Does Turbulent Convection Feel the Shape of the Container?

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Temperature and vertical velocity fluctuations are measured in turbulent Rayleigh-Bénard convection for Rayleigh numbers $2 \times 10^8 < R < 4 \times 10^9$ in cells with cylindrical and square geometries with approximately unit-aspect ratios. The geometries are constructed such that they have the same height and cross sectional area. We find, very unexpectedly, that both the magnitudes and scalings of fluctuations as a function of R depend strongly on the geometry. This latter result implies a possible nonuniversality in internal fluctuations and poses significant difficulties for existing model descriptions of turbulent convection.

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Turbulent Rayleigh-Bénard convection has been one of the most studied problems in fluid turbulence over the past decade [1,2]. The source of this abundant interest is that turbulent convection is the fundamental process that drives flows in the atmosphere and oceans, in the earth's mantle, and in gaseous planets and stars. To be relevant to these natural phenomena that occur on large horizontal length scales, it is essential to understand laterally extended systems. Laboratory experiments, however, have been conducted in systems of comparable lateral and vertical extent, the advantage being that larger forcing can be reached and better experimental control can be achieved than in equivalent laterally extended systems. One important question regarding this system is whether results and theoretical descriptions for small boxes apply to laterally large systems. In other words, which aspects of convection are universal and which depend on the details of lateral confinement? Theoretical models [1–5] have sought to characterize turbulent convection by its universal aspects and have broken the problem into interacting boundary layer and bulk regions with or without the influence of large scale shear. Is this a valid procedure that captures the essential elements of the problem or is it too simple to describe the real experiments?

Turbulent convection is often characterized by the globally averaged heat transport N as a function of the buoyancy forcing parameter, the Rayleigh number R. Early theories and experiments seemed to be consistent with $N \sim R^\beta$ with $\beta = 1/3$, but experiments in cryogenic helium gas and subsequent scaling theories suggested that $\beta = 2/7$ [1,2]. Recently, new experiments [6–9] that span a larger range of R and σ (the fluid Prandtl number) showed that the situation is more complicated. Significant agreement with these results was found using a model [5] that involves multiple scaling regions and crossover effects in the parameter space of R and σ. Heat transport $N(R)$ is, however, a somewhat insensitive measure of the properties of turbulent convection. For example, the total variation of β in power-law fits to available heat transport data for fluids with $\sigma > 0.5$ is of order 10%. Further, the addition of rough boundaries [10], rotation [11], and internal obstructions [12,13], all of which cause large changes in boundary-layer and turbulent-interior structures, barely modify the heat transport scaling exponent [14].

A perspective that nicely complements heat transport is obtained by measuring local quantities such as temperature and velocity fluctuations in the deep interior of the cell. Scaling theories suggest that, far from the walls, the scaling of fluctuations should be roughly independent of the details of the container apart from a nonuniversal scale factor that could depend, for example, on the ratio of lateral-to-vertical length scales. Experimental measurements of internal fluctuations, on the other hand, have shown significant variability with quantitatively different results reported for different convection cells and fluids [8,13,15–20].

We measured temperature and velocity fluctuations at moderate R in a large, square convection cell filled with water and with inserts of square and circular geometries that have the same cross sectional area, 345 cm^2; see Fig. 1. The ratios of the length of a horizontal side for the square and of the diameter for the cylinder to the cell height $d = 26.4$ cm define, respectively, the aspect ratios $\Gamma_s = 0.70$ and $\Gamma_c = 0.79$, where subscripts s and c denote square and cylinder. Amazingly, the fluctuations deep in the interior feel the geometry of the container, not just in overall magnitude but in very different power-law scaling

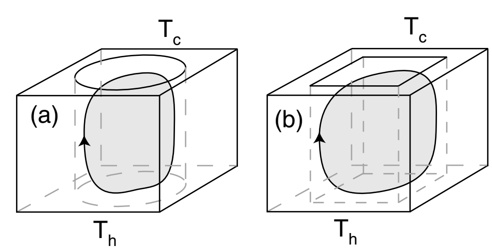

FIG. 1. Schematic illustration of the convection cell geometry with (a) cylindrical insert and (b) square insert. The shaded regions with arrows are illustrative of the large-scale circulatory flow.

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with R. These results demonstrate that convection in unit-aspect-ratio containers is more complicated than previously assumed and that the fluctuations result from some intricate coupling between the boundary-layer plumes, the large-scale circulation, and the inhomogeneous turbulence in the cell interior. Existing scaling theories cannot account for such behavior.

