Time-Domain Atom Interferometry across the Threshold for Bose-Einstein Condensation

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We have performed time-domain interferometry experiments with matter waves trapped in a harmonic potential above and below the Bose-Einstein phase transition, by means of the method of separated oscillating fields, with a variable time delay T. We observe the oscillations of the population between two internal Zeeman states versus the delay T to be rapidly depleted both below and slightly above Bose-Einstein condensation. We give a quantitative explanation in terms of the phase evolution due to the entanglement between the internal and external degrees of freedom.

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The question about the coherence of Bose-Einstein condensates (BECs) [1] and the characterization of their phase properties has drawn considerable attention in recent literature.

The first evidence of a definite phase for weakly interacting condensates dates back to the experiment of the MIT group [2], where high contrast matter-wave interference fringes were observed in the density distribution of two freely expanding BECs. Subsequent experiments performed at JILA [3,4] measured the relative phase of two condensates in different internal (hyperfine) states experiencing almost the same trapping potential.

Other experiments have further investigated this subject [5–8]. In particular, the NIST group has recently measured the evolution of the spatial profile of the phase of BECs, by using a Bragg interferometer [7].

In a recent Letter [9], we reported an experimental method for a sensitive and precise investigation of the interaction between two Bose-Einstein condensates [10]. Here we present an interferometry experiment performed in the time domain to study the phase evolution of the same system. At the same time, this work allows one to gain a deeper insight into decoherence in Ramsey interferometry [11] with ultracold atoms across the BEC phase transition. It is well known that Ramsey fringes are readily observable with a sample at room temperature. The reason is that Ramsey signals rely on the persistence of coherence between two distinct atomic levels, no matter what the state is of the particle's center-of-mass. This is true only when the internal and external degrees of freedom are uncoupled. Whenever the latter condition fails, a depletion of the Ramsey fringes visibility can occur and has indeed been observed [12]. Thus, the entanglement of external and internal states of atoms trapped in a magnetic potential is the basis for the use of the Ramsey method to characterize the phase properties of a condensate and a thermal cloud near BEC. In the JILA experiment [3], such an entanglement caused the observed Ramsey fringes in the population of the two hyperfine levels to undergo a loss of contrast, as a consequence of the reduced spatial overlap between the two BECs, due to their mutual repulsion.

In our experiment we assist to a similar depletion but on a much faster time scale, of the order of tens of μs, that cannot be explained with the reduction of spatial overlap. We prepare, and subsequently probe the system with a sequence of two identical radio frequency (rf) pulses, separated by a delay time T. The relative phase accumulated by the condensates produces Ramsey fringes in the population of each level, as a function of the delay T. We observe a strong reduction of the fringe amplitude even at very short times, when the condensates are still almost completely overlapping, due to the relative velocity acquired by the condensates. We compare this reduction with that of a cloud of thermal atoms, at a temperature 3 times larger than the phase-transition temperature $\Theta \sim 3\Theta_c$.

We prepare a condensate of typically 2×10^5 ^{87}Rb atoms in the $|F = 2, m_f = 2\rangle$ hyperfine level, confined in a four-coils Ioffe-Pritchard trap elongated along the z symmetry axis [13,14]. The axial and radial frequencies for the $|2\rangle$ state are $\omega_{z2} = 2\pi \times 13$ Hz and $\omega_{\perp 2} = 2\pi \times 131$ Hz, respectively, with a magnetic field minimum of 2.86 G. Then, we apply an rf pulse to split the initial condensate into a coherent superposition of different Zeeman $|m_f\rangle$ sublevels of the $F = 2$ state. The atoms transferred in a different sublevel move away from the $|2\rangle$ equilibrium position with an acceleration that depends on their m_f value. Thus, the state of motion becomes entangled with the internal atomic state.

