Proposal for Detection of QED Vacuum Nonlinearities in Maxwell's Equations by the Use of Waveguides

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(Received 11 July 2001; published 4 October 2001)

We present a novel method for detecting nonlinearities, due to quantum electrodynamics through photon-photon scattering, in Maxwell's equation. The photon-photon scattering gives rise to self-interaction terms which are similar to the nonlinearities due to the polarization in nonlinear optics. These self-interaction terms vanish in the limit of parallel propagating waves, but if, instead of parallel propagating waves, the modes generated in waveguides are used, there will be a nonzero total effect. Based on this idea, we calculate the nonlinear excitation of new modes and estimate the strength of this effect. Furthermore, we suggest a principal experimental setup.

DOI: 10.1103/PhysRevLett.87.171801 PACS numbers: 12.20.Ds, 42.50.Vk

According to QED, the nonclassical phenomenon of photon-photon scattering can take place due to the exchange of virtual electron-positron pairs. The observation of this scattering is a long-standing experimental challenge which has yet to be met. Photon-photon scattering is a second order effect (in terms of the fine structure constant $\alpha \equiv e^2/2\varepsilon_0 hc \approx 1/137$), and it can in standard notation be formulated in terms of the Euler-Heisenberg Lagrangian density [1,2]

$$\mathcal{L} = \varepsilon_0 \mathcal{F} + \xi(4\mathcal{F}^2 + 7\mathcal{G}^2), \tag{1}$$

where

$$\xi \equiv \frac{20\alpha^2\varepsilon_0^2\hbar^3}{45m_e^4c^5},$$

$\mathcal{F} \equiv \frac{1}{2}(E^2 - c^2B^2)$, $\mathcal{G} \equiv c\mathbf{E}\cdot\mathbf{B}$, and $\mathcal{F}^2$ and $\mathcal{G}^2$ are the QED corrections. Here e is the electron charge, c the velocity of light, h the Planck constant, and m_e the electron mass. We note that $\mathcal{F} = \mathcal{G} = 0$ in the limit of parallel propagating waves. The latter terms in (1) represent the effects of vacuum polarization and magnetization, and the QED corrected Maxwell's vacuum equations take the classical form using

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}, \qquad \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M},$$

where $\mathbf{P}$ and $\mathbf{M}$ are of third order in the field amplitudes $\mathbf{E}$ and $\mathbf{B}$, and $\mu_0 = 1/c^2\varepsilon_0$. Furthermore, they contain terms $\mathcal{F}$ and $\mathcal{G}$ such that $\mathbf{P} = \mathbf{M} = 0$ in the limit of parallel propagating waves. It is therefore necessary to use other waves in order to obtain an effect from these QED corrections. Several attempts have been presented in the literature over the years [3–8], where Refs. [3–6] mainly focused on principal issues, whereas the experimental possibilities for detection have been discussed in [7,8]. Soljacic and Segev concluded that, using their mechanism [7], the detection of the QED nonlinearities will be technologically viable within 10 to 15 years, provided that the laser power increases steadily. In this work we suggest the use of waveguides as an experimental setup, something which, as far as the authors know, has not been discussed previously for this purpose. The idea of using a waveguide is to achieve a resonant coupling between the parallel propagating waves of different frequencies. We calculate the generated electromagnetic field for a rectangular waveguide, using the TE_{01} and TE_{10} modes as pump waves. In the proposal for the experimental setup, the waveguide is replaced by a cavity, in order to prevent the convective loss of energy, and thereby maximize the saturated amplitude of the excited mode. The saturation level due to a finite conductivity of the cavity walls is estimated. We find that the excited mode can be detected even for moderate levels of the pump mode amplitudes, i.e., for field strengths that can be supported by the cavity walls.

