Electron-Spin Precession in a Plane Electromagnetic Wave

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It is shown that a departure of phase velocity v_{ph} of a circularly polarized plane electromagnetic wave from the speed of light c gives rise to the new effect of electron-spin precession in this wave exceeding by several orders of magnitude the known effects caused by radiative corrections. This effect reveals a property of the electron interaction with a polarized electromagnetic wave which is consistent with the symmetry considerations, however, it vanishes at $v_{ph} = c$ and is not described by the solution of the Dirac equation obtained by Volkov.

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Introduction.—The electron behavior in the field of a plane electromagnetic wave is described by the Dirac equation solution obtained by Volkov [1,2]. The phase of Volkov's solution does not depend on an electron-spin direction. As a consequence, the electron states with opposite spin projections on the wave vector $\mathbf{k}$ have the same (quasi)energy and momentum. Such a degeneracy of the electron-spin states predetermines the absence of the electron-spin precession about the vector $\mathbf{k}$ of a circularly (elliptically in the general case) polarized electromagnetic wave.

Meanwhile this degeneracy looks unnatural from the symmetry point of view. Indeed, let ξ_2 be the Stokes parameter, characterizing the degree of wave circular polarization, and $\mathbf{S}$ be the spin of some quantum object. The combination $\xi_2(\mathbf{kS})$ is a true scalar which can enter the scattering amplitude and effective potential of the quantum object interaction with the wave. In particular, this combination describes the splitting of atomic energy levels in the field of a circularly polarized electromagnetic wave (CPW) [3].

The absence of the similar splitting in the case of the electron can be explained only by an "accidental" compensation of, in fact, very large contributions of CPW magnetic and electric fields to the energy splitting of electron-spin states and electron-spin precession frequency. This accidental compensation takes place, first, for a "Dirac's electron" (an electron with magnetic moment equal to the Bohr magneton $\mu_0 = |e|\hbar/2mc$) and, second, if the amplitudes of the CPW electric and magnetic fields are equal.

If any of these two conditions is violated, the electron-spin precession about the vector $\mathbf{k}$ has to arise. Indeed, radiative corrections [4,5] give rise to the electron-spin precession by means of violation of the first condition of Dirac's electron. The effect caused by the radiative corrections is, naturally, limited in value.

The second condition of equality of the magnetic and electric field amplitudes is automatically satisfied for a plane CPW propagating in vacuum. However, it can be easily violated by changing the CPW phase velocity v_{ph} using some medium or waveguide structure. We will show that a departure of v_{ph} from c gives rise to a much more pronounced effect of electron-spin precession than that caused by the radiative corrections [4,5].

Semiclassical picture of the electron-spin evolution in CPW.—Let us proceed from the equation (we use the system of units in which $\hbar = c = 1$)

$$\frac{d\mathbf{a}}{dt} = \frac{2\mu m}{\varepsilon}\{[\mathbf{aH}] + (\mathbf{av})\mathbf{E}\} + O(\mu') \simeq \frac{e}{\varepsilon}\{[\mathbf{aH}] + (\mathbf{av})\mathbf{E}\} \quad (1)$$

of the evolution of the spatial part $\mathbf{a}$ of the 4-vector of spin [1]. Here $\mathbf{H}$ and $\mathbf{E}$ are magnetic and electric field strengths; m, ε, $\mu = \mu_0 + \mu' \simeq e/2m$, and μ' are the electron mass, energy, total, and anomalous magnetic momenta, respectively. Since we discuss the effect which has nothing in common with radiative corrections, we will completely neglect μ', using the right-hand side of Eq. (1).

We are going to analyze the electron-spin evolution in the field of the electromagnetic wave with phase velocity $v_{ph} \neq c$. Such a wave can propagate in refracting medium ($v_{ph} < c$) or plasma ($v_{ph} > c$) as well as in vacuum inside a hollow ($v_{ph} > c$) or dielectric-lined ($v_{ph} < c$) cylindrical waveguide. We will assume in the last case that the electron beam is thin and propagates along the axis of the waveguide. The interaction of such a beam with the wave does not differ considerably from that with a *plane* wave having $v_{ph} \neq c$.