In this experiment, we use a 24 cycles rf pulse at 2 MHz, which quickly leaves the $|2\rangle$ state with 41% of the initial number of atoms, transferring an equal part of atoms (41%) to the $|1\rangle$ state, 15% to the $|0\rangle$, 3% to the $|-1\rangle$, and 0% to the $|-2\rangle$ states, respectively.

Even though all five levels should be taken into account in the early stages of the evolution (before the atoms in the $|0\rangle$, $|-1\rangle$, and $|-2\rangle$ states leave the trap), the basic features can be explained by considering only the dynamics of the two most populated levels, i.e., $|1\rangle$ and $|2\rangle$. We will discuss later how the other three levels affect the overall behavior of the system.

We describe our double condensate system by a spinor wave function, where the upper and lower components refer to states $|2\rangle$ and $|1\rangle$, respectively. Immediately after the first rf pulse, the wave function can be written as

$$\Psi(\boldsymbol{r}; t = 0) = \frac{\sqrt{N_0}\, u(\boldsymbol{r}; 0)}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad (1)$$

where $\sqrt{N_0}\, u(\boldsymbol{r})$ is the equilibrium wave function of the N_0 atoms of the initial $|2\rangle$ condensate, in the Thomas-Fermi regime [15]. As the rf pulse is much shorter than the oscillation periods of the harmonic trap, we could safely take the spatial wave function $u(\boldsymbol{r})$ to remain unchanged and flip only the internal state.

In the subsequent free evolution, the relative phase between the two components accumulates with a rate proportional to the difference of chemical potentials $\mu_2 - \mu_1$. Moreover, $u(\boldsymbol{r}; 0)$ is no longer the equilibrium wave function for $|2\rangle$ nor for $|1\rangle$: The spatial wave functions evolve as dictated by two coupled Gross-Pitaevskii equations [10] into $u(\boldsymbol{r}; t)$ and $v(\boldsymbol{r}; t)$, respectively.

By applying a second rf pulse, identical to the first, after a time delay T, we suddenly mix the two components,

$$\psi(\boldsymbol{r}; T) = \frac{\sqrt{N_0}}{2} \begin{pmatrix} u(\boldsymbol{r}; T) e^{-i\omega_0 T} + i v(\boldsymbol{r}; T) \\ i u(\boldsymbol{r}; T) e^{-i\omega_0 T} + v(\boldsymbol{r}; T) \end{pmatrix}, \quad (2)$$

with $\omega_0 = (\mu_2 - \mu_1)/\hbar$.

We then separate the two internal states by holding them in the magnetic trap for a suitable time to exploit their different dynamics. In absorption images taken after releasing the trap, the two internal states appear as distinct clouds, allowing the determination of the atom number in each state. The fractional populations show Ramsey fringes at frequency ω_0 [16] (see Fig. 1).

For delay times $T < 0.1$ ms, i.e., much shorter than the harmonic periods, the only relevant effect of the external degrees-of-freedom evolution is that, due to the differential gravitational "sagging," the $|1\rangle$ condensate acquires a downward time-dependent momentum $-\hbar q(t)$. In particular, we can neglect the loss of spatial overlap arising from the $|1\rangle$ displacement. As we will discuss later, this approximation is justified by the results of numerical simulations based on the Gross-Pitaevskii equation (GPE). Then, taking $u(\boldsymbol{r}; T) = u(\boldsymbol{r}; 0)$, $v(\boldsymbol{r}; T) = i u(\boldsymbol{r}; 0) \exp[-i q(T) y]$ and given the normalization $\int |u(\boldsymbol{r}; 0)|^2\, dy = 1$, we have

$$\begin{aligned} N_2(T) &= N_0 \tfrac{1}{2}[1 - A_c(T)\cos(\omega_0 T)], \\ N_1(T) &= N_0 - N_2(T), \end{aligned} \quad (3)$$

with the slow time-dependent amplitude,

$$A_c(T) = \int |u(\boldsymbol{r}; 0)|^2 \cos[q(T) y]\, d\boldsymbol{r}\,. \quad (4)$$

The amplitude of the quadrature component vanishes because $|u(\boldsymbol{r}; 0)|^2$ is an even function of the y coordinate.

