y}}$ plane as a function of \mathbf{r} ,

$$\langle \mathbf{s}(\mathbf{r}) \rangle_{\mathbf{Q}} = m_Q [\cos(\mathbf{Q} \cdot \mathbf{r}) \hat{\mathbf{x}} + \sin(\mathbf{Q} \cdot \mathbf{r}) \hat{\mathbf{y}}], \quad (2)$$

i.e., it forms a spiral spin state. As Q increases, the order parameter's spatial dependence will [5] cause a decrease in the magnetic condensation energy and in the magnitude of the order parameter. *Dependence of the ground-state energy E on Q implies that these many-particle eigenstates have finite current densities with equal magnitude and opposite direction for up and down spins,*

$$\mathbf{j}_\uparrow = \frac{c}{V} \frac{\partial E(\mathbf{A}_\uparrow, \mathbf{A}_\downarrow)}{\partial \mathbf{A}_\uparrow} \Big|_{\mathbf{A}_\uparrow = -\mathbf{A}_\downarrow} = \frac{e}{\hbar} \frac{\partial \epsilon(\mathbf{Q})}{\partial \mathbf{Q}} = -\mathbf{j}_\downarrow, \quad (3)$$

where $\epsilon(\mathbf{Q})$ is the energy per unit volume.

As we discuss at greater length below, easy-plane anisotropy ascribes a topological character to the wave vector \mathbf{Q} , so that these currents can decay only by phase slip processes that have large barriers, i.e., *these are dissipationless supercurrents.*

Equation (3) is similar to the connection between the exchange coupling of ferromagnets separated by a tunnel junction and the spin currents that flow between them [6]. Our proposal for supercurrents in ferromagnets is related to Anderson's discussion of superconductivity [7] in terms of magnetic order in an effective spin model; the physics of the two ordered states appears similar if a particle-hole transformation is made in one of the spin subspaces. The supercurrents we propose are also related to those supported in double-layer quantum Hall systems [8–12], where ordered states form that are describable either as pseudospin ferromagnets or as electron-hole-pair condensates [13,14]. In fact, the role of magnetic anisotropy in controlling the observability of these supercurrents is connected in part with the role of band hybridization terms in controlling the observability of collective electron-hole-pair transport in excitonic insulators [15–18].

To illustrate these ideas, we consider the simplest possible microscopic model of a metallic ferromagnet, a fermion gas with delta-function repulsive particle-particle interactions $U\delta(\mathbf{r}_i - \mathbf{r}_j)$ treated in a mean-field approximation. The unrestricted Hartree-Fock Hamiltonian for the ordered state with wave vector $\mathbf{Q} = Q\hat{\mathbf{x}}$ is

$$\mathcal{H}^{\text{HF}} = \frac{Vh^2}{U} + \sum_{\mathbf{k}} (c_{\mathbf{k}+\mathbf{Q}/2,\uparrow}^\dagger c_{\mathbf{k}-\mathbf{Q}/2,\downarrow}^\dagger) \begin{pmatrix} \epsilon_{\mathbf{k}+\mathbf{Q}/2} & -h \\ -h & \epsilon_{\mathbf{k}-\mathbf{Q}/2} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}+\mathbf{Q}/2,\uparrow} \\ c_{\mathbf{k}-\mathbf{Q}/2,\downarrow} \end{pmatrix} = \frac{Vh^2}{U} + \sum_{\mathbf{k},\pm} E_{\mathbf{k},\pm} a_{\mathbf{k},\pm}^\dagger a_{\mathbf{k},\pm}, \quad (4)$$

where $\epsilon_{\mathbf{k}} = \hbar^2 k^2 / (2m)$, the ordered-state quasiparticle energies are $E_{\mathbf{k},\pm} = [\epsilon_{\mathbf{k}+\mathbf{Q}/2} + \epsilon_{\mathbf{k}-\mathbf{Q}/2} \pm \sqrt{(\epsilon_{\mathbf{k}+\mathbf{Q}/2} - \epsilon_{\mathbf{k}-\mathbf{Q}/2})^2 + 4h^2}] / 2$, and the effective magnetic field which splits the quasiparticle bands by $\pm h$ with $h = Um_Q$ is determined self-consistently by

$$\frac{h}{U} = \frac{1}{V} \sum_{\mathbf{k}}' \frac{h}{\sqrt{(\epsilon_{\mathbf{k}+\mathbf{Q}/2} - \epsilon_{\mathbf{k}-\mathbf{Q}/2})^2 + 4h^2}}. \quad (5)$$

The prime on the sum in Eq. (5) indicates restriction to those wave vectors for which only the lower-energy quasiparticle state is occupied.