In a medium with polarization $\mathbf{P}$ and magnetization $\mathbf{M}$ the general wave equations for $\mathbf{E}$ and $\mathbf{B}$ are

$$\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \nabla^2\mathbf{E} = -\mu_0\left[\frac{\partial^2\mathbf{P}}{\partial t^2} + c^2\nabla(\nabla\cdot\mathbf{P}) + \frac{\partial}{\partial t}(\nabla\times\mathbf{M})\right], \tag{2}$$

and

$$\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} - \nabla^2\mathbf{B} = \mu_0\left[\nabla\times(\nabla\times\mathbf{M}) + \frac{\partial}{\partial t}(\nabla\times\mathbf{P})\right]. \tag{3}$$

Furthermore, the effective polarization and magnetization in vacuum due to photon-photon scattering induced by the exchange of virtual electron-positron pairs are given by (see, e.g., Ref. [7])

$$\mathbf{P} = 2\xi[2(E^2 - c^2B^2)\mathbf{E} + 7c^2(\mathbf{E}\cdot\mathbf{B})\mathbf{B}],$$

and

$$\mathbf{M} = -2c^2\xi[2(E^2 - c^2B^2)\mathbf{B} + 7(\mathbf{E}\cdot\mathbf{B})\mathbf{E}].$$

Next we consider propagation in a rectangular waveguide with dimensions x_0 and y_0 (i.e., the region $0 \le x \le x_0$, $0 \le y \le y_0$ is vacuum surrounded by walls that, as a starting point, are assumed to be perfectly conducting). We assume that the TE_{10} and TE_{01} modes act as pump waves with distinct frequencies. To lowest order (i.e., neglecting the vacuum nonlinearities) the fields are

$$B_{1z} = \tilde{B}_{1z} \cos\left(\frac{\pi x}{x_0}\right) \exp[i(k_1 z - \omega_1 t)] + \text{c.c.}, \tag{4a}$$

$$B_{1x} = -\left(\frac{i k_1 x_0}{\pi}\right) \tilde{B}_{1z} \sin\left(\frac{\pi x}{x_0}\right) \exp[i(k_1 z - \omega_1 t)] + \text{c.c.}, \tag{4b}$$

$$E_{1y} = \left(\frac{i x_0 \omega_1}{\pi}\right) \tilde{B}_{1z} \sin\left(\frac{\pi x}{x_0}\right) \exp[i(k_1 z - \omega_1 t)] + \text{c.c.}, \tag{4c}$$

together with $\omega_1^2 = k_1^2 c^2 + \pi^2 c^2 / x_0^2$ for the TE_{10} mode, and

$$B_{2z} = \tilde{B}_{2z} \cos\left(\frac{\pi y}{y_0}\right) \exp[i(k_2 z - \omega_2 t)] + \text{c.c.}, \tag{5a}$$

$$B_{2y} = -\left(\frac{i k_2 y_0}{\pi}\right) \tilde{B}_{2z} \sin\left(\frac{\pi y}{y_0}\right) \exp[i(k_2 z - \omega_2 t)] + \text{c.c.}, \tag{5b}$$

$$E_{2x} = -\left(\frac{i y_0 \omega_2}{\pi}\right) \tilde{B}_{2z} \sin\left(\frac{\pi y}{y_0}\right) \exp[i(k_2 z - \omega_2 t)] + \text{c.c.}, \tag{5c}$$

together with $\omega_2^2 = k_2^2 c^2 + \pi^2 c^2 / y_0^2$ for the TE_{01} mode, where c.c. stands for complex conjugate. Here we have denoted the wave amplitudes, which to lowest order are constants, by $\tilde{B}_{z1}$ and $\tilde{B}_{z2}$, respectively. Substituting the linear expression for the fields into the cubic nonlinear terms, we note that there will be perturbations with frequency and wave number (ω_3, k_3), where the possible combinations are $(\omega_3, k_3) = (\omega_1, k_1)$, (ω_2, k_2), $(2\omega_1 \pm \omega_2, 2k_1 \pm k_2)$, and $(2\omega_2 \pm \omega_1, 2k_2 \pm k_1)$. If there is a small perturbation (e.g., of the order of 10^{-15}) of the amplitude of any of the original TE modes, it would probably be a too difficult task to measure such an effect, whereas the appearance of a distinctly new frequency, although with small amplitude, will be easier to detect. We therefore concentrate on the two latter combinations, which are physically equivalent. Furthermore, if any of these combinations satisfies the dispersion relation for a natural mode of the waveguide, the amplitude of this resonantly driven mode will be much larger than the others. We therefore, for definiteness, choose to consider the matching condition

$$(\omega_3, k_3) = (2\omega_1 - \omega_2, 2k_1 - k_2), \tag{6}$$

where the dimensions of the waveguide are assumed to be chosen such as to make (ω_3, k_3) a natural mode of the waveguide.