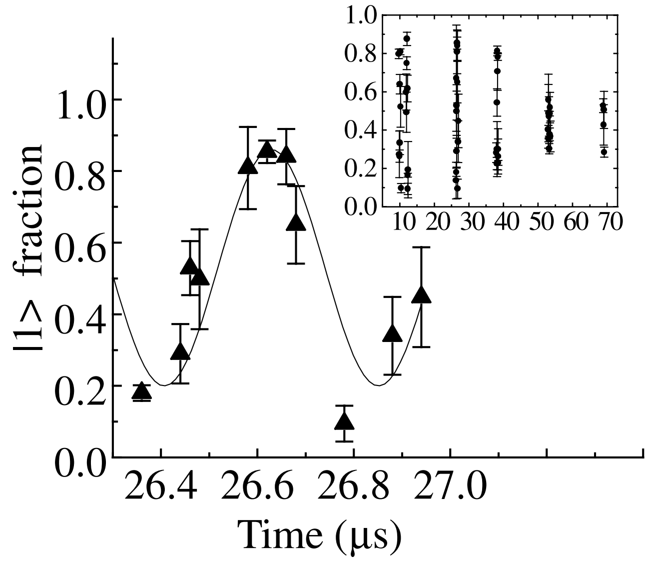

FIG. 1. Experimental data: $N_1/(N_2 + N_1)$ fraction versus the time delay between the two rf pulses for $T \sim 26\ \mu$s. Likewise, we have sampled several oscillation periods at different delay times T (the inset shows all data).

The amplitude $A_c(T)$ decays as the relative velocity increases and q^{-1} becomes of the order of the vertical extension of the original condensate. Eventually, Ramsey fringes are completely washed out. The relative displacement would give the same result, but only at later times, for T of the order of ms.

In Fig. 2 we plot the experimental data of the fringe amplitudes for the fraction $N_2/(N_1 + N_2)$ versus the time delay T. To measure the fringe amplitude at T, we sample one fringe period with about ten points (see inset of Fig. 1). Each of these points is averaged over a few (typically five) experimental runs. Upon fitting with a sine wave, we extract the amplitudes and their error bar. To compare with the values predicted by Eq. (4), we rescale the latter

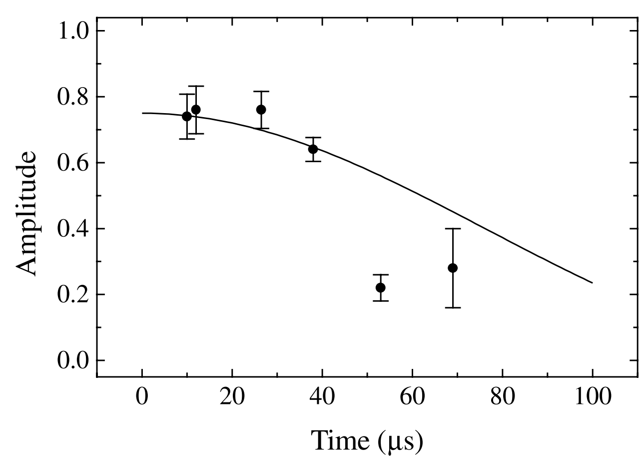

FIG. 2. Peak-to-peak amplitude of the oscillating observable $N_2/(N_2 + N_1)$, i.e., the $|2\rangle$ condensate population normalized to the total number of atoms in $|2\rangle$ and $|1\rangle$: experiment and calculation (solid line, rescaled by 0.75).

by a factor 0.75, chosen to match the experimental data around $T = 0$. Our theoretical analysis, despite being very simple, predicts a damping time (94 μs at $1/e$) that agrees with experimental results within 30%. However, there appears a significant discrepancy at $T = 50$ μs that we cannot fully explain.