The procedure described above provides a mean-field approximation to the ferromagnet spin-supercurrent states. In Fig. 1 we plot quasiparticle bands for a typical model of this type at a moderately large value of $Q = 0.5k_F$, where k_F is the Fermi wave vector for zero order parameter, and ϵ_F is the Fermi energy [19]. The product $UD(\epsilon_F)$ was chosen to be close to experimental values for Co and Fe (taken from Ref. [20]). In this case the Stoner criterion [21] for mean-field ferromagnetism, $UD(\epsilon_F) > 1$, is satisfied. In Fig. 2 we plot the order parameter m_Q , the magnetic condensation energy ϵ_{cond} , and the spin-supercurrent density j as a function of the ordering wave vector Q . Note that the current density is proportional to the derivative of the condensation energy in agreement with the more general discussion above.

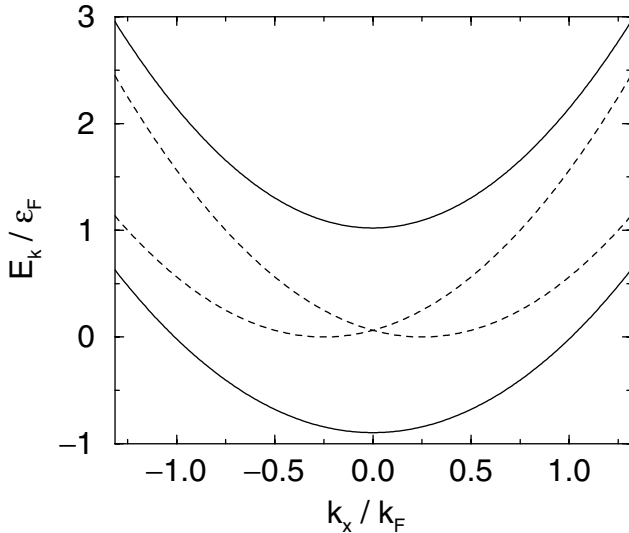


FIG. 1. Quasiparticle bands $E_{k,+}$ (upper solid curve) and $E_{k,-}$ (lower solid curve) for $Q = 0.5k_F$, $UD(\epsilon_F) = 1.5$, and $k_y = k_z = 0$. For comparison we also show the dispersion $\epsilon_{\mathbf{k}+\mathbf{Q}/2}$ and $\epsilon_{\mathbf{k}-\mathbf{Q}/2}$ for zero order parameter (dashed lines).

Our calculations demonstrate that spin supercurrents are possible in states with equilibrium quasiparticle populations; elastic scattering from occupied to unoccupied states cannot provide the current decay mechanism familiar from the standard theory of metallic transport. To establish the stability of the spin currents it is, however, still necessary to show that the spin-supercurrent state is stable against infinitesimal distortions of its order-parameter field. In what follows, we demonstrate that magnetic anisotropy is necessary for stability. Since real metallic ferromagnets are much more complex than the toy model system discussed above, we now turn to a phenomenological approach that will allow us to relate critical currents to known micromagnetic parameters.

We consider a generalized Landau-Ginzburg model for the dependence of an easy-plane ferromagnet's free-energy density [20,22] on its magnetic state:

$$f = -|\alpha|\mathbf{M} \cdot \mathbf{M} + \frac{\beta}{2} (\mathbf{M} \cdot \mathbf{M})^2 + \tilde{A}|\nabla\mathbf{M}|^2 + \tilde{K}M_z^2. \quad (6)$$