The experimental convection cell was constructed of anodized aluminum plates with 1.27-cm-thick polycarbonate side walls. The top plate (1.9-cm thick) was temperature regulated to within ± 0.003 °C and constant heater power was applied to the bottom plate (3.8-cm thick). The mean temperature of each plate was measured as the average of four thermistors embedded at half height and distributed in a laterally even pattern. The thermistors were calibrated with respect to an NBS traceable platinum thermometer, and the temperature difference across the fluid was corrected for the thermal conductivity of the aluminum and the anodized aluminum coating. Heat supplied to the bottom heater maintained a temperature difference ΔT across the layer, resulting in a Rayleigh number, $R = g\alpha d^3 \Delta T/\nu\kappa$, where g is the acceleration of gravity, d is the cell height, α is the thermal expansion coefficient, ν is the kinematic viscosity, and κ is the thermal diffusivity. The maximum temperature difference was 8.61 °C. Our experiments did not deviate significantly from Boussinesq conditions as measured by temperature drops across the top and bottom boundary layers: the ratio $x = \Delta T_{\mathrm{top}}/\Delta T_{\mathrm{bot}}$ was $0.92 < x < 1.04$. The mean cell temperature was maintained at 29.78 °C corresponding to a Prandtl number $\sigma = \nu/\kappa = 5.46$. The convection cell was insulated on the vertical and horizontal surfaces with 2.54-cm-thick solid foam insulation and 5 cm of fiberglass insulation. The inserts were 3.2-mm-thick acrylic and were attached to the bottom plate with a thin layer of silicon rubber adhesive. An O ring sealed the insert to the top plate, and a 1.6 mm hole in each insert equalized the pressure between the fluid in the insert and the surrounding fluid in the larger square container. Because the fluid in the outer region was nearly isothermal owing to the turbulent convection, it acted as another insulating layer and minimized lateral heat flow at the insert side walls. The main effect of the inserts was to confine the large scale circulation.

The rms temperature fluctuations σ_T were measured using a glass encapsulated semiconductor thermistor attached at the end of a 2.5-mm-diam stainless steel tube. The probe was positioned at the cell center to within 1 mm. The data were sampled at 16.67 Hz. Each time sequence consisted of 768 K points. Despite the insulation of the convection cell and the regulated room temperature (± 1 °C), the probe signal was weakly correlated with room temperature variations. These trends were corrected by subtracting a linear trend over 8 K point intervals. This procedure changed slightly the overall magnitude of σ_T, but did not affect the trend with respect to R. We normalize σ_T by ΔT. The rms vertical velocity fluctuations σ_V were determined using a commercial laser Doppler velocimetry system. (This technique works well [15,21,22] since fluid in the cell interior is nearly isothermal owing to strong turbulent mixing.) Uniform 1.53 μm latex spheres were used to seed the flow. As with the temperature measurements, the vertical velocities were measured at the cell center and sampled at 16.67 Hz. Each data sequence consisted of 128 K points. The mean velocity for the data sets was always less than 0.5 mm/sec which was significantly less than σ_V. We normalize σ_V by ν/d.

The normalized fluctuations $\sigma_T/\Delta T$ and $\sigma_V d/\nu$ are shown in Fig. 2 as a function of R. The large difference in the variation of $\sigma_T/\Delta T$ with R in the cylindrical and square inserts is apparent; a power-law fit to the data yields exponents of -0.10 ± 0.02 for the cylindrical insert and -0.48 ± 0.03 for the square insert. Theoretical predictions for this scaling exponent are $-1/7$ and $-1/9$ (see Ref. [3], and references therein). Measurements of power-law exponents in cylindrical convection cells using helium gas and water are consistent with $-1/7$ [8,13,16,17]. We compare our measurements to those in water [13,17] in Fig. 2a. Given the somewhat different aspect ratios and Prandtl numbers of these experiments, our measurements are consistent, up to a multiplicative constant, with these experiments. Similarly, our measurements in the square insert are consistent with other square-geometry experiments [18,19] that are also shown

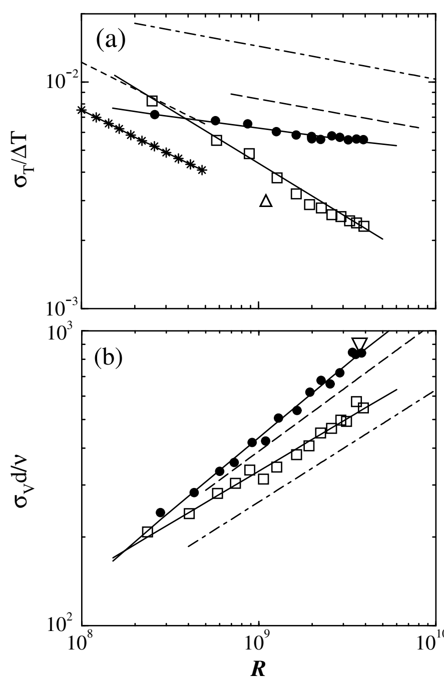

FIG. 2. Normalized (a) temperature $\sigma_T/\Delta T$ and (b) vertical velocity fluctuations $\sigma_V d/\nu$ measured at the cell center for square (□) and cylindrical (•) convection cell geometries vs R. The solid lines are power-law fits to our measurements. All other lines are power-law fits to other data. (a) Cylindrical cells: dash-dotted line (Ref. [13]) and dashed line (Ref. [17]); square cells: dotted line, $* - *$ (Ref. [18]), and △ (Ref. [19]). (b) Cylindrical cells: dashed line (Ref. [20]) and ▽ (Ref. [22]); square cell: dash-dotted line (Ref. [24]).