Actually, for the double condensate we can refine the above model to include the mean-field repulsion in the evolution of the wave functions: To this end, we numerically integrate two coupled GPEs, according to the model described in Ref. [10]. On general ground, in the first stages of the evolution the phase of each component can be written as a quadratic form in the spatial coordinates [7,15], describing the center-of-mass motion (linear terms) and the mean-field expansion/contraction in the Thomas-Fermi regime [15] (quadratic terms). The results of the numerical simulations, in the time range considered here, show the following: (i) as the $|2\rangle$ condensate is at rest, its phase remains almost uniform, and only for later times ($t \simeq 0.5$ ms) develops a negative curvature, due to the contraction caused by the transfer of atoms to the other levels; (ii) the phase of the $|1\rangle$ condensate is dominated by a linear term, representing its overall motion along the vertical y direction,

$$\phi_1(\boldsymbol{r};t) \simeq \frac{m}{\hbar}[v_0(t) + \delta v(t)]y\,, \tag{5}$$

where $v_0(t)$ is the velocity acquired during the fall in the trapping potential,

$$v_0(t) = -\frac{g}{\sqrt{2}\,\omega_{\perp 2}}\sin\!\left(\frac{\omega_{\perp 2}t}{\sqrt{2}}\right), \tag{6}$$

and $\delta v(t)$, negligible for $t < 0.5$ ms, is due to the mutual repulsion between the two condensates. Thus, the Gross-Pitaevskii (GP) simulations confirm that, at short times, the simple model adopted above correctly describes the basic features of the system.

The phase term (5) is responsible for washing out completely the Ramsey fringes even at very short times, when the condensates are still almost completely overlapping [as shown in Fig. 3 the overlap between the two wave packets is substantial even at T as long as 0.5 ms].

Physically, in a Ramsey experiment the transfer of population at the second interaction is driven by the relative phase between the two involved states. When this phase varies across the spatial extension of the wave packet, there are regions of alternated positive and negative interference, with a vanishing net transfer of atoms.

For a thermal cloud, we can consider the system to be in a given quantum state and then take an ensemble average over all the accessible states. This way, we need only to replace the time-dependent amplitude (4) with

$$A_{\rm th}(T) = \frac{1}{N_0}\sum_{\{n\}}\int f_{\{n\}}|\Psi_{\{n\}}(\boldsymbol{r})|^2\cos[q(T)y]\,d\boldsymbol{r}\,, \tag{7}$$

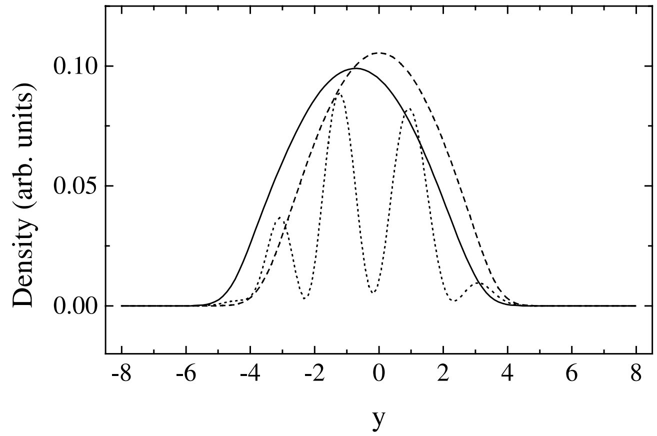

FIG. 3. Overlap of the two condensates at the second rf pulse. Density profiles along the vertical axis y of condensates $|2\rangle$ (dashed line) and $|1\rangle$ (solid line) before the pulse, and of condensate $|1\rangle$ (dotted line) immediately after. The curves are obtained by solving the GP equations for the two-state model in Ref. [10]. Lengths are given in units of $a_{\perp 2} = [\hbar/(m\omega_{\perp 2})]^{1/2} = 0.94$ μm; $T = 0.5$ ms.

