## *Tc* **Suppression in Co-Doped Striped Cuprates**

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We propose a model that explains the reduction of  $T_c$  due to the pinning of stripes by planar impurity co-doping in cuprates. A geometrical argument about the planar fraction of carriers affected by stripe pinning leads to a linear  $T_c$  suppression as a function of impurity concentration *z*. The critical value  $z_c$ for the vanishing of superconductivity is shown to scale like  $T_c^2$  in the incompressible stripe regime and becomes universal in the compressible regime. Our theory agrees very well with the experimental data in single- and bilayer cuprates co-doped with Zn, Li, Co, etc.

One of the most striking properties of high temperature superconductors (HTSC) is the sensitivity of the critical temperature,  $T_c$ , to planar impurities which are introduced with the substitution of Cu atoms in CuO<sub>2</sub> planes.  $T_c$  is suppressed with a few percent of doping almost independently of the magnetic nature of the impurity [1]. In particular, it has been shown experimentally that the HTSC undergo a superconductor to insulator transition due to Zn doping [2]. The fast depression of  $T_c$  with Zn has been studied extensively by nuclear magnetic resonance (NMR), muon spin resonance  $(\mu SR)$  [3,4], and infrared techniques [5] and a heated debate has been developed on the interpretation of the experimental data [6]. It has also been established that this suppression is more robust in the underdoped compounds [7]. The reduction of  $T_c$  has been assigned to formation of magnetic defects [8,9], electron scattering by disorder in the presence of a *d*-wave order parameter [10,11], unitary scattering [12], local variation of the superconducting gap [13], and weak localization [14]. However, the existing theories cannot fully describe the experimental data.

In this work we introduce a scenario for the destruction of superconductivity which is based on the stripe picture. NMR studies of the effect of Zn co-doping in cuprates have associated the enhancement of antiferromagnetic moments in local patches to charge localization [15]. Transport measurements also point to the same direction [2,5,16,17]. We propose that Zn pins stripes and slows down their dynamics in a region of size *R* in its surrounding. This effect resembles the "swiss cheese"-like model with Zn sites creating voids in the superconducting mesh and destroying superfluid density [4]. In our model, however, the superfluid density remains unaltered upon Zn co-doping, whereas the stripe inertia increases locally. Since the stripe mass density  $\rho$  is inversely proportional to the hopping energy *t* [18], a local decrease of *t* by pinning will enhance the stripe effective mass  $\rho$ .  $T_c(x, z)$  is a function of the carrier density *x* and *z* and it is determined by the ratio between the 3D superfluid density  $n<sub>s</sub>$  and the

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*ab*-plane charge carriers effective mass *m ab*. Within the stripe picture we must have  $T_c \propto n_s/\rho$ . Hence, the pinning of stripes in the vicinity of the impurity implies a reduction of  $T_c$  by the enhancement of  $\rho$ . As a result of our model we obtain  $T_c(x, z)/T_c(x, 0) = 1 - z/z_c(x)$ , where  $z_c(x)$  is the critical concentration required to completely suppress superconductivity. In the incompressible stripe regime we find that  $z_c(x) \propto x^2$  and scales with  $T_c^2(x,0)$ in contrast with the case of ordinary superconductors. On the other hand, in the compressible stripe regime  $z_c(x)$  is independent of *x* and exhibits universal behavior. Our results give a very good description of the experimental data in La<sub>2-x</sub>Sr<sub>x</sub>CuO<sub>4</sub> (LSCO), YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\epsilon$ </sub> (YBCO), and  $Bi_2Sr_2Ca_{1-y}Y_yCu_2O_{8+\epsilon}$  (BSCO) co-doped with different impurities, such as Zn, Li, Co, etc.

Striped phases of holes have been observed experimentally in the superconductor  $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$  [19] and in  $\text{La}_{2-x}\text{Sr}_x\text{NiO}_{4+y}$  [20], which is an antiferromagnetic insulator. Moreover, stripe formation provides a simple explanation for the observed magnetic incommensurability in LSCO [21] and YBCO [22]. The magnetic incommensurability appears in neutron scattering by the splitting of the commensurate peak at  $\mathbf{Q} = (\pi/a, \pi/a)$  ( $a \approx 3.8$  Å is the lattice spacing) by a quantity  $\delta$ , which is inversely proportional to the average interstripe distance  $\ell$  [23]. In LSCO with  $x < 0.12$  the charge stripes behave as an incompressible quantum fluid, i.e.,  $\delta$  is proportional to x and therefore  $\ell = a/2x$  [23,24]. In this incompressible stripe regime  $T_c$  scales with  $\delta$  [23]

$$
T_c(x) = \frac{569}{\ell(x)} \propto \delta \,, \tag{1}
$$

where  $\ell$  is given in Å. For  $x > 0.12$  the incommensurability  $\delta$  saturates to a constant value and  $\ell$  becomes nearly independent of *x* [23] (compressible stripe regime). To date, there is strong evidence for stripe formation in LSCO compounds [19,23]. Recent ion-channeling [25], NMR [26],  $\mu$ SR [27], and neutron scattering data [28] also indicate that stripes might be present in YBCO, whereas a lack of experimental data still persists for BSCO. In what follows we will concentrate on the physics of HTSC co-doped with planar impurities, such as  $\text{La}_{2-x}\text{Sr}_x\text{Cu}_{1-z}\text{Zn}_z\text{O}_4$ . We will verify that our results can describe equally well YBCO and BSCO co-doped with Zn, Li, and Co.

For simplicity, we assume that the Zn atoms are located half the distance between superconducting stripes and suppress stripe fluctuations within a circle of radius *R* around their position, as shown in Fig. 1. This distance  $R$  is the only free parameter in our model. It is set by the zero-temperature in-plane magnetic correlation length,  $R \sim \xi(x, T = 0)$  [15]. Recently, an empirical expression was proposed for describing the dependence of  $\xi$ on the doping concentration *x* within the spin-glass and underdoped regimes,  $\xi(x, T = 0) = Ca/(x - x_c)^{\eta}$ . It was found that the best fit for the data was obtained for  $C \approx 0.5$ ,  $x_c = 0$ , and  $\eta \approx 1$ , i.e.,  $\xi = a/2x$  [29]. Since  $\ell = a/2x$  in the underdoped regime, it is natural to assume that  $R \propto \ell$ . Thus, we will parametrize  $R = \gamma \ell/2$ , with  $\gamma > 1$  denoting a doping independent phenomenological parameter and will later check for the consistency of this assumption.

As shown in Fig. 1, the stripe length which is pinned per Zn atom is  $4\sqrt{R^2 - \ell^2/4} = 2\ell\sqrt{\gamma^2 - 1}$ . If we assume that all the  $N_{Zn}$  Zn atoms take part in pinning the stripes, the total pinned length is  $N_{\text{Zn}} 2\ell \sqrt{\gamma^2 - 1}$ . The transverse kinetic energy density of the stripe is  $t/a$ , where *t* is the single hole kinetic energy. Thus, the suppressed energy density in the plane is  $\delta \tau = (t/a)2\ell \sqrt{\gamma^2 - 1} N_{\text{Zn}}/L^2$ , where  $L$  is the sample size. Defining the impurity fraction  $z = N_{\text{Zn}}(a/L)^2$  and the planar kinetic energy density  $\tau = t/(a\ell)$ , we find that the suppressed energy density reads

$$
\delta \tau = \frac{2z \ell t \sqrt{\gamma^2 - 1}}{a^3} = \frac{2z \ell^2 \sqrt{\gamma^2 - 1}}{a^2} \tau(x, 0). \tag{2}
$$

The energy density of the co-doped system is given by



FIG. 1. Effect of Zn on the stripe grid. Dark dashed lines mark the regions of stripes with finite superfluid density. The circle shows the region where hopping is suppressed and the superfluid effective mass is increased.

 $\tau(x, z) = \tau(x, 0) - \delta \tau$ , which leads to

$$
\frac{\tau(x,z)}{\tau(x,0)} = 1 - \frac{z}{z_c(x)},\tag{3}
$$

where

$$
z_c(x) = \frac{1}{2\sqrt{\gamma^2 - 1}} \left[ \frac{a}{\ell(x)} \right]^2.
$$
 (4)

Superconducting long range order, and therefore  $T_c$ , is obtained when the stripe array attains phase coherence. This coherence can be achieved by the Josephson coupling between stripes which may occur via exchange of Cooper pairs [30] or  $dx^2 - y^2$  bosons [31]. In this case the finite transition temperature is in the 2D *XY* universality class (stabilized by the interplanar coupling), i.e.,

$$
T_c = \frac{\pi \hbar^2 L_c}{2k_B} \frac{n_s}{m_{ab}^*},\tag{5}
$$

where  $L_c$  is the interplanar distance. Notice that  $n_s \sim$  $1/[2a\ell(x)L_c]$  (Ref. [19]) and  $m_{ab}^*$  is proportional to the stripe linear mass density  $\rho = \hbar^2/(ta^3)$  [18]. Therefore we conclude that

$$
T_c(x,z) \propto \frac{t(z)}{\ell(x)} \propto \tau(x,z). \tag{6}
$$

Combining (3) with (6) we find

$$
\frac{T_c(x,z)}{T_c(x,0)} = \frac{\tau(x,z)}{\tau(x,0)} = 1 - \frac{z}{z_c(x)}.
$$
 (7)

It is very illuminating to compare the expressions for  $z_c(x)$ , that is, the critical Zn doping for which  $T_c$  vanishes, in the various regimes. In the incompressible regime, neutron scattering data show that for LSCO  $T_c(x,0) \propto \delta$  [23]. Comparing (4) with (1), one finds an unforeseen result, which is

$$
z_c(x) = \left[\frac{a(\mathring{A})}{805}\right]^2 \frac{T_c^2(x,0)}{\sqrt{\gamma^2 - 1}}.
$$
 (8)

Moreover, in the compressible regime the average stripe separation saturates to a value  $\ell_s$ , and therefore we expect *zc* to be a universal constant, independent of *x*,

$$
z_c = \frac{1}{2\sqrt{\gamma^2 - 1}} \left(\frac{a}{\ell_s}\right)^2. \tag{9}
$$

In Fig. 2, we plot the experimental data for LSCO co-doped with Zn [34]. By comparing the best fit to the data corresponding to  $x = 0.10$  in Fig. 2 (dashed line across filled circles) with the theoretical results given by Eqs. (7) and (8), we determine the only fitting parameter in our model to be  $\gamma \sim 1.42$ . The dotted line describing the universal regime is then obtained from Eqs. (7) and (9) *without* any fitting parameter. In addition, in Fig. 3 we show the experimental data for  $z_c \times T_c$ , as well as the theoretical prediction (8) using the same value for  $\gamma$ .

The excellent agreement between theory and experiment is, however, not restricted to the Zn doped lanthanates. In Fig. 4 we display the experimental data for YBCO (filled



FIG. 2.  $T_c(x, z)/T_c(x, 0)$  versus the Zn doping concentration *z* for the lanthanate compound. Dashed and dotted lines correspond to Eq. (7) with  $z_c$  given by Eqs. (8) and (9), respectively. ( $\bullet$ ) denote  $x = 0.10$  [32], and the opened symbols are (+)  $x = 0.15$  [32]; (O)  $x = 0.15$  [1]; ( $\Diamond$ )  $x = 0.18$  [33];  $(\triangle)$  *x* = 0.20 [33]; ( $\square$ ) *x* = 0.20 [32].

symbols) co-doped with Li (squares) and Zn (circles) [35], as well as for BSCO co-doped with Co (open symbols) [34]. The universal behavior at high doping is very clear (Fig. 4a) as well as the different slopes characteristic of the underdoped compounds (Fig. 4b). Observe that YBCO and BSCO exhibit exactly the same features in terms of co-doping, with the same value of  $z_c$  in the compressible regime. We assign this behavior to the fact that both systems are bilayers and both impurities have spin zero. Moreover, it has been shown recently that the relationship of  $T_c$  and  $\delta$  for YBCO is similar to the one in LSCO [36]. Using the value of the incommensurability  $\delta = 0.11$ measured by neutron scattering in  $YBCO<sub>6.6</sub>$  [28], which has  $T_c = 60$  K, we can determine the stripe separation in this underdoped compound,  $\ell(x) \sim 4.55a$ . Then, by fitting the data from Ref. [35] for the same compound (filled circles in Fig. 4b,  $z_c = 0.04$ ) to Eqs. (4) and (7), we determine the value of our free parameter for bilayer systems,  $\gamma = 1.17$ , corresponding to  $R \sim 10$  Å. NMR measurements in Zn doped YBCO<sub>6.7</sub> found  $R \sim 10-15$  Å [15], in excellent agreement with our predictions. Now, we use this value of  $\gamma$  to analyze the compressible regime. The universal line in Fig. 4a defines the critical doping in over-



FIG. 3.  $z_c \times T_c$ : the continuous line is given by Eq. (8). The data points are experimental values for underdoped La compound: ( $\blacksquare$ ) Ref. [11]; ( $\spadesuit$ ) Ref. [34]; ( $\spadesuit$ ) Ref. [1].

doped bilayer cuprates,  $z_c \sim 0.13$ . By replacing  $z_c$  and  $\gamma$ in Eq. (9), we can estimate the saturation value of the average stripe distance. We then find  $\ell_s \sim 3a$ , corresponding to  $R = \gamma \ell_s/2 \sim 6$  Å. Inelastic neutron scattering data in overdoped YBCO with Zn impurities [37] indicate that Zn induces a magnetic perturbation on a range of  $R \sim 7$  Å, very close to our estimates. Moreover, we predict that the saturation of the incommensurability in YBCO should start at optimal doping, where  $\delta_s \sim 0.2$  and  $\ell_s \sim 3a$ , in contrast with the case of LSCO where the saturation starts at  $x \sim 0.12$  and  $\ell_s \sim 4a$ . This result agrees with recent neutron scattering data in YBCO [36] and establishes a very important relationship between two different kinds of experiments, namely, magnetization and neutron scattering. By determining the doping concentration *x* beyond which the  $T_c(x, z)/T_c(x, 0)$  versus *z* line becomes universal, one is automatically finding the value of *x* beyond which a saturation should be reached in the  $\delta$  versus *x* plot. It is also a straightforward conclusion of the data in Fig. 4 that BSCO has a behavior very similar to YBCO, although there is no



FIG. 4. NMR and  $\mu$ SR data for yttrium- and bismuth-based compounds. Filled symbols correspond to  $YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub>$  [35] co-doped with Li (squares) and Zn (circles), whereas open symbols correspond to  $Bi_2Sr_2Ca_{1-y}Y_y(Cu_{1-z}Co_z)_2O_{8+x}$  [34]. (a) Optimally doped and overdoped compounds; (b) underdoped materials.

direct evidence for incommensurability or stripes in BSCO. It is interesting to notice that  $T_c(x, z)/T_c(x, 0)$  when plotted versus the in-plane 2D residual resistance,  $\rho_0^{2D}$ , exhibits universal behavior in the underdoped but not in the overdoped regime [2]. However, as a function of *z*, as we have shown here, the universal behavior is found in the optimally and overdoped regimes. This fact is due to the linear dependence of  $\rho_0^{\text{2D}}$  with *z* [2].

We finally would like to point out that the prediction that  $z_c \propto T_c^2(x, z = 0)$  is highly unusual. In BCS-like models the Abrikosov-Gor'kov [38] approach predicts that  $T_c$  is suppressed when the scattering rate  $1/\tau_s \propto z$  becomes of the order of the superconducting gap  $\propto T_c(x, 0)$  and therefore one would expect  $z_c \propto T_c(x, z = 0)$  in disagreement with the data.

In conclusion, we presented a "geometrical" model for the suppression of  $T_c(x, z)$  by planar impurities within the stripe model. It is based on the following assumptions: (a)  $T_c$  is proportional to  $n_s/m_{ab}^*$ ; (b) Zn impurities work as local pinning centers and suppress the transverse kinetic energy density of the stripes within a radius *R* in their immediate vicinity, therefore increasing the effective charge carriers mass, since  $m_{ab}^* \sim t^{-1}$ . As a result, we obtain a linear  $T_c$  suppression with doping, with a slope inversely proportional to the critical impurity concentration  $z_c$ , as given by Eq. (7). By comparing our results with neutron scattering data for  $\ell$ , we find that  $z_c$  exhibits a nontrivial behavior in the incompressible stripe regime,  $z_c(x) \propto$  $T_c^2(x)$ , and a universal one,  $z_c$  = const in the compressible regime. Our model contains only one free parameter,  $\gamma$ , which depends on the compound ( $\gamma$  is different for single and bilayer cuprates) and on the nature (spin) of the impurity. By fitting one set of data for a fixed value of  $x$ , we determine  $\gamma$  and obtain the next results without any fitting parameter. Our findings agree very well with the experimental data and allow us to establish a connection between neutron scattering and magnetization measurements.

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