in Fig. 2a. The $* - *$ line is a power-law fit to $\sigma_T/\Delta T$ measured at a fixed distance from the sidewall of a square cell [18]. Since the σ_T are larger at the cell boundary, we have reduced them by a factor of 3 and compared them with our data. Note that $\sigma_T/\Delta T$ measured at the cell center or at the wall has the same power-law scaling with R. To our knowledge, there are no other systematic measurements of the Rayleigh-number dependence (at similar R and σ) of σ_T at the cell center in cells with square cross section and approximately unit-aspect ratio. However, the variation of $\sigma_T/\Delta T$ with R in the square insert is dramatically different from that in the cylinder and begs explanation.

The variation in $\sigma_V d/\nu$ with R between the square and cylindrical inserts is less pronounced than that for $\sigma_T/\Delta T$. The slopes are, however, distinctly different with power-law exponents of 0.50 ± 0.03 for the cylindrical insert and 0.36 ± 0.05 for the square insert. Two different theoretical derivations for this scaling exponent yield 3/7 and 4/9 (see Ref. [3], and references therein). Other experiments in cylindrical water cells are consistent with power-law scaling with an exponent of 4/9 [15,20]. The data of Refs. [20] and [22] are compared to our data in Fig. 2b. Once again, considering the different aspect ratios and Prandtl numbers, the measurements are consistent. We are unaware of experiments in square cells of approximately unit-aspect ratio with water in which σ_V were measured at the cell center. However, experiments in square cells, with SF_6 at $27 < \sigma < 93$, give an exponent of 0.42 ± 0.02 [23]. On the other hand, measurements in water, but at the sidewall where the fluctuations have a maximum rms, give an exponent of 0.38 ± 0.01 [24], comparing favorably to our measurements. The σ_V at the center are always smaller than at the sidewall. Using a factor of 2 reduction, we compare the data of Ref. [24], to our measurements in Fig. 2b. Overall, the systematic uncertainties in the fluctuations and the relatively short range in R of about 1.3 decades preclude making a definitive statement about the precise values of the scaling exponents for either $\sigma_T/\Delta T$ or $\sigma_V d/\nu$. Nevertheless, it is the significant difference between the inserts that interests us here.

The markedly different behavior between the circular and square inserts demonstrates that the state of thermal turbulence in the interior of approximately unit-aspect-ratio convection cells and at moderate R has important nonuniversal features. It is well known that along the cell perimeter there is a large-scale circulatory (LSC) flow. Plumes irregularly erupt and interact with the LSC. A common assumption in models of turbulent convection is that the interior regions of the cell are insulated from boundaries by the LSC or by an intermediate mixing zone. As a result the properties of turbulence in the interior are assumed to depend only on the gross features of the boundary layers (BLs) or the LSC. At present, none of the turbulent convection models prescribe any dependence of the properties of the BLs or the LSC on detailed lateral constraints. If one were to subscribe to these models, then one must conclude that the necessary features of the BLs and the LSC are the same for the circular and square geometries, and thus the fluctuations in the interior must be independent of geometry. Our results manifestly refute this assumption.

How can we understand the differences in scaling behavior for temperature and velocity fluctuations between square and cylindrical convection cells? The first observation is that, for $R > 3 \times 10^8$, fluctuations in the square are always smaller than in the cylinder. Second, the ratio of fluctuations in the square to those in a cylinder decreases with increasing R: $\sigma_T^s/\sigma_T^c \sim R^{-0.38}$ and $\sigma_V^s/\sigma_V^c \sim R^{-0.12}$. From other measurements [16,24], we know that the mean LSC velocity $(\bar{V})$ scales as $R^{1/2}$ in both square and cylindrical cells. Our results combined with other velocity fluctuation measurements [24,25] show that boundary and interior vertical velocity fluctuations scale together, but with different exponents for square $\sigma_V^s d/\nu \sim R^{-3/8}$ and cylindrical $\sigma_V^s d/\nu \sim R^{-1/2}$ geometry. This means that the relative velocity fluctuations, i.e., the ratio $\sigma_V/\bar{V}$, become small for a square but remain constant in a cylinder as R becomes large. Similar evidence for temperature fluctuations in a square [18] show that the scaling is the same at the sidewall and in the center. We will then assume that temperature fluctuations at the sidewall and in the center of a cylindrical cell also scale the same way. What seems to be happening is that the coupling of temperature fluctuations from the thermal boundary layers and the LSC is more synchronous for square geometry—increasingly so as R increases—so that the flow fluctuates less in both velocity and temperature. Some characteristic of the cylinder makes the flow less steady, inducing larger fluctuations in the LSC and in the interior quantities. Although the origin of this coherence is not clear, the sharp differences in scaling of fluctuations for different geometries show that one cannot consider the LSC and the boundary layers as decoupled quantities as is assumed in existing theories. For example, the theory of Grossman and Lohse [5] makes the assumption that small-scale velocity fluctuations are driven completely by the LSC and thus should scale with R in the same way.

Given the large differences in the magnitudes of the temperature and velocity fluctuations in square and cylindrical geometry, it is interesting to compare the probability density functions (PDFs) of those fluctuations. In Fig. 3, we show the PDFs of temperature and vertical velocity fluctuations for the different cell geometries at $R = 2 \times 10^9$. Despite the large changes in the fluctuation magnitudes, the PDFs are almost indistinguishable once the temperature and velocity fluctuations are normalized by σ_T and σ_V, respectively. Similarly, the corresponding power spectra reveal no marked differences between the square and cylindrical inserts. Further, measurements of the heat transport in unit-aspect-ratio cylindrical and cubical cells are, to within experimental uncertainty, indistinguishable in both magnitude and scaling [24]. Although the heat transport, the normalized PDFs, and

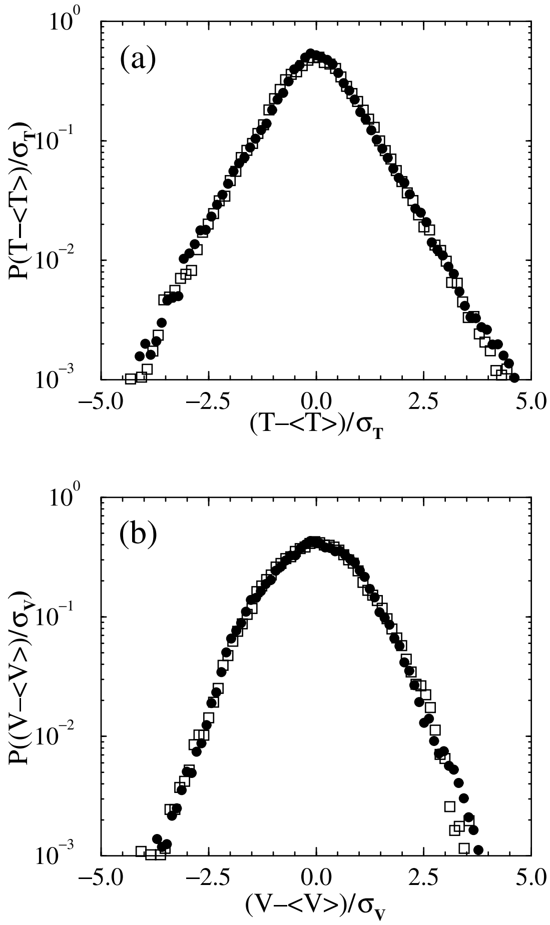

FIG. 3. Probability distribution functions of (a) temperature and (b) vertical velocity differences normalized, respectively, by their standard deviations for square (□) and cylindrical (•) geometries for $R = 2 \times 10^9$ and $\sigma = 5.46$.

the power spectra do not differ significantly between the geometries, the local temperature and velocity fluctuations vary greatly.

We have demonstrated that the Rayleigh-number dependence of the temperature and vertical velocity fluctuations at the cell center is strongly modified by the geometry; interior turbulent fluctuations are nonuniversal in low-aspect ratio convection. It is not known whether this nonuniversality is characteristic of the moderate Rayleigh numbers of our experiments and/or of the low-aspect ratio. Turbulent convection in nature typically has significantly higher Rayleigh numbers and occurs on much larger horizontal length scales. We conclude that the local properties of turbulence measured in small convection cells and at moderate R cannot be extrapolated to the laterally extended case. Similarly, models of turbulent convection that neglect lateral constraints provide an incomplete description of laboratory experiments; our measurements in the square insert are in clear disagreement with these models. Our second conclusion is that different local states of turbulent convection may lead to an identical response in global properties, and thus measuring global characteristics alone yields an incomplete description of the flow. In the future, the coupling of the interior to the periphery must be investigated. This will invariably require detailed measurements over both small and large spatial regions. An understanding of this mechanism would provide new input for the development of successful models of turbulent Rayleigh-Bénard convection. Finally, it is important to test whether interior fluctuations at higher Rayleigh numbers and in significantly higher aspect-ratio cells are dependent on the lateral geometry.

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