Scaling Analysis of Magnetic-Field-Tuned Phase Transitions in One-Dimensional Josephson Junction Arrays

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We have studied the magnetic-field-induced superconductor-insulator quantum phase transition in one-dimensional arrays of small Josephson junctions. We found that the critical magnetic field that separates the two phases corresponds to the onset of Coulomb blockade of Cooper pairs tunneling in the current-voltage characteristics. The resistance data are analyzed in the context of the superfluid-insulator transition in one dimension, and a finite-temperature scaling analysis is performed to extract the critical exponents. The dynamical exponents z are determined to be close to 1, and the correlation length exponents ν are found to be approximately 0.3 and 0.45 in the two groups of measured samples.

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A 1D array of small Josephson junctions (JJ) provides an ideal testing ground for the quantum phase transition because of controllable sample parameters and continuously tunable Josephson coupling strength. Although there have been many theoretical advances [1] for this system, only a few experiments have been reported [2]. While scaling analysis on 2D JJ arrays indicated a power law dependent correlation length and critical exponents νz of the order of unity [3], no scaling analysis on 1D arrays has been reported to date. Theoretically, the kinetic energy term in the Hamiltonian is generalized to a spacelike coupling term on the imaginary time. With this additional dimension, a d-dimensional JJ array can be mapped to a classical $(d + 1)$-dimensional XY model. However, the types of governing phase transitions may not be the same in different dimensions [1,4]. In the $(1 + 1)$D XY model, for instance, the exponentially dependent correlation length of the Kosterlitz-Thouless-Berezinskii (KTB) transition [5] should lead to scaling properties different from those in a $(2 + 1)$D model. Furthermore, the effect of dissipation is also predicted to change the isotropic XY model to an anisotropic one [6]. These properties can be explored by utilizing the scaling analysis. In this study we investigate the scaling behavior in 1D JJ arrays by using a magnetic field to continuously tune the Josephson coupling strength across the critical point. From the analysis, we find a power law dependent correlation length and determine the correlation length exponent ν and the dynamical critical exponent z.

As shown in Fig. 1(a), the measured arrays are composed of I-shaped aluminum islands, whose sizes as defined by electron beam lithography, are on the order of 1 μm. Each Al island has two tunnel junctions connected in parallel to its nearest neighbors, and forms a SQUID which is referred to as a unit cell. Being fabricated on the same chip, each group of measured arrays (denoted as groups A and B) having different cell numbers, N, should have similar controlled parameters. Thus, length dependence of the phase transition can be investigated. The normal state resistances R_T of each cell for the arrays, as listed in Table I, are determined at high bias, $V > N(2\Delta_0/e)$. The resistances for arrays in a group are quite similar, confirming the uniformity of the fabricated arrays. When magnetic field B is applied perpendicularly to SQUID loops with area A, the Josephson coupling energy E_J can be tuned periodically as $E_J = E_J^0 \cos(\pi AB/\Phi_0)$, where Φ_0 is the flux quantum. The zero field Josephson coupling E_J^0 is determined using the $T = 0$ Ambegaokar-Baratoff formula $E_J^0 = (\Delta_0/2)(R_Q/R_T)$ with a superconducting gap Δ_0 of 200 μeV, and a resistance quantum R_Q of $h/4e^2 \approx 6.5$ kΩ. Accordingly, the E_J^0 values are approximately 130 μeV for arrays in group A, and 650 μeV for arrays in group B. From the scanning electron microscopy

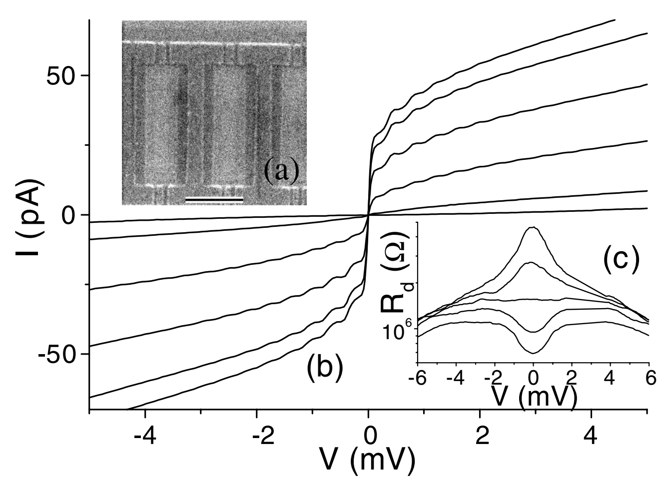

FIG. 1. (a) The SEM image of an 1D JJ array. The overlapping areas between "I"-shaped islands are the tunnel junctions. The scale bar at the bottom of the image is 1 μm. (b) Evolution of IV characteristics for arrays A2, from supercurrentlike structure to Coulomb blockadelike structure, at selected filling numbers between 0.0 and 0.5 with step $\Delta f = 0.1$. (c) Dynamic resistance R_d as a function of bias voltage shows a crossover from a superconducting (bottom, $f = 0.42$) to an insulating behavior (top, $f = 0.50$) with $\Delta f = 0.02$.