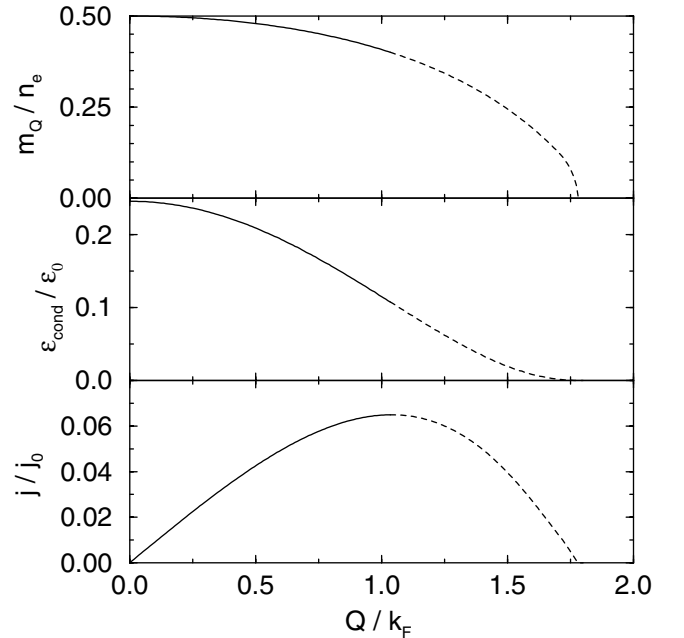


FIG. 2. The order parameter m_Q normalized to electron density n_e , the magnetic condensation-energy density ϵ_{cond} normalized to the energy density of the disordered state, $\epsilon_0 = (3/5)n_e\epsilon_F$, and the spin supercurrent density $j = j_1 = -j_l$ normalized to $j_0 = en_e\hbar k_F/m$ as a function of the ordering wave vector Q for $UD(\epsilon_F) = 1.5$. The dashed lines indicate an instability regime against phase separation into regions with larger and smaller Q .

The free energy of this model is minimized by a constant magnetization in the \hat{x} - \hat{y} easy plane with magnitude $M_0 = \sqrt{|\alpha|/\beta}$. The resulting dependence of energy density on magnetization orientation at fixed magnitude allows us to identify $A = \tilde{A}M_0^2$ with the exchange constant and $K = \tilde{K}M_0^2$ with the uniaxial anisotropy coefficient of the ferromagnet's micromagnetic energy functional [20]. The spin-supercurrent state has magnetization $\mathbf{M}_Q(x) = M_Q[\cos(Qx), \sin(Qx), 0]$, where $M_Q^2 = (|\alpha| - \tilde{A}Q^2)/\beta$ is decreasing with Q as in our microscopic calculations. Using Eq. (3), we find that the spin-supercurrent density is

$$j(Q) = \frac{2e\tilde{A}Q}{\hbar\beta} (|\alpha| - \tilde{A}Q^2), \quad (7)$$

reaching a maximum at Q_{ph} , where $\tilde{A}Q_{\text{ph}}^2 = |\alpha|/3$.

Expanding around the spin-supercurrent state free-energy extremum, we find that

$$\begin{aligned} \delta f = & 2\beta M_Q^2 M_a^2 + \tilde{A}|\nabla M_a|^2 + \tilde{A}|\nabla M_{\text{ph}}|^2 \\ & + 2Q\tilde{A}(M_a\partial_x M_{\text{ph}} - M_{\text{ph}}\partial_x M_a) \\ & + (\tilde{K} - \tilde{A}Q^2)M_z^2 + \tilde{A}|\nabla M_z|^2, \end{aligned} \quad (8)$$

where M_a and M_{ph} are the amplitude and phase fluctuations of the easy-plane magnetization (the projections along and perpendicular to \mathbf{M}_Q), while M_z is the hard-axis fluctuation. The translationally invariant kernel of this quadratic form has three wave vector (\mathbf{p}) dependent eigenvalues:

$$K_{\pm} = \beta M_Q^2 + \tilde{A}p^2 \pm \sqrt{\beta^2 M_Q^4 + 4\tilde{A}^2 Q^2 p_x^2}, \quad (9)$$

$$K_{zz} = \tilde{K} - \tilde{A}Q^2 + \tilde{A}p^2. \quad (10)$$

It follows from Eq. (9) that the spin-supercurrent state is stable against easy-plane fluctuations provided that Q is smaller than Q_{ph} ; at larger values of Q , energy can be lowered by phase separation into regions with larger and smaller Q . For the soft ferromagnets we have in mind, however, it is the out-of-plane fluctuations, described by Eq. (10), that become unstable first. For $Q > Q_z = \sqrt{K/A}$, the spin supercurrent can relax by tilting out of the easy plane to one of the poles and unwinding phase with no energy cost. In Table I we list Q_z values and the corresponding critical current densities $j_{\text{crit}} = j(Q_z)$ for some common soft thin film magnets, including only the shape (magnetostatic) contribution $K_{\text{shape}} = \mu_0 M_0^2/2$ to K . From our model calculation and the results shown in Fig. 2, we conclude that Q_{ph} is typically of the order of the Fermi wave vector k_F , i.e., much larger than Q_z . To estimate j_{crit} we can, therefore, linearize Eq. (7) in Q ,

