

Transition from Collisionless to Hydrodynamic Behavior in an Ultracold Fermi Gas

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Relative motion in a two-component, trapped atomic gas provides a sensitive probe of interactions. By studying the lowest frequency excitations of a two-spin-state gas confined in a magnetic trap, we have explored the transition from the collisionless to the hydrodynamic regime. As a function of collision rate, we observe frequency shifts as large as 6% as well as a dramatic, nonmonotonic dependence of the damping rate. The measurements agree qualitatively with expectations for behavior in the collisionless and hydrodynamic limits and are quantitatively compared to a classical kinetic model.

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Because dilute quantum gases have controllable interactions that can be described from first principles they provide an ideal testing ground for many-body theories of quantum fluids. Elementary excitations, in particular, play a key role in understanding the behavior of quantum fluids. The nature of these excitations varies drastically between two regimes, collisionless and hydrodynamic, based on the relative strength of interactions. In spite of their promise of controllable interactions, atomic gas experiments have had difficulty exploring this full range of behavior, in part because the high densities required to reach the hydrodynamic regime also lead to large inelastic collision rates and consequent rapid number loss. The recently created Bose-Einstein condensates in metastable He, on the other hand, are deeply in the hydrodynamic regime [1,2]. In this work we have utilized relative motion in a two-component alkali atom gas to observe the transition from the collisionless to the hydrodynamic regime along one spatial dimension.

In general, the underlying character of excitations, as well as their frequencies and damping rates, depends on the relative strength of the interactions in the system. In the collisionless regime, the collision rate Γ_{coll} is much smaller than the excitation frequency ω . In this limit, there are few scattering events per oscillation. Classically, the motion is described by the single-particle Hamiltonian and collisions tend to damp excitations. For a quantum Fermi gas, this is the regime of zero sound, which is a collective excitation due to the self-consistent mean field of a large number of particles. The opposite limit of large collision rate, $\Gamma_{\text{coll}} \gg \omega$, is called the hydrodynamic or collisional regime. Here, the motion consists of collective excitations in which the high collision rate maintains local equilibrium throughout the gas. This is the regime of first sound waves, which, unlike single-particle excitations, are only weakly damped by collisions.

For an atomic gas confined in a harmonic potential the excitation frequency scales as the trap frequency while the collision rate can also be controlled by varying the gas density and temperature. Excitations of a trapped classical gas

have been treated theoretically for both the collisionless and hydrodynamic regimes [3–7]. However, experiments have typically been in the collisionless regime or between the two regimes [8]. In order to reach the hydrodynamic regime at typical densities for alkali atom experiments, we excite the lowest frequency excitation of a two-component, magnetically confined gas. The excitation mode involves center-of-mass motion along the weaker, axial direction of a cylindrically symmetric magnetic potential. Using excitations in the weakest direction of the trap has the advantage that the strength of the interactions relative to the excitation frequency can be increased by tightly confining the gas radially. For a single-component gas this dipole mode, or “slosh,” is unaffected by interactions since collisions between atoms cannot alter center-of-mass momentum. However, for a two-component gas, consisting of atoms in two different spin states for example, collisions can impact the relative motion of the two gases and this lowest frequency mode becomes a useful probe of interactions. Excitations of this type have been observed in studies of two-component Bose-Einstein condensates [8–10] and have been explored theoretically for degenerate Fermi gases [11,12].

For this experiment the gas consisted of ^{40}K atoms in the internal states $f = 9/2$, $m_f = 9/2$ and $f = 9/2$, $m_f = 7/2$ (denoted here by $|9/2\rangle$ and $|7/2\rangle$, respectively), where f is the total atomic spin and m_f is the magnetic quantum number. Because of their different magnetic moments, atoms in these two states have slightly different single-particle oscillation frequencies in the magnetic trap. This fact is essential to these experiments because we find that the crossover to the hydrodynamic regime for the slosh mode occurs at a collision rate set by the small difference in the axial trap frequencies for the two components.