TABLE I. Some important parameters of measured arrays. The arrays that are fabricated on the same chip, and have thus similar junction parameters, are categorized into one group. Note the closeness between the critical filling numbers f^* and the correlation length exponents ν for arrays in a group.

Sample	A1	A2	A3	B1	B2	B3	B4
N	49	29	14	100	75	50	30
R_T(kΩ)	0.9	1.1	0.9	5.0	4.7	4.3	4.9
f^*	0.41	0.41	0.45	0.20	0.25	0.26	0.27
K^*	1.0	0.9	0.7	0.72	0.70	0.71	0.66
νz	0.3	0.3	0.3	0.45	0.5	0.45	0.45

(SEM) image one can estimate the junction area, and infer a junction capacitance C of 3.0 ± 0.8 fF by using a specific capacitance of 45 fF$/\mu$m^2 [7]. With this capacitance, the charging energy for Cooper pairs, $E_{\mathrm{CP}} \equiv (2e)^2/2C$, is 106 ± 35 μeV, giving E_J^0/E_{CP} values of about 6 and 1.3 for arrays in groups A and B, respectively.

Transport measurements were conducted in a dilution refrigerator at temperatures ranging between 40 mK and 1.5 K. Zero bias resistance R_0 was taken from the slope of $I(V)$ characteristics at a very small bias, and it was further confirmed by using a lock-in technique at a frequency of 1.7 Hz with an excitation of 20 nV. Sweeping a wide range of magnetic fields at 40 mK, we determined the periodicity of the magnetoresistance oscillation to be 9.15 G. With this period ΔB, we denote the field as a dimensionless filling number $f = B/\Delta B$, which represents an average number of flux quantum in one cell. At integer values of f, the arrays are most conductive with R_0 at a minimum, while at half integer f values, the arrays become most resistive with R_0 at a maximum.

Figure 1(b) shows $I(V)$ characteristics for array A2 measured at $f = 0.0$–0.5. For array A2 at $f = 0$ (i.e., the most superconducting curve), deviations of the supercurrent of consisting junctions are quite small, reaffirming the uniformity of these arrays. However, even at $f = 0$, the array is not truly superconducting, but has a finite zero bias resistance of 2 kΩ. The supercurrent decreases with f and diminishes at $f \simeq 0.45$, above which the supercurrent-type structure turns into a Coulomb-blockade-type structure, with zero bias resistance reaching a maximum value of 5.5 MΩ at $f = 0.5$. The evolution from one structure to the other is best represented by Fig. 1(c), which shows a smooth crossover from dip to hump structure with differential resistance $R_d \equiv dV/dI$ as a function of bias voltage, and with a flat $R_d(V)$ curve separating the two limits.

Because of a smaller E_J^0/E_{CP} value, the $I(V)$ characteristics for array B1, as depicted in Fig. 2(a), have a higher R_0 of about 200 kΩ at $f = 0$ and 50 GΩ at $f = 0.5$. The evolution of $I(V)$ characteristics, from the supercurrent-type structure to Coulomb-blockade-type structure, for this array is quite different from that of array A2: As f increases beyond $f = 0.20$, a small