where $f_{\{n\}} = \{\exp[(\epsilon_{\{n\}} - \mu)/k\Theta] - 1\}^{-1}$ is the Bose mean occupation number of the harmonic trap eigenstate $\Psi_{\{n\}} = \psi_{n_1}(x)\psi_{n_2}(y)\psi_{n_3}(z)$ with energy $\epsilon_{\{n\}}$, μ is the chemical potential, given by the normalization $\sum_{\{n\}} f_{\{n\}} = N_0$, and Θ is the temperature. By carrying out the integration over x, z and the sum over the corresponding quantum numbers n_1, n_3, we find

$$A_{\rm th}(T) = \frac{(k\Theta)^2}{N_0\hbar^2\omega_x\omega_z}\sum_{n_2} g_2\!\left[\exp\!\left(\frac{\mu - n_2\hbar\omega_y}{k\Theta}\right)\right] \times \int |\psi_{n_2}(y)|^2\cos[q(T)y]\,dy\,, \tag{8}$$

the $g_2(x) = \sum_{l=1}^{\infty} x^l/l^2$ function being the result of replacing the discrete sum over n_1, n_3 with a double integral. In principle, one should allow for the rf pulses to act differently on the different harmonic oscillator eigenstates as the detuning varies. This would introduce $\{n\}$-dependent weights in Eq. (7). However, we have verified that, for the relevant levels at $\Theta \simeq 0.4$ μK, these weights are equal within a few percent.

We note here that Eq. (7) can be rewritten:

$$A_{\rm th}(T) = \frac{1}{N_0}\int n(\boldsymbol{r})\cos[q(T)y]\,d\boldsymbol{r}\,, \tag{9}$$

where $n(\boldsymbol{r})$ is the spatial density. In this form, it is evident that the thermal cloud behaves as a coherent wave packet, the reason being that the displacement occurring between the two rf pulses is much less than the thermal coherence length ($\lambda_{\rm th} = h/\sqrt{2\pi mk\Theta} \simeq 0.3$ μm).

The above analysis relies on the fact that, for the noncondensed sample, where typical densities are a factor 30 lower than those of BEC, we can neglect the atom-atom interactions and use single-particle eigenfunctions $\Psi_{\{n\}}$.

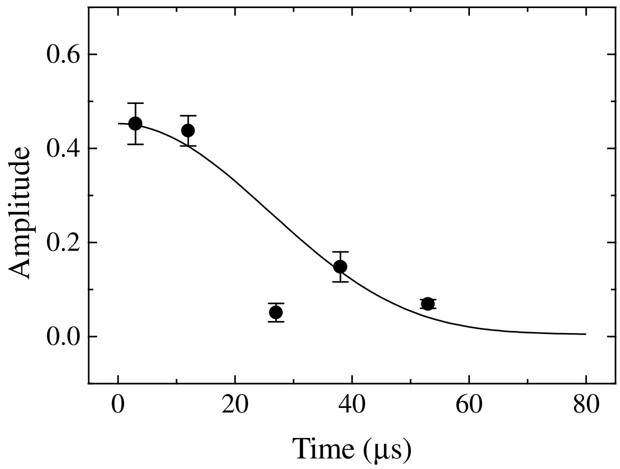

FIG. 4. Peak-to-peak amplitude of the oscillating observable $N_2/(N_2 + N_1)$, for thermal atoms at $\Theta = 0.4\ \mu\text{K} \simeq 3\Theta_c$: experiment and calculation (solid line, rescaled by 0.45).