As described in previous work [13], the atoms are pre-cooled in a vapor-cell magneto-optical trap and then loaded into a magnetic trap where they are further cooled by forced evaporation. For this experiment a gas of between 0.35 and 3.5 million atoms, with a spin mixture of 45% $|9/2\rangle$ (55% $|7/2\rangle$), was cooled to a temperature between

0.5 and 2 μK . The magnetic trap strength corresponded to an axial frequency of 19.84 Hz for a $|9/2\rangle$ atom. The radial trap frequency, which was varied to access different relative interaction strengths in the gas, was set to either 135 or 256 Hz [14]. ^{40}K atoms are fermions; however, the measurements described in this Letter were performed in the nondegenerate regime by keeping the temperature of the gas above the Fermi temperature.

To excite a slosh mode we applied an additional magnetic field that shifted the trap center along the axial direction. After 28 ms (roughly half of the period of oscillation in the trap) this external field was switched off, and the motion of the atoms was allowed to evolve freely in the magnetic trap for time t . After this delay time, the trap was switched off and the gas ballistically expanded for 11 ms before an absorption image of the atoms was taken. During the ballistic expansion an inhomogeneous magnetic field was applied in order to spatially separate atoms in the two spin states through the Stern-Gerlach effect [15,16]. By using Gaussian fits to the images recorded on a CCD camera, we extracted the temperature, number, and density of both components. The time evolution was mapped out by repeating this sequence for different times t and recording the position of the center of mass of each component gas after expansion.

For a spin-polarized gas of either spin state the motion of the cloud center fit a sine function, with no damping observed over 1 s. The axial trap frequency was thus determined to be $\omega_9/2\pi = 19.84$ Hz for the $|9/2\rangle$ atoms and $\omega_7/2\pi = 17.44$ Hz for the $|7/2\rangle$ atoms. In contrast, excitations of a two spin-state gas in the magnetic trap depended strongly on collisions in the gas. Because the atoms are fermions, s -wave collisions are forbidden between identical atoms. Additionally, p -wave (and higher order) collisions are energetically forbidden at the temperatures of interest [13]. However, collisions can occur between atoms in different spin states and it is these collisions that impact relative motion in the two-component system [17].

Typical data are shown in Fig. 1. The vertical axis plots the center positions of the two atom clouds, after 11 ms of free expansion. The data in Fig. 1(a) are for a small number of atoms, resulting in a relatively low collision rate. The $|7/2\rangle$ and $|9/2\rangle$ clouds oscillate close to their respective bare trap frequencies, but the motion is clearly damped. In Fig. 1(c), we look at the cloud motion in the hydrodynamic regime. Here a larger number of atoms and tighter radial confinement are used to reach a much higher collision rate. The $|7/2\rangle$ and $|9/2\rangle$ clouds now oscillate synchronously at an intermediate frequency between the two single-particle frequencies. This collective mode also damps, at a rate similar to the low collision rate data in Fig. 1(a). Data in Fig. 1(b) show the cloud motion in the transition region between the collisionless and hydrodynamic limits. Here the motion of the two clouds is coupled and the frequencies of motion are significantly shifted from the single-particle trap frequencies. Furthermore, the excitation is subject to strong damping.

