Nodal Quasiparticles in Stripe Ordered Superconductors

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We study the properties of a quasi-one-dimensional superconductor which consists of an alternating array of two inequivalent chains. This model is a simple caricature of a striped high temperature superconductor, and is more generally a theoretically controllable system in which the superconducting state emerges from a non-Fermi-liquid normal state. Even in this limit, "d-wave-like" order parameter symmetry is natural, but the superconducting state can either have a complete gap in the quasiparticle spectrum, or gapless "nodal" quasiparticles. We also find circumstances in which antiferromagnetic order (typically incommensurate) coexists with superconductivity.

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A key feature of the cuprate high temperature superconductors is that the "normal" state is not well described as a Fermi liquid. Therefore, to understand the physics of the transition temperature, the brilliantly successful BCS theory, which presupposes [1] that the normal state is a Fermi liquid, must be modified. Although the cuprate high temperature superconductors are layered materials, self-organized one-dimensional structures [2], or "stripes," have been widely observed, making plausible the idea that at intermediate scales these materials can be thought of as quasi-one-dimensional. The only theoretically well understood example of a superconducting system with a non-Fermi-liquid (NFL) normal state is a quasi-one-dimensional superconductor. Here the "normal state" is governed by the quantum critical physics of a decoupled set of one-dimensional electron gases (1DEG); superconducting long-range order is triggered by interchain coupling, and accompanied by a crossover to higher dimensional physics [3].

Several salient features of the high temperature superconductors are naturally understood from this viewpoint [3]. The fact [4] that in under and optimally doped materials, quasiparticles in the gap antinodal regions of the Brillouin zone (BZ) exist only in the superconducting state and have a quasiparticle weight $[Z(T,x)]$ which vanishes as the transition point is approached, either as a function of the doped hole concentration, x, or of the temperature, T, can, in our opinion, be understood only if the normal state has no well defined quasiparticle excitons. The extraordinarily one-dimensional dispersion apparent [5] in the quasiparticle spectrum in this region of the BZ is independent evidence of a quasi-1D origin for the NFL behavior. That the zero temperature superfluid density, and with it T_c, is roughly proportional to x is also simply understood in this way, as is the fact that the pairing scale, Δ_0, and the superconducting T_c appear to be distinct energy scales in the problem.

We thus propose studying the high temperature superconductors by adiabatic continuity from the quasi-one-dimensional limit. Conceptually, we imagine introducing an explicit symmetry breaking field of strength h into the physical Hamiltonian, so that for h large the problem is literally quasi-one-dimensional, and can hence be *solved*

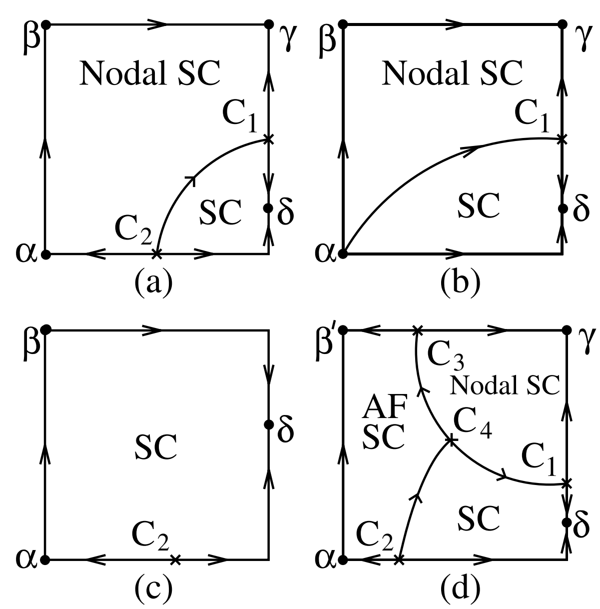

FIG. 1. Schematic phase diagrams and qualitative RG flows of an array of alternating A and B type chains with $\Delta_s^A > 0$ and $\Delta_c^A = \Delta_c^B = \Delta_s^B = 0$. The figures represent cuts through a multidimensional parameter space. Interactions that couple neighboring A and B chains increase along the x axis, while the A to A and B to B couplings increase along y. The RG flows in the neighborhood of the fixed points (except C_4) follow from the analysis presented in the text; the phase boundaries and global flows are qualitative renderings. The unstable fixed points represent decoupled chains (α), the two fluid state with a 2D gapped superconductor on A and an anisotropic Fermi liquid on B (β), and various critical points (C_j). The phases controlled by the stable fixed points are coexisting superconducting and antiferromagnetic order with a full quasiparticle gap (β'), a 2D nodal superconductor (γ), and a 2D fully gapped superconductor (δ).

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using the powerful nonperturbative methods developed for the theory of the 1DEG. Then, so long as there is no phase transition as a function of h, the results should be qualitatively correct, even as $h \to 0$. Moreover, so long as the isotropic system has substantial local stripe order, many of these results should even be quantitatively reasonable. Empirical evidence which suggests that such explicit symmetry breaking is innocuous comes from experiments in the strongly orthorhombic materials, $Y_2BaCu_3O_{7-\delta}$ and $Y_2Ba_4Cu_8O_{7-\delta}$, where anisotropies in the in-plane superfluid density as large as a factor of 10 can be induced without, apparently, affecting the qualitative physics of high temperature superconductivity.