In Fig. 4 we plot the amplitude of the observed $N_2/(N_2 + N_1)$ fringes for several values of the time delay T between the two rf pulses, and we compare with the corresponding predictions given by Eq. (7), rescaled by a factor 0.45 to match the experimental values around $T = 0$. As for the damping time, we have a satisfactory agreement: Besides one point lying far outside the theoretical curve, we believe that the cause of the fringes' loss of contrast is well understood. As for the observed amplitudes being smaller than the predictions, we remind that part of the atoms end up in the $m_f = 0, -1, -2$ Zeeman sublevels after the second rf pulse; as a consequence, the peak-to-peak amplitude of the $N_2/(N_1 + N_2)$ fraction cannot exceed 0.87, both for the condensates and the thermal clouds. However, this represents only a partial explanation of the discrepancy.

As a last remark, we point out that our system is suitable to study the possibility of a revival of the Ramsey fringes after the condensates have been spatially well separated. As the center of mass of the $|1\rangle$ condensate undergoes harmonic oscillations around its equilibrium position [9,10], it comes back to rest at its initial position. According to the above description, one should expect a revival of the fringes in the relative population when the condensates $|1\rangle$ and $|2\rangle$ come to overlap again with almost vanishing velocity. The numerical solution of the GP equations of the two-level model shows indeed that over time scales of tens of ms the Ramsey fringes are characterized by collapse and revival [17].

In summary, we have studied the Ramsey interference of atomic clouds across the BEC phase transition, with a system where the two involved states have equilibrium positions far apart. We have shown that the phase pattern created on the moving wave packet by its acquired velocity washes out the Ramsey fringes of the fractional populations well before the spatial overlap decreases. By repeating the same experiment on a thermal cloud at 3 times the condensation temperature, we have observed that the same mechanism is responsible for an even faster damping of fringe contrast. In this respect, there is no substantial difference between a thermal cloud and a Bose condensate. The analogy with light optics is straightforward: To observe interference between two paths, one needs only the difference between the path lengths not to exceed the coherence length.

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[1] M. H. Anderson *et al.,* Science **269**, 1989 (1995); K. B. Davis *et al.,* Phys. Rev. Lett. **75**, 3969 (1995); C. C. Bradley, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. **78**, 985 (1997).
[2] M. R. Andrews *et al.,* Science **275**, 637 (1997).
[3] D. S. Hall, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **81**, 1543 (1998).
[4] E. A. Cornell, D. S. Hall, M. R. Matthews, and C. E. Wieman, cond-mat/9808105.
[5] E. W. Hagley *et al.,* Phys. Rev. Lett. **83**, 3112 (1999).
[6] Y. Torii *et al.,* Phys. Rev. A **61**, 041602(R) (2000).
[7] J. E. Simsarian *et al.,* Phys. Rev. Lett. **85**, 2040 (2000).
[8] K. Bongs *et al.,* Phys. Rev. A **63**, 031602 (2001).
[9] P. Maddaloni *et al.,* Phys. Rev. Lett. **85**, 2413 (2000).
[10] M. Modugno *et al.,* Phys. Rev. A **62**, 063607 (2000).
[11] N. F. Ramsey, *Molecular Beams* (Clarendon, Oxford, 1956).
[12] P. D. Featonby *et al.,* Phys. Rev. Lett. **81**, 495 (1998).
[13] C. Fort *et al.,* Europhys. Lett. **49**, 1 (2000).
[14] F. Minardi, C. Fort, P. Maddaloni, and M. Inguscio, in *Bose-Einstein Condensates and Atom Lasers,* edited by S. Martellucci *et al.* (Kluwer Academic Plenum Publishers, New York, 2000).
[15] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
[16] We note that, unlike usual Ramsey experiments where the signal oscillates at the frequency difference between the atomic frequency and the interrogation frequency, here we record oscillations at the atomic frequency. The reason is that the phase of the second pulse remains the same, no matter what the time delay T. Thus, varying T, we just scan the phase of the atomic dipole.
[17] We defer a more complete discussion on this point to a forthcoming publication.