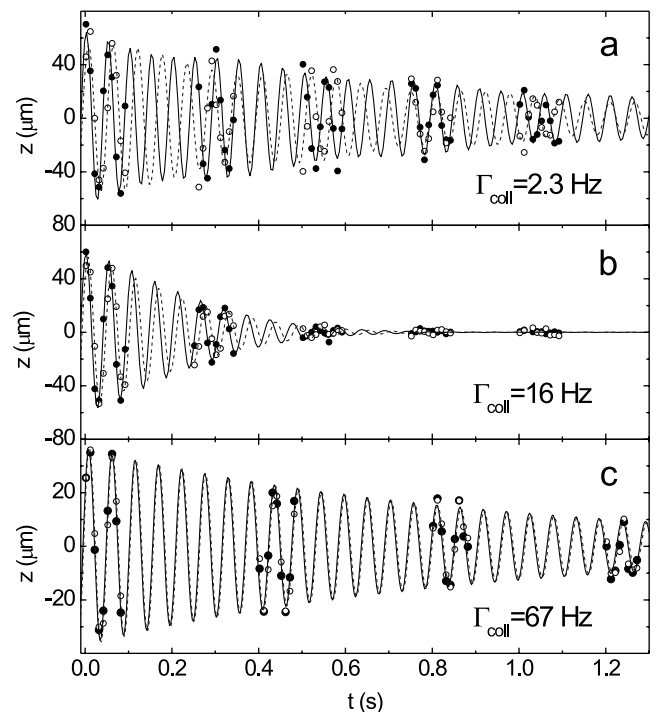


FIG. 1. Typical data showing the motion of the $|9/2\rangle$ (●) and $|7/2\rangle$ (○) cloud centers (z) following an abrupt shift of the trap potential. The cloud centers were measured following 11 ms of expansion from the magnetic trap. Traces (a) and (c) correspond to the collisionless and hydrodynamic regimes, respectively, while (b) is in the transition region. The lines are fits to a superposition of two damped, harmonic oscillator modes, with the solid and dashed lines corresponding to the motion of the $|9/2\rangle$ (●) and $|7/2\rangle$ (○) cloud centers, respectively.

The lines in Fig. 1 are fits to two modes of damped, harmonic oscillator motion. Because the gas has two components, the motion consists of a superposition of two normal modes, each with an oscillation frequency ω and an exponential damping time τ . The time-dependent center positions of the $|9/2\rangle$ and $|7/2\rangle$ gases, $z_9(t)$ and $z_7(t)$, respectively, are simultaneously fit to the following function:

$$\begin{aligned} z_9(t) &= A_1 e^{-t/\tau_1} \sin(\omega_1 t + \phi_1) \\ &\quad + A_2 e^{-t/\tau_2} \sin(\omega_2 t + \phi_2), \\ z_7(t) &= B_1 e^{-t/\tau_1} \sin(\omega_1 t + \theta_1) \\ &\quad + B_2 e^{-t/\tau_2} \sin(\omega_2 t + \theta_2). \end{aligned} \quad (1)$$

The measured frequencies, $\omega_1/2\pi$ and $\omega_2/2\pi$, for different collision rates Γ_{coll} are plotted in Fig. 2. The collision rate was varied primarily by changing the trapped gas density, either by changing the total number of atoms or by changing the radial trap strength. The average collision rate per atom in the trap is $\Gamma_{\text{coll}} = 2n\sigma v/N$, where N is the total number of atoms. The density overlap integral $n = \int n_9(\mathbf{r})n_7(\mathbf{r})d^3\mathbf{r}$ and the mean relative speed $v = 4\sqrt{\frac{k_B T}{\pi m}}$ were determined from two-dimensional Gaussian fits to the absorption images of the expanded

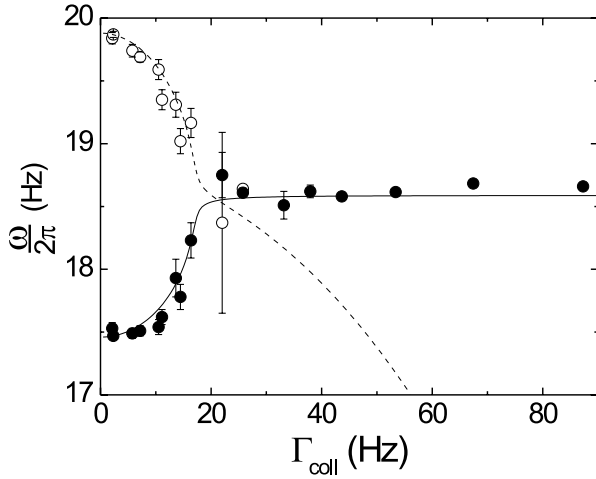


FIG. 2. Frequencies of the two excitation modes as a function of collision rate Γ_{coll} . The two mode frequencies correspond to the bare trap frequencies for the two spin states in the limit of low collision rate. As Γ_{coll} grows the mode frequencies shift away from the bare trap frequencies. However, a change in behavior occurs at $\Gamma_{\text{coll}} \approx \frac{3}{2}(\omega_9 - \omega_7)$, after which only one mode, with a frequency that is independent of Γ_{coll} , is observed. Results of a classical kinetic model (lines) are compared to the data.