Using the pump modes (4) and (5), we can get source terms in the wave equations either for a TE_{01} mode or for a TM_{01} mode. For the latter case, however, the frequency matching conditions and dispersions relations cannot be fulfilled simultaneously for real values of all wave numbers, and thus we consider the excitation of a TE_{01} mode, i.e., we let

$$\omega_3^2 = k_3^2 c^2 + \frac{\pi^2 c^2}{y_0^2}.$$

Thus, using Eqs. (4) and (5), we can evaluate the source terms in Eqs. (2) and (3). For a waveguide of finite length fulfilling the boundary condition of no incoming wave with frequency ω_3 at the waveguide starting point $z = 0$, we then have spatial growth. The ansatz $B_{3z} = \tilde{B}_{3z}(z) \exp[i(k_3 z - \omega_3 t)] + \text{c.c.}$ in (3) thus gives a linear spatial growth

$$\tilde{B}_{3z}(z) = \frac{izV}{2k_3} \tilde{B}_{1z}^2 \tilde{B}_{2z}^*, \tag{7}$$

provided k_3 is not too small. Here the coupling constant V is

$$V \equiv \frac{4\xi}{\varepsilon_0}\left(4\omega_3^2 + \frac{k_3^2 c^2}{2} + 2\omega_3\omega_2 - \frac{7x_0^2}{2y_0^2} k_1^2 c^2 - k_2 k_3 c^2\right),$$

and the star in (7) denotes complex conjugation.

When designing parameters for an experiment, it might be tempting to choose parameters such that the dispersion relation is fulfilled for $k_3 = 0$, in which case we instead get a quadratic spatial growth of the amplitude. However, the case $k_3 = 0$ is not the most interesting choice for two reasons. First, the group velocity of and thereby the energy flux of the excited mode is proportional to k_3. Second, it is impossible for $\omega_3 > \omega_1, \omega_2$ to hold when $k_3 = 0$. The reason for requiring the excited wave to have a higher frequency than the others is that we then have the possibility to use waveguide filtering (see below) in order to measure the excited wave without any disturbing signals from the pump waves. Since k_3 cannot be small as compared to $1/x_0$ for $\omega_3 > \omega_1, \omega_2$ to hold, the spatial growth in (7) may be too slow for practical purposes, and we are therefore motivated to consider a cavity rather than a waveguide. Since the waves propagating in positive and negative directions are identical in that cavity, the boundary conditions imply temporal rather than spatial growth. If we assume that all waves have $k > 0$ (are propagating in the positive z direction) in the waveguide example, the coupling coefficient in a cavity can be found from the waveguide result, simply by noting that the positive propagating part of the standing pump waves couples to the positive propagating part of the excited standing wave, and vice versa. Since V is a quadratic function of the wave numbers, the same coupling strengths apply for standing waves $\sim\sin(n\pi z/L)\exp[-i\omega t]$, where L is the length of the cavity, as for propagating waves $\sim\exp[i(kz - \omega t)]$, if we just let the wave number be $k = \pm n\pi/L$ where the sign corresponds to the direction of propagation, and n is a positive integer. For design purposes we must keep in mind, however, that we now have additional constraints relating the frequencies and dimensions since the wave numbers cannot be chosen continuously. If we let $\tilde{B}_z$ represent the standing wave amplitude, and modify the ansatz such that the excited amplitude depends on time, we find that the temporal growth in a cavity is

$$\tilde{B}_{3z}(t) = \frac{itc^2V}{2\omega_3} \tilde{B}_{1z}^2 \tilde{B}_{2z}^* .$$