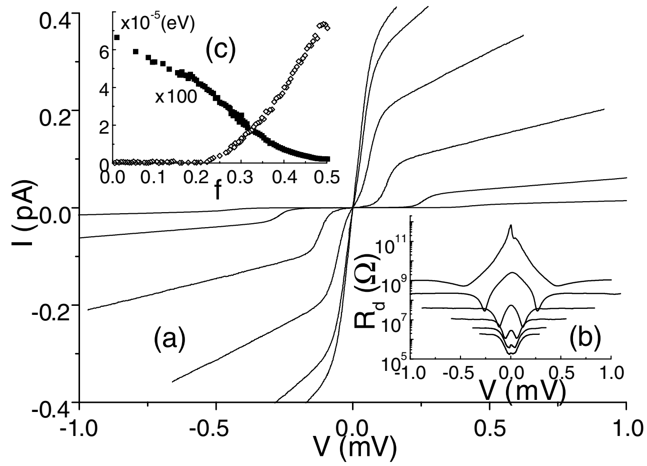

FIG. 2. (a) $I(V)$ characteristics and (b) differential resistance $R_d(V)$ for array B1 at $f = 0$–0.5 at 40 mK ($\Delta f = 0.1$). Notice the coexistence of hump and dip structures; this is not seen in arrays in group A [cf. Fig. 1(c)]. The curves in (b) are shifted by 0.25 decade each curve for clarity. The scale labels are for curve with $f = 0$. (c) The f dependence of supercurrent (solid square) and threshold voltage (open diamond) for array B1. Both are converted to the energy scale, i.e., $\hbar I_C/2e$ and eV. The supercurrent is magnified 100 times.

Coulomb gap in the $I(V)$ characteristic appears, signifying competition between Josephson coupling and Coulomb blockade of Cooper pair tunneling. This behavior can be clearly seen in Fig. 2(b), which shows a coexistence of dip and hump structures in the $R_d(V)$ curves for intermediate filling numbers. From these $R_d(V)$ curves, supercurrent and Coulomb blockade thresholds are plotted as a function of f as shown in Fig. 2(c). Note that the two curves cross each other, a feature different from that reported in Ref. [2], with a considerably larger E_J^0/E_{CP} value (about 6.1) than those of our arrays in group B.

The temperature dependences of zero bias resistance at various filling numbers for arrays A2 and B1 are depicted in Fig. 3. For the most insulating case, the conductance between 1 K and 150 mK fits the Arrhenius form with an energy barrier of 120 μeV, which is very close to E_{CP}. Similar behaviors are also reported in experiments on 2D JJ arrays [8]. This suggests that the dynamics is dominated by simple thermal activation of Cooper pairs, because the strength of the Josephson coupling is suppressed to the minimum. At even lower temperatures, shorter arrays show a saturation of resistance probably due to the finite size effect. For the most superconducting case (i.e., $f = 0$), the resistance decreases with decreasing temperature and levels off at low temperatures. We emphasize that this leveling-off behavior is not due to any unwanted high-frequency noises in the measurement system for the following reasons: (1) the leveling-off temperatures for S (superconductor) and I (insulator) sides are not the same, and (2) the leveling-off temperatures for arrays in the two groups are not the same. This leveling-off behavior cannot be accounted for by heating effects because the

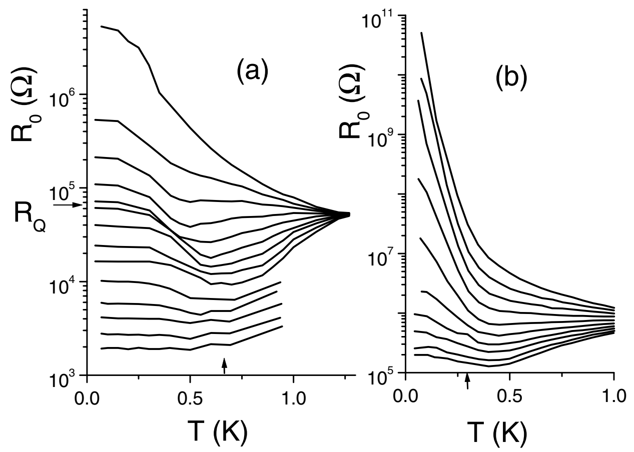

FIG. 3. The $R_0(T)$ as a function of the filling number f for arrays A2 (a) and B1 (b). The f values are, from the bottom, 0.0, 0.20, 0.24, 0.26, 0.28, 0.32, 0.34, 0.37, 0.39, 0.41, 0.43, 0.46, 0.48, and 0.50 in (a) and 0.0, 0.125 to 0.500 with $\Delta f = 0.042$ in (b). At $T > T_{\rm cr}$, the array displays an f-tuned SI transition, whereas at low temperatures the $R_0(T)$ curves in the S side level off or rise. The vertical arrows indicate the calculated $T_{\rm cr}$.