However, there is also experimental evidence of gapless "nodal" excitations at low temperatures, deep in the superconducting state. In all the simplest realizations of a quasi-one-dimensional superconductor, for example, an array of weakly interconnected doped two-leg $t - J$ or Hubbard ladders, the superconducting state has a gap to all spin-carrying excitations, including quasiparticles. The gapless and fully gapped phases of a superconductor are necessarily separated by a quantum phase transition. In order to substantiate the claim that the quasi-one-dimensional limit is adiabatically connected to the physics of the cuprates, it is at least *necessary* to determine whether, and under what circumstances, gapless nodal excitations exist.

It is important to stress that the issue of the "d-wave-like" character of the order parameter (i.e., whether the expectation value of the pair creation operator changes sign under a 90° rotation) is distinct from the issue of the existence of nodal quasiparticles. Even in a weak coupling (BCS) superconductor, if the Fermi surface does not close around the $\vec{k} = (0,0)$ or $\vec{k} = (\pi,\pi)$ points in the BZ, it is possible to have perfect d-wave symmetry without nodal quasiparticles. For the two-leg ladder of the above cited example, the superconducting pairing is known [6] to be d-wave-like but fully gapped.

We will study the simple model effective Hamiltonian, $H = H^* + H'$, of a quasi-one-dimensional system which consists of an array of alternating, inequivalent (A and B type) "chains." For zero interchain coupling, $H' = 0$ (i.e., $h \to \infty$) , the problem is solved exactly using standard bosonization methods; we refer to this limit as the decoupled fixed point, although in reality it is a high-dimensional manifold of fixed points, parametrized by the various Luttinger exponents of the 1DEG's. It is indicated by the point α in Fig. 1. By studying the perturbative renormalization-group (RG) flows in the vicinity of the decoupled fixed point, as well as the behavior at various anisotropic 2D fixed points, we determine the qualitative phase diagram and the character of the excitations in the limit of small, but nonzero interchain coupling.

The decoupled fixed point Hamiltonian H^* consists of a sum of terms for each decoupled chain. This problem can be solved exactly using methods of bosonization [7] to express the Hamiltonian for each chain as a sine-Gordon field theory for the spin and charge degrees of freedom, respectively. The electronic field operators on each chain can be expressed in terms of the charge and spin fields, $\phi_{n,s}$ and $\phi_{n,c}$, and their duals, $\theta_{n,s}$ and $\theta_{n,c}$, as

$$\begin{aligned} \Psi_{n,\sigma}(x) &= e^{ik_F^n x}\Psi_{n,\sigma,+}(x) + e^{-ik_F^n x}\Psi_{n,\sigma,-}(x)\,, \\ \Psi_{n,\sigma,\pm}(x) &= \mathcal{N}_{n,\sigma} e^{i\sqrt{\pi/2}[\theta_c^n + \sigma\theta_s^n \pm \phi_c^n \pm \sigma\phi_s^n]}, \end{aligned} \tag{1}$$

where the Klein factors obey $\{\mathcal{N}_{n,\sigma}, \mathcal{N}_{n',\sigma'}\} = \delta_{nn'}\delta_{\sigma\sigma'}/\pi a$, a is the short-distance cutoff, $\sigma = \pm 1$ for up and down spins, k_F^n is the Fermi wave number of chain n, and the bosonic fields satisfy $[\phi_{n,\alpha}(y), \partial_x\theta_{m,\alpha'}(x)] = i\delta_{n,m}\delta_{\alpha,\alpha'}\delta(x-y)$. For n even, the A chains, which represent the stripe regions where the concentration of mobile "doped" charges is high, are metallic but with a spin gap $\Delta_s^{(A)} > 0$, i.e., a Luther-Emery liquid. Microscopically, one can imagine that each A chain represents the low energy physics of a doped two-leg or three-leg $t - J$ or Hubbard ladder [8]. The Fermi wave number, $k_F^{(A)}$, is therefore far from the commensurate value, $\pi/2$; experiment [5] suggests that in a variety of cuprate superconductors, $k_F^{(A)} \approx \pi/4$. The remaining quantities which characterize the A chains are a charge Luttinger parameter, $K_c^{(A)}$, and the charge and spin velocities, $v_c^{(A)}$ and $v_s^{(A)}$. The B chains represent the more lightly doped, locally antiferromagnetic strips between stripes. We will be interested in the case in which some doped holes have leaked into these strips, so they have no charge gap, but because they are still nearly Mott insulating [9], $k_F^{(B)} \approx \pi/2$, $K_c^{(B)} \approx 1/2$, and $v_c^{(B)} \ll v_c^{(A)}$. We will, however, also consider the cases in which umklapp scattering opens a charge gap, $\Delta_c^{(B)}$, on chains B, which, in addition, may or may not have a spin gap. In the more interesting gapless case, spin rotation invariance implies that at low energies the spin Luttinger exponent, $K_s^{(B)} \to 1$.

The