Saturation occurs when the amplitude is large enough for linear damping due to a finite conductivity to balance the driving term. The saturated equilibrium amplitude is found to be

$$|\tilde{B}_{3z\mathrm{eq}}| = \frac{c^2V}{2\omega_3\Gamma} |\tilde{B}_{1z}^2| |\tilde{B}_{1z}|, \tag{8}$$

where Γ is the linear damping rate of mode 3. If the linear damping is due to a finite conductivity σ of the walls, we have

$$\Gamma = U \frac{\omega_3 R}{\mu_0 c}$$

in the regime $\Gamma/\omega_3 \ll 1$, where $\delta \equiv \mathrm{Re}[(i\mu_0\sigma\omega_3)^{1/2}]$ is the skindepth, $R = \mathrm{Re}(\sigma^{-1})/\delta$ is the surface resistance of the cavity walls, and U is a function of the geometry that is of the order of unity.

In order to avoid problems due to the large amplitude pump signals when trying to measure the excited mode, it is convenient to use a slightly modified cavity rather than an idealized cubic cavity. Such a cavity could consist of two parts; one cavity (I), in which the signal is generated, attached to another cavity (II), acting as a waveguide filter. The dimensions of cavity I should be chosen such as to keep the frequency of the excited mode above cutoff in cavity II, whereas the pump modes should be below cutoff. By measuring the excited signal far enough into cavity II (in practice 30–40 pump decay lengths), the pump waves effectively vanish, and we can forget about the disturbing influence of the pump waves. However, note that a certain fine tuning of the length of cavity II might be necessary to keep a maximally efficient phase relation between the excited eigenmode and the pump waves in cavity I.

Next, we demonstrate that the experimental model setup gives signals that can be detected with presently available technology. High performance, i.e., large electromagnetic fields combined with low dissipative losses, can be found in superconducting cavities, which among other things are used for particle accelerator purposes [9]. Adopting data from these experiments, we assume that the pump waves have a field strength $E_{\mathrm{crit}} \sim 30$ MV/m, i.e., close to the maximum that can be tolerated by the walls without field emissions. For a cavity resistance $R \sim 1$ nΩ, corresponding to superconducting niobium at a temperature 1.4 K and a frequency $\omega_3 \sim 2 \times 10^{10}$ rad/s, we find from Eq. (8) that the saturated energy flux P_3 of mode 3 is of the order of $P_3 \sim 10^{-6}$ W/m^2. Here we have used the simple order of magnitude estimate that all wavelengths are comparable to the dimensions x_0 and y_0 when evaluating V. Clearly this energy flux is above the detection level by several orders of magnitude. Note, however, the importance of the superconducting walls for the output level of the excited mode. For copper at room temperature, the cavity resistance increases by a factor of $\sim 10^7$ as compared to the above example, and consequently the energy flux of the excited mode falls by a factor $\sim 10^{-14}$. In this latter case it is questionable whether the excited signal can be detected.

To our knowledge, it has not been possible to verify the Euler-Heisenberg Lagrangian density experimentally. The above calculation of the QED mode coupling strength and the subsequent estimations suggest that it can be very suitable to use two pump modes in a superconducting cavity for this purpose. The parameters of the problem should be designed such as to simultaneously fulfill the dispersion relations of each mode together with the matching condition (6). Naturally, care must be taken when drawing the conclusions, since there are certain effects that we have not yet addressed. For example, in the present model the conductivity of the walls is linear, but in principle there might be a nonlinear contribution to the conductivity that could give rise to a small signal at the same frequency as the QED contribution. On the other hand, to our knowledge, there are no theoretical or experimental reports of such effects. Second, in reality the vacuum in the waveguide will not be perfect, and in principle this may lead to dielectric breakdown for the pump field strength considered in the above estimate. However, we do not expect this particular effect to be a serious threat against our proposal, since similar electric field strengths have been reached in present experiments [9], and the pump field strength that is actually needed is much less than the one considered in the estimate. Thus we conclude that it is likely that the effect of photon-photon scattering in vacuum due to the exchange of virtual electron-positron pairs can be measured using existing technology.

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