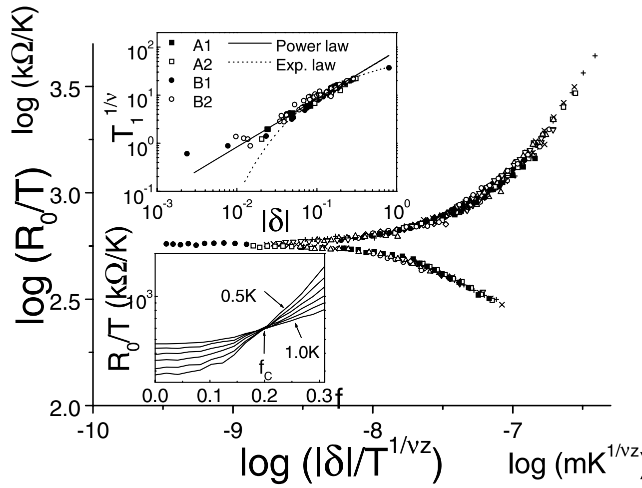

FIG. 4. The $R_0(\delta, T)$ scaling plot from 50 $R_0(T)$ curves for array B1 for $0.5 < T < 1$ K and $0.0 < f < 0.46$ ($|\delta| < 0.6$). The scaled $R_0(T)$ curves for various f are plotted in various symbols. The upper inset shows the scaled $T_1^{1/\nu}$ dependence of δ as described in the text. Note that four arrays A1, A2, B1, and B2 are scaled into a single curve which is better fitted with a power law (solid line) than with an exponential law (dotted curve). The lower inset shows a plot of R_0/T dependence of f at temperatures ranging from 0.5 to 1.0 K for array B1 with $\Delta T = 0.1$ K.

measured resistance is almost the same for both ac and dc measurements.

Such leveling-off behaviors, similar to those seen in 2D JJ array experiments [8], can be explained in the context of vortex macroscopic quantum tunneling (MQT) [9]. Above crossover temperature $T_{\rm cr}$, the thermally activated vortex motion dominates, while below $T_{\rm cr}$ the vortex MQT prevails [10]. This $T_{\rm cr}$ is approximately $\hbar\omega_p/2\pi k_B$, where $\omega_p = \sqrt{2E_{\rm CP}E_J}$ is the vortex plasma frequency. Using the E_J and $E_{\rm CP}$ values for the measured arrays, we find a good consistency between the calculated and measured $T_{\rm cr}$ values. The measured $T_{\rm cr}$ for arrays in group A is about 0.75 K, and for arrays in group B, because of smaller E_J^0 values, it is about 0.5 K. The leveling-off resistance is found to be lower for arrays with stronger vortex dissipation. The strength of dissipation α is inversely proportional to R_T and is related to R_Q as $\alpha = R_Q/R_T$. A close inspection reveals a separation point $R_0(T \to 0) \simeq R_Q$, above which $R_0(T)$ curves move upward with decreasing temperature (leveling off for the case of shorter arrays) and below which $R_0(T)$ curves simply level off. Similar results were also reported by Haviland and co-workers [2].

According to the theory of superfluid-insulator transition in 1D systems [11], at the critical point $f = f^*$, the resistance is linearly proportional to the temperature. Experimentally, f^* is identified as the filling number where the extrapolation of the $R_0(T)$ curve passes through $R_0 = 0$, $T = 0$ point. This is best seen in the inset of Fig. 4 which shows a good crossing for all $R_0(T)/T$ curves at the critical filling number. As a supplementary clue, we notice that at the base temperature this critical filling number corresponds to an onset of Coulomb blockade threshold voltage [see Fig. 2(c)]. We thus interpret these phenomena as evidence of a magnetic field-tuned SI phase transition with a critical filling number f^* in our measured arrays.

In a noninteracting model, the Hamiltonian of a 1D array of small JJ can be mapped to a classical 2D XY model. Theoretically, this mapping relates $E_{\rm CP}$ and E_J to a dimensionless coupling constant K in the XY model as $K = \sqrt{E_J/2E_{\rm CP}}$ [1]. In the 2D XY model, below the KTB transition temperature $T_{\rm KTB}$ the spins form vortex-antivortex pairs, while above $T_{\rm KTB}$ the pairs dissociate and the whole system becomes a vortex plasma [5]. Note that the topological spin vortex in the 2D XY model represents the phase slip in 1D JJ arrays. In the system of 1D JJ array, the corresponding KTB transition takes place at a critical coupling energy $E_J^* = E_J^0 \cos(\pi f^*)$, which is achieved by the tuning of external magnetic field. In the region $E_J < E_J^*$, the phase fluctuations render insulating 1D JJ arrays. According to the model, the transition takes place at $K^* = \sqrt{E_J^*/2E_{\rm CP}} = 2/\pi \simeq 0.64$ [4]; however, the experimental values are slightly larger than the theoretical one. Despite the scattering of K^* values in group A, the values in group B are well consistent with one another.