gas. The collision cross section is given by $\sigma = 4\pi a^2$, where the triplet scattering length for ^{40}K is $a = 169a_0$ (a_0 is the Bohr radius) [18].

As the collision rate was increased the measured frequencies shifted from the single-particle values. This corresponds to leaving the collisionless limit where the interactions do not significantly affect the atomic motion. At larger Γ_{coll} , the observed frequencies do not depend on the collision rate. Furthermore, the frequency and damping time could only be extracted for one normal mode. This marks the transition to the regime of hydrodynamic or first sound, where Γ_{coll} is high enough to give rise to a collective excitation in the nondegenerate gas. As can be seen in Fig. 1(c) the observed collective mode corresponds to an in-phase motion of nearly equal amplitude for the two-component gases. The frequency of this mode lies between the two bare frequencies with a shift of approximately 6% compared to either ω_9 or ω_7 .

The measured exponential damping times, τ_1 and τ_2 , are plotted in Fig. 3. We observe a minimum in τ (or, correspondingly, a maximum in the damping rate) near the point where the collective mode emerges. This minimum marks the transition between the collisionless and hydrodynamic regimes and is well known in the context of quantum fluids [19]. In this transition regime the collisions are strong enough to impede single-particle excitations but are not sufficient to give rise to a well-defined sound wave. As expected the damping rate $1/\tau$ increases linearly with Γ in the collisionless regime (see inset of Fig. 3) and decreases as $1/\Gamma$ in the hydrodynamic regime.

In Figs. 2 and 3 we also compare the data to the results of a classical kinetic model. We model the system with the

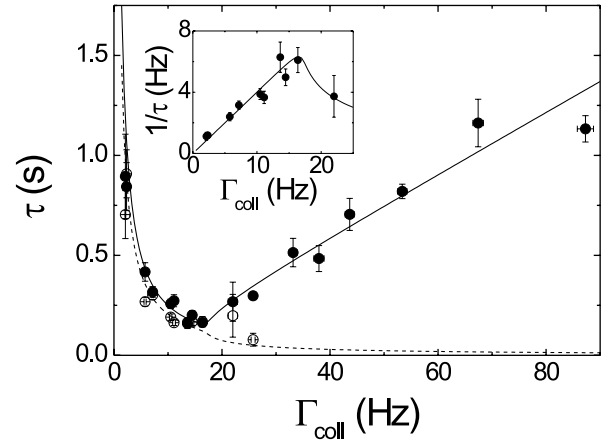


FIG. 3. Exponential damping times for the two modes of excitation. The damping times exhibit a minimum at the transition from the collisionless to the hydrodynamic regime. In the hydrodynamic regime, the measured τ is seen to depend linearly on Γ_{coll} . The inset shows the damping rate $1/\tau$ which scales linearly with Γ_{coll} in the collisionless regime.

following coupled equations of motion:

$$\begin{aligned}\ddot{z}_9 &= \frac{-F_d}{N_9 m} - \omega_9^2 z_9, \\ \ddot{z}_7 &= \frac{F_d}{N_7 m} - \omega_7^2 z_7,\end{aligned}\quad (2)$$

where

$$F_d = \frac{1}{3} m N \Gamma_{\text{coll}} (\dot{z}_9 - \dot{z}_7).$$

Here N_9 and N_7 are the numbers of atoms in each spin state, and m is the atom mass. The viscous damping force due to the collisional interactions in the gas, F_d , was derived by considering the effect of individual collision events on the cloud's center-of-mass motion and averaging over all possible collisions. The expression F_d assumes a classical gas and a small amplitude for the slosh. This model reproduces the previous results by Vichi and Stringari [11] but can also accommodate different trap frequencies for the two spin states as well as different numbers of atoms in the two components.