$$j_{\text{crit}} = 2\frac{e}{\hbar}AQ_z = 2\frac{e}{\hbar}\sqrt{AK_{\text{shape}}}, \quad (11)$$

to obtain the large critical currents listed in Table I.

We have thus far neglected magnetocrystalline anisotropy, since it is much weaker than shape anisotropy in the situation we have in mind. It does, however, break

TABLE I. Saturation moment $\mu_0 M_0$, exchange constant A , cubic anisotropy constant $K_1^{(c)}$, critical wave vector Q_z , critical spin current density j_{crit} , and ratio of minimum to critical spin current density for common soft thin film magnets. The values for $\mu_0 M_0$, A , and $K_1^{(c)}$ are taken from Ref. [20].

	Fe	Co	Ni
$\mu_0 M_0$ [T]	2.15	1.81	0.62
A [pJ m ⁻¹]	8.3	10.3	3.4
$K_1^{(c)}$ [MJ m ⁻³]	0.048		-0.005
Q_z [nm ⁻¹]	0.47	0.36	0.21
j_{crit} [A cm ⁻²]	1.19×10^9	1.11×10^9	2.2×10^8
$j_{\text{min}}/j_{\text{crit}}$	0.013		0.015

rotational symmetry within the easy plane and has the tendency to fix the phase and, thus, to suppress the supercurrents. When an in-plane anisotropy term is included in the energy-density functional, extrema at small phase winding rates consist of weakly coupled solitons in which the magnetization goes from one in-plane minima to another. ($Q = \theta N_s/L$, where θ is the angle between in-plane minima and N_s/L is the soliton density.) The energy density at small Q is proportional to the number of solitons. As a consequence, the minimum spin-current density j_{min} , that can be supported by a spin-supercurrent state, is nonzero. To estimate j_{min} for cubic materials, we include the leading-order bulk cubic anisotropy [20] in the energy density, $K_1^{(c)} \sin^2\varphi \cos^2\varphi$, where φ is the angle of the order parameter within the easy plane. For small Q the functional is minimized by a kink soliton solution. By evaluating the energy of in-plane solitons of this model, we find from Eq. (3) that

$$\frac{j_{\text{min}}}{j_{\text{crit}}} = \frac{1}{4\pi} \sqrt{\frac{K_1^{(c)}}{K_{\text{shape}}}}, \quad (12)$$

which is of the order of 1.5% (see Table I). $\langle 100 \rangle$ hcp cobalt thin films with in-plane easy axis will typically have still smaller values of j_{min} because of the higher hexagonal symmetry. From these considerations, we conclude that spin supercurrents will be observable at moderate current densities only in materials that have weak magnetic anisotropy within the easy plane. Because of their extremely weak magnetocrystalline anisotropies, homogeneous permalloy samples might be ideal candidates for the experiments proposed below. Although the coarse grained in-plane magnetic anisotropy can, in principle, be fine-tuned to zero, spin-rotational invariance in the easy plane will always be broken by disorder terms in the microscopic Hamiltonian. Since dissipationless spin supercurrents will not occur if these disorder terms are too strong, the effects we propose are more likely to be observable in homogeneous alloys.

One possible experimental arrangement in which this collective transport phenomena could be detected is illustrated schematically in Fig. 3. An easy-plane thin film

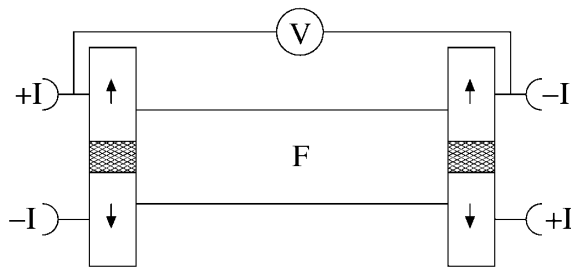


FIG. 3. Schematic illustration of one possible experimental setup to prepare a spin-supercurrent state.