Although the leveling-off behavior marks the breakdown of global superconductivity at low temperatures, the theory of finite-temperature scaling allows an appreciation of quantum phase transition using finite-temperature data. The finite-temperature scaling law of quantum phase transitions asserts that $\mathcal{O}(k, \omega, K, L_T) = L^{d_O/z}\mathcal{O}(kL_T^{1/z}, \omega L_T, L_T/\xi_T)$, where $L_T = \hbar\beta$ is a finite length on the imaginary time axis.

Some of the terms can be neglected: the wave vector k is assumed to be zero, the scaling dimension $d_{\mathcal{O}} = 2 - d = 1$ [11], and $\omega = 0$ in dc measurements. We thus obtain a concise finite-size scaling form for zero bias resistance, $R_0(f,T) = T^{1/z}\tilde{R}_0(1/T\xi^z)$, where ξ is a function of f. To examine the correlation length dependence, we rewrite the scaling form as $R_0(\delta,T) = T^{1/z}\tilde{R}_0(T_1/T)$ where $\delta = (K - K^*)/K^*$ is the distance from the transition point and T_1 is the field dependent scaling parameter (see [12] for details). In this way, as shown in the inset of Fig. 4, one obtains a clear power law dependence of T_1 on δ over 1 order of magnitude, suggesting a power law dependence of the correlation length. Based on this analysis, the correlation length ξ can be written as $\xi = |\delta|^{-\nu}$ where ν is a critical exponent. Finally, the scaling law can be written with a scaling function $\tilde{R}_0$ as $R_0(f,T) = T^{1/z} \times \tilde{R}_0(\frac{\delta}{T})1/\nu z$.

The exponent νz is determined from the slope of the power law fitting. For the seven arrays measured, we find that νz varies from 0.30 to 0.50 for different arrays. Nevertheless, the same values of νz are obtained from both S and I sides. Assuming a z value of unity and using νz and K^* values obtained from above, one can derive the scaling curves such as that shown in Fig. 4 for array B1. In addition, the scaling function form is found to be $\tilde{R}_0(x) \propto e^{\alpha x}$, with $\alpha \simeq \pm 2.3$ and ± 2.5 (in unit of $K^{1/\nu z}$, "+" for the I phase and "−" for the S phase), for arrays in groups A and B, respectively. This form for resistance is similar to results deduced from the variable-range hopping (VRH) of Bose glass [13]. However, the VRH mechanism should not be accounted for in our system since the exponent on T is larger than 1. The fact that the scaling function in one phase is symmetrical to that in the other phases suggests that S and I phases play a dual role at zero bias. To refine the critical filling number and the scaling exponents, one begins with a trial scaling form, namely, $R_0(f,T) = AT^{1/z}\exp[\kappa(f)/T^{1/\nu z}]$. By noting that A is an f-independent constant and that κ is zero at f^*, the νz and f^* values can be unambiguously determined. The only adjustable parameter, z, can then be determined from the scaling curves. This parameter can be determined to a reasonable accuracy; a smaller z would give better scaling on the S side, whereas a larger z would result in better scaling on the I side. This fine-tuning procedure slightly modifies νz and f^* values, and gives a z value of 0.85 ± 0.05 for array B1. Furthermore, by comparing the trial scaling form and the finite-size scaling form $T^{1/z}\tilde{R}_0(1/T\xi^z)$, and using the known $\kappa(f)$, one can obtain a relationship between ξ and δ, which is converted from f. In this way, we find a power law dependence of ξ on δ, reaffirming the previous analysis on $T_1(\delta)$.

In summary, we have observed a magnetic field-tuned SI phase transition in the system of 1D small Josephson junction arrays, and have, for the first time, made scaling analysis on such a system. The observed phase transitions are associated with the $I(V)$ characteristics in a manner that the critical magnetic field which separates the superconducting phase from the insulating phase corresponds to the onset of Coulomb blockade threshold for Cooper pairs. The exponents νz are found to be 0.3 to 0.5, with z close to 1, implying an isotropy in spatial and time dimensions. The value of correlation length exponent ν contradicts what is expected under the current theory of the 1D boson system, $\nu \geq 2/d$ [13]. This discrepancy and the breakdown of scaling at low temperatures depart from the scope of current SI transition theory. These experimental results suggest that certain important physics has not been unearthed in this system.

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