Thus, to illustrate the idea, we can consider the electron interaction with a *plane* CPW having $v_{ph} \neq c$. Let us direct a Cartesian z axis along the vector $\mathbf{k}$ and describe the plane CPW by the transverse 3-dimensional vector potential

$$\mathbf{A} = A_0(\mathbf{n}_x \cos\omega\eta + \xi_2\mathbf{n}_y \sin\omega\eta), \quad \eta = t - nz, \quad (2)$$

where the Stokes parameter $\xi_2 = \pm 1$ determines the CPW polarization (left and right, respectively), t is time, A_0 is the amplitude of the 3-dimensional vector potential (2), ω is classical ($\omega \ll m$) frequency, and $\varphi = \omega\eta$ is the phase of the CPW. The ratio

$$n = k/\omega = c/v_{\mathrm{ph}}\,, \tag{3}$$

which, in a sense, generalizes the notion of refraction index to the waveguide case, will be widely used below to characterize the departure of v_{ph} from c both in media and waveguides. We assume that this departure can be comparable with c. Note also that the amplitudes $H_0 = kA_0$ and $E_0 = \omega A_0$ of the magnetic and electric fields of the CPW differ at $n \neq 1$.

Since in most cases the CPW field strength is limited by the breakdown considerations we assume that the amplitude of the 3-dimensional vector potential (2) obeys the restriction

$$eA_0 \ll m\,, \tag{4}$$

making our consideration more transparent. To simplify it further let us pass to the reference system in which the electron is at rest before it gets into the CPW. The Hamilton-Jacobi method [6] allows one to evaluate the energy and the momentum,

$$\varepsilon \simeq m + \frac{e^2A_0^2}{2m}\,, \qquad \mathbf{p} \simeq -e\mathbf{A} + \mathbf{n}_z\frac{e^2A_0^2}{2m}\,, \tag{5}$$

which an electron acquires in the field of CPW with the amplitude adiabatically increasing from zero up to some constant value A_0.

Equations (5) determine the electron velocity $\mathbf{v} = \mathbf{p}/\varepsilon$ allowing one to evaluate the electron-spin evolution using Eq. (1). Both electric and magnetic fields should be taken at the electron location point $z = v_z t$ in Eq. (1). Since the longitudinal electron velocity $v_z = p_z/\varepsilon$ is constant at $A_0 =$ const, the CPW frequency

$$\omega' = \omega - kv_z = \omega(1 - nv_z) \simeq \omega\left[1 - \frac{1}{2}\left(\frac{neA_0}{m}\right)^2\right] \simeq \omega \tag{6}$$

measured at the electron location point is constant as well. Since, according to Eq. (4), $v_z \simeq (eA_0/m)^2/2 \ll 1$, the frequency ω' and variable $\eta = \omega' t$ are very close, respectively, to the CPW frequency ω and time t in the laboratory system. Since $\omega' \simeq \omega$, the resonance effects which manifest themselves when the Cherenkov condition $\omega' = 0$ is satisfied will not be important in our consideration.

More important is the fact that the CPW phase at the electron location point

$$\varphi = \omega\eta = \omega' t \tag{7}$$

is linear in time when A_0 is constant. Equation (7) makes it possible to simplify Eq. (1) by passing to the reference system rotating about the vector $\mathbf{k} = k\mathbf{n}_z$ with the angular velocity $\xi_2\omega'$ (compare with Ref. [7]). Equation (1) takes the form of the system of three differential equations with constant coefficients for the components $a_i, i = 1,2,3$ of the vector $\mathbf{a}$ in this rotating system. The substitution $a_i = a_i \exp i\Omega$ transforms these differential equations into the algebraic ones. The condition of solvability of the obtained system of algebraic equations determines the (cyclic) frequencies of the electron-spin evolution,

$$\Omega_0 = \xi_2\omega', \qquad \Omega_1 = \xi_2\omega'[1 + \sqrt{1 + (e/\varepsilon\omega')^2(H_0^2 - E_0^2)}] \simeq \xi_2(2\omega' - \Omega_{\mathrm{pr}})\,,$$
$$\Omega_2 = \xi_2\omega'[1 - \sqrt{1 + (e/\varepsilon\omega')^2(H_0^2 - E_0^2)}] \simeq \xi_2\Omega_{\mathrm{pr}}\,, \tag{8}$$

where the approximations (4)–(6) were used as well as the frequency

$$\Omega_{\mathrm{pr}} = \frac{1}{2\omega'}\left(\frac{e}{\varepsilon}\right)^2(E_0^2 - H_0^2) \simeq \left(\frac{eA_0}{m}\right)^2(1 - n^2)\,\frac{\omega}{2} \tag{9}$$

was introduced.