ferromagnet (F) is connected to four spin-selective leads (full spin polarization in the leads is optimal but not required) that feed opposing up and down spin currents, where “up” and “down” refer to the direction *perpendicular* to the thin film. We emphasize that, even with recent advances in transition metal ferromagnet spintronics, realizing a system with this geometry represents an experimental challenge. In this setup, a quasiparticle current would flow dissipatively between upper and lower leads on both the left- and right-hand side of the thin film ferromagnet. A sizable voltage drop (measured, e.g., between the upper leads), proportional to the injected currents, would result. Its exact value depends on the resistivity of the ferromagnet and on details of the geometry and is not a concern here. If the collective transport effect predicted here occurred, however, currents with opposite spin would flow without a voltage drop across the sample, from left to right and vice versa. Dissipationless current flow in the bulk could still be masked by resistance in the film-lead contacts or by collective spiral wave phase-slip processes. Our uncertainty in the magnitude of the contact resistances compared to the quasiparticle resistance makes our proposal somewhat speculative. A collective element to the spin transport could be unambiguously identified by driving the critical current density j through either the maximum or the minimum current, j_{crit} or j_{min} , or by reversing the spin orientations of the leads on one side of the sample. The later change would have no effect on the measured voltage if the current were carried entirely by quasiparticles but would increase the voltage if part of the current was carried collectively.

In conclusion, we have examined circumstances under which dissipationless spin supercurrents, associated with spiral magnetic order, can occur in thin film ferromagnets.

We have estimated critical values of these supercurrents and proposed an experiment to generate and detect this new collective transport behavior.

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- [1] M. N. Baibich *et al.*, Phys. Rev. Lett. **61**, 2472 (1988).
- [2] G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn, Phys. Rev. B **39**, 4828 (1989).
- [3] M. Julliere, Phys. Lett. **54A**, 225 (1975).
- [4] J. S. Moodera, L. R. Kinder, T. M. Wong, and R. Meservey, Phys. Rev. Lett. **74**, 3273 (1995).
- [5] If the ground state is a unidirectional spin-density wave, the minimum of $\epsilon(Q)$ will occur at a finite value of Q , rather than at $Q = 0$.
- [6] J. C. Slonczewski, Phys. Rev. B **39**, 6995 (1989).
- [7] P. W. Anderson, Phys. Rev. **112**, 1900 (1958).
- [8] X.-G. Wen and A. Zee, Phys. Rev. Lett. **69**, 1811 (1992).
- [9] K. Yang *et al.*, Phys. Rev. Lett. **72**, 732 (1994).
- [10] K. Moon *et al.*, Phys. Rev. B **51**, 5138 (1995).
- [11] J. Kyriakidis, D. Loss, and A. H. MacDonald, Phys. Rev. Lett. **83**, 1411 (1999); **85**, 2222 (2000).
- [12] J. Kyriakidis and L. Radzihovsky, cond-mat/0010329.
- [13] Yu. E. Lozovik and V. I. Yudson, Zh. Eksp. Teor. Fiz. **71**, 738 (1976) [Sov. Phys. JETP **44**, 389 (1976)].
- [14] S. I. Shevchenko, Sov. J. Low Temp. Phys. **2**, 251 (1976).
- [15] L. V. Keldysh and Yu. V. Kopae, Sov. Phys. Solid State **6**, 2219 (1965).
- [16] D. Jérôme, T. M. Rice, and W. Kohn, Phys. Rev. **158**, 462 (1967).
- [17] B. I. Halperin and T. M. Rice, Rev. Mod. Phys. **40**, 755 (1968).
- [18] R. R. Guseinov and L. V. Keldysh, Zh. Eksp. Teor. Fiz. **63**, 2255 (1972) [Sov. Phys. JETP **36**, 1193 (1973)].
- [19] We find that there is no excitation gap in the quasiparticle spectrum, similar to, e.g., *d*-wave superconductors, which show gapless superconductivity.
- [20] R. Skomski and J. M. D. Coey, *Permanent Magnetism* (IOP, Bristol, 1999).
- [21] E. C. Stoner, Proc. R. Soc. London A **154**, 656 (1936).
- [22] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics* (Butterworth-Heinemann, Oxford, 1995), Vol. 9.