The solution of these equations consists of two normal modes whose frequencies and exponential damping times are shown as the lines in Figs. 2 and 3. For these theory lines the bare trap frequencies, as well as the measured temperature, number, and spin composition (45% $|9/2\rangle$), are input to the model. In addition, we allow a single free parameter that is a multiplicative factor that scales the collision rate axis. The best fit scaling corresponds to 1.30 ± 0.02 , which implies that the experimentally determined collision rates are low. This scaling factor is consistent with our estimated $\pm 50\%$ systematic uncertainty in extracting N from absorption images.

The model gives excellent agreement with the data for both frequency shifts and damping times. The model reveals the existence of a second, strongly damped collective mode in the hydrodynamic regime. This collective

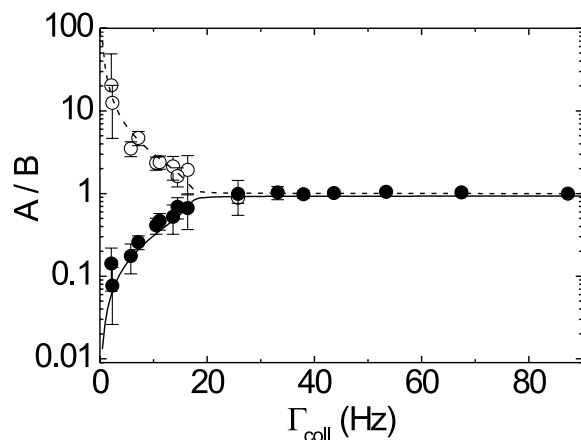


FIG. 4. Ratio of the amplitudes for $|9/2\rangle$ and $|7/2\rangle$ motion. The ratios of the amplitudes, A_1/B_1 and A_2/B_2 , are plotted versus collision rate Γ_{coll} . As Γ_{coll} increases the motion of the two gases becomes increasingly coupled until a collective mode, consisting of equal amplitude motion of the two gases, emerges in the hydrodynamic regime.

mode corresponds to the spin-dipole mode studied by Vichi and Stringari for a degenerate Fermi gas [11] and is overdamped in the hydrodynamic regime. This type of excitation is also observed as the giant dipole resonance in nuclei. The model can also answer the question of what sets the scale for the transition from the collisionless to the hydrodynamic regime in the two-component system. Varying the bare trap frequencies in the model reveals that the maximum in the damping rate, which marks the emergence of hydrodynamic behavior, scales with the frequency difference of the bare modes [20].

Finally in Fig. 4 we show the emergence of coupled motion in the two-component gas by plotting the amplitude ratios A_1/B_1 and A_2/B_2 extracted from the fits to Eq. (1). Each normal mode has some amplitude ratio for the two degrees of freedom, i.e., motion of the two-component gases. A finite ratio corresponds to a nontrivial combination and Fig. 4 reveals that the motion of the two species becomes coupled as Γ_{coll} increases.

In conclusion, by exploiting relative motion in a two-component trapped atom gas we have examined the transition from the collisionless to the hydrodynamic regime. The ability to access the full range of excitation behaviors demonstrated here will be extremely useful for studying elementary excitations of quantum gases, whether they be Bose-Einstein condensates or Fermi degenerate gases. In the Fermi system, for example, quantum statistical reduction in the collision rate could be revealed in a study of excitations in the hydrodynamic regime [21]. Although it has been previously suggested that this regime would be difficult to reach for ^{40}K experiments [11,21], the results of this work prove otherwise and in the near future we plan to explore excitation dynamics in the degenerate regime. The one-dimensional hydrodynamic regime for slosh excitations requires only moderate collision rates which are in

fact reached during our standard evaporation. Excitations in the hydrodynamic regime could also be used to reveal the onset of superfluidity [22,23]. In addition, spin excitations such as described here are interesting in their own right for quantum degenerate gases.

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