Solving the system of algebraic equations with the frequencies (8) and passing back to the laboratory system one obtains

$$a_x = a_\perp^0\left[\cos(\Omega_{\mathrm{pr}}t + \varphi_0) + \left(\frac{eA_0}{2m}\right)^2(1 - n^2)\cos(2\omega' t - \varphi_0)\right] + a_z^0 n\,\frac{eA_0}{m}\cos\omega' t + O\left(\left(\frac{eA_0}{m}\right)^3\right),$$
$$a_y = \xi_2 a_\perp^0\left[\sin(\Omega_{\mathrm{pr}}t + \varphi_0) + \left(\frac{eA_0}{2m}\right)^2(1 - n^2)\sin(2\omega' t - \varphi_0)\right] + \xi_2 a_z^0 n\,\frac{eA_0}{m}\sin\omega' t + O\left(\left(\frac{eA_0}{m}\right)^3\right) \tag{10}$$

for transverse components of the vector $\mathbf{a}$. The values $a_\perp^0$, a_z^0, and φ_0 specify the last

$$\mathbf{a}^0 = a_\perp^0(\mathbf{n}_x\cos\varphi_0 + \mathbf{n}_y\sin\varphi_0) + a_z^0\mathbf{n}_z \tag{11}$$

before the electron gets into the CPW. The new effect of electron-spin precession in a CPW with $v_{\mathrm{ph}} \neq c$ is described by the first terms of Eqs. (10). This precession occurs about the wave vector $\mathbf{k} = k\mathbf{n}_z$ with the frequency (9) in the direction of rotation of the CPW vector potential (2).

Equations (10) should be compared with their particular case at $v_{\mathrm{ph}} = c$,

$$a_x = a_\perp^0\cos\varphi_0 + a_z^0\,\frac{eA_0}{m}\cos\omega' t\,,$$
$$a_y = \xi_2 a_\perp^0\sin\varphi_0 + \xi_2 a_z^0\,\frac{eA_0}{m}\sin\omega' t \tag{12}$$

which is described by Volkov's solution of the Dirac equation. Note that Eqs. (12) do not describe any directed evolution which could cause a continuous accumulation of the electron-spin variation in time. The first terms of Eqs. (10), on the opposite, describe the electron-spin rotation which does accumulate in time and remains observable when the CPW action is terminated.

Magnitude of the effect.— In principle, the spin precession frequency (9) can reach its own CPW frequency ω at $eA_0 \sim m$, if the problem of electric breakdown is bypassed. At a given CPW intensity the frequency (9) greatly exceeds that of the electron-spin precession caused by radiative corrections. Indeed, the anomalous magnetic moment μ' gives rise to the electron-spin precession in vacuum with frequency

$$2(\mu' A_0)^2\omega \simeq \left(\frac{\alpha}{2\pi}\right)^2\left(\frac{eA_0}{m}\right)^2\frac{\omega}{2} \tag{13}$$

having 2 more orders of α than (9). Equation (13) was obtained in [4] and is discussed in detail in [5]. The frequency (9) exceeds (13) $(\mu_0/\mu')^2|1-n^2| \simeq 0.74\times 10^6|1-n^2|$ times (2.2×10^6 times at $v_{\rm ph} = 0.5c$ and 0.80×10^5 times at $v_{\rm ph} = 0.95c$, for example). The frequency (9) also $|1-n^2|\times(m/\omega)^2/\alpha\ln(m/\omega)$ times (10^{10} times at $\omega = 1.6eV$ and $v_{\rm ph} = 0.995c$) exceeds that of the electron-spin precession arising [5] because of the first order radiative correction to the Compton scattering amplitude.

Such an increase of the precession frequency at $v_{\rm ph} \neq c$ makes it possible to change the electron-spin direction by short electromagnetic pulses almost instantly. For example, a pulse of duration t will reverse the electron-spin direction if the condition $\Omega_{\rm pr}t = \pi$ is fulfilled. To do this the energy of the pulse with cross section S has to be equal to

$$\frac{E_0^2}{4\pi}tcS = \frac{\pi}{(1-n^2)}\frac{mc^2}{\lambda r_e}S = \frac{0.91S[\mathrm{cm}^2]}{(1-n^2)\lambda[\mathrm{cm}]}[\mathrm{J}] \tag{14}$$

($r_e = e^2/mc^2$), or to about 1 J at $\lambda \sim 1$ cm, $S \sim 1$ cm^2, or 10^{-3} J at $\lambda \sim 10$ μm, $S \sim 100$ μm^2. Such pulses, which can readily be obtained, could be useful to change the electron beam polarization direction or electron-spin state population in electron traps.

The relation $2\mu_0 H_{\rm eff} = \hbar\Omega_{\rm pr}$ allows one to introduce the strength

$$H_{\rm eff} = \frac{5.4\times 10^3(1-n^2)}{\lambda[\mathrm{cm}]}\left(\frac{eA_0}{m}\right)^2[\mathrm{G}] \tag{15}$$

of the effective magnetic field acting on the electron spin as a CPW with phase velocity $v_{\rm ph} = c/n$ and vector potential amplitude A_0. The effective field (15) can be very large in the absence of breakdown limitations, reaching tens of megagauss in sufficiently dense plasma at $eA_0 \sim m$ and $\lambda \sim 1$ μm. In more usual situations the CPW effective magnetic field, pulsed or continuous, can be used instead of the real one in experiments of ESR or the spin echo type with quasifree electrons in gases or pores of irradiated or amorphous semiconductors. In addition, the energy splitting of the electron-spin states in the effective field (15) opens up a new way to separate electrons with opposite longitudinal spin projections.

The Dirac equation solution.—We will show now how the effect of electron-spin precession with the frequency (9) is described by the solution of the Dirac equation in the field of a CPW with $v_{\rm ph} \neq c$. This solution can be represented in the form $\psi = \exp(-i\varepsilon_0 t + i\mathbf{p}_0\mathbf{r})F(\omega\eta)$ where $\varepsilon_0 = \sqrt{p_0^2 + m^2}$ and $\mathbf{p}_0$ are, respectively, the energy and momentum of the electron before it gets into the CPW. The substitution

$$F = \exp(i\Omega\eta)\begin{pmatrix}\exp(-i\xi_2\omega\eta/2)\psi_1\\ \exp(+i\xi_2\omega\eta/2)\psi_2\\ \exp(-i\xi_2\omega\eta/2)\psi_3\\ \exp(+i\xi_2\omega\eta/2)\psi_4\end{pmatrix} \tag{16}$$

transforms the Dirac equation in the field (2) to the system of algebraic equations with constant coefficients which has the form,

$$\begin{aligned}(\Omega - \xi_2\omega/2)\psi_1 - n(\Omega - \xi_2\omega/2)\psi_3 - eA_0\psi_4 &= 0,\\ (\Omega + \xi_2\omega/2)\psi_2 + n(\Omega + \xi_2\omega/2)\psi_4 - eA_0\psi_3 &= 0,\\ (\Omega - \xi_2\omega/2 - 2m)\psi_3 - n(\Omega - \xi_2\omega/2)\psi_1 - eA_0\psi_2 &= 0,\\ (\Omega + \xi_2\omega/2 - 2m)\psi_4 + n(\Omega + \xi_2\omega/2)\psi_2 - eA_0\psi_1 &= 0,\end{aligned} \tag{17}$$

in the standard representation in the considered case of $\mathbf{p}_0 = 0$ and $\varepsilon_0 = m$. The last allows one to solve the system (17) in the most direct way. Indeed, excluding ψ_1 from the first and third equations and substituting ψ_3 from the second one, we obtain a quadratic equation for Ω, corresponding to the upper sign in equality,

$$\Omega^2 - \frac{2m}{1-n^2}\Omega - \frac{\omega^2}{4} - \frac{e^2A_0^2 \pm \xi_2 m\omega}{1-n^2} = 0. \tag{18}$$

That one corresponding to the lower sign is obtained by excluding ψ_2 from the second and fourth equations (17) and substituting ψ_4 from the first one. However only two,

$$\begin{aligned}\Omega_\pm &= \frac{m}{1-n^2} - \left[\frac{m^2}{(1-n^2)^2} + \frac{\omega^2}{4} + \frac{e^2A_0^2 \pm \xi_2 m\omega}{1-n^2}\right]^{1/2}\\ &\simeq \mp\xi_2\frac{\omega}{2} - \frac{e^2A_0^2}{2m} + \frac{\omega^2}{8m^2}(1-n^2)(1-\xi_2^2) \pm \xi_2\left(\frac{eA_0}{m}\right)^2(1-n^2)\frac{\omega}{4},\end{aligned} \tag{19}$$

of the four roots of Eq. (18) should be taken into consideration in the case of adiabatic CPW amplitude variation at the electron location point. Indeed, the roots (19) tend to $\mp\xi_2\omega/2$ and lead to the adequate time dependence $\psi \sim \exp(-imt)$ of the electron wave function at $A_0 \to 0$ [see Eq. (20) below]. Meanwhile another pair of roots which corresponds to the "plus" sign of the square root in Eq. (19) leads to $\psi \sim \exp[-im(1+n^2)t/(1-n^2)]$ at $A_0 \to 0$ and should be ignored in the case of adiabatic CPW field growth starting from $A_0 = 0$. This pair of roots, however, should be taken into consideration for electrons emergent inside a wave, for example, in the process of ionization.

In order to demonstrate the correspondence of the Dirac equation solution to the semiclassical spin evolution picture, the second row of Eq. (19) has been obtained by expansion of the first one up to terms quadratic in small quantities ω/m and eA_0/m. The considered electron-spin precession is evidently described by the fourth term of this expansion proportional to the spin precession frequency (9) and depending both on polarization sign and wave intensity.

The substitution of the frequencies Ω_+ and Ω_- into Eqs. (17) yields the pairs of equations $\psi_4 = 0$, $\psi_1 = n\psi_3$ and $\psi_3 = 0$, $\psi_2 = -n\psi_4$, respectively, which determine two solutions of the system (17). Their linear combination

$$\psi = \exp\left(-i\varepsilon t + ipz - i\frac{e^2A_0^2}{2m}\eta\right) \times \begin{pmatrix} c_+ e^{-i\frac{\Omega_{\mathrm{pr}}}{2}\eta} - c_-\frac{eA_0 n}{2m}e^{-i\xi_2\omega\eta + i\frac{\Omega_{\mathrm{pr}}}{2}\eta} \\ c_- e^{i\frac{\Omega_{\mathrm{pr}}}{2}\eta} + c_+\frac{eA_0 n}{2m}e^{i\xi_2\omega\eta - i\frac{\Omega_{\mathrm{pr}}}{2}\eta} \\ -c_-\frac{eA_0}{2m}e^{-i\xi_2\omega\eta + i\frac{\Omega_{\mathrm{pr}}}{2}\eta} \\ -c_+\frac{eA_0}{2m}e^{i\xi_2\omega\eta - i\frac{\Omega_{\mathrm{pr}}}{2}\eta} \end{pmatrix} \quad (20)$$

gives the required solution of the Dirac equation in a plane CPW with $v_{\mathrm{ph}} \neq c$. The coefficients c_+ and c_- of this linear combination are the amplitudes of probability to find an electron, before it gets into a CPW, to be polarized along and opposite the wave vector $\mathbf{k} = k\mathbf{n}_z$, respectively. The different dependence of the leading terms $c_\pm e^{\mp i(\Omega_{\mathrm{pr}}/2)\eta}$ (the first terms of the two upper lines) of the bispinor (20) on $\eta \simeq t$ reflects the electron energy level splitting arising at $v_{\mathrm{ph}} \neq c$ and giving rise to the predicted effect of electron-spin precession in a CPW. Note that the solution (20) takes the form of that obtained by Volkov in a plane CPW with $v_{\mathrm{ph}} = c$ and leads to Eqs. (12).

Equation (20) allows one to evaluate the components of the 4-vector of electron spin [2]

$$a_\mu = \bar\psi\gamma_5\gamma_\mu\psi/\bar\psi\psi, \quad (21)$$

where γ_5 and $\gamma_\mu, \mu = 0,1,2,3,$ are the Dirac matrixes and

$$\bar\psi\psi = (|c_+|^2 + |c_-|^2)[1 + (eA_0/2m)^2(n^2-1)] + O((eA_0/m)^4)$$

is the normalizing factor already used to obtain Eqs. (10). A direct substitution of Eq. (20) into Eq. (21) again yields Eqs. (10) with the well-known relations [7]

$$a_\perp^0 \exp i\varphi_0 = \frac{2c_+^* c_-}{|c_+|^2 + |c_-|^2}, \qquad a_z^0 = \frac{|c_+|^2 - |c_-|^2}{|c_+|^2 + |c_-|^2},$$

of the classical spin components with the quantum amplitudes $c_\pm$. Thus, the solution (20) of the Dirac equation in the plane CPW of low field strength (4) reproduces the semiclassical picture of the electron-spin evolution rigorously confirming the prediction of the new effect of electron-spin precession at $v_{\mathrm{ph}} \neq c$. This solution also describes specific quantum features of the electron-spin evolution will manifest themselves in a CPW with $eA_0 \geq m$, $\omega \geq m$, and $v_{\mathrm{ph}} \neq c$.

In conclusion, it was shown that a departure of a phase velocity of a circularly (elliptically in the general case) polarized electromagnetic wave from the speed of light in vacuum gives rise to the effect of electron-spin precession about the wave vector with a precession frequency exceeding that caused by the radiative corrections by 6 orders of magnitude and more. This effect opens up new possibilities to control the electron polarization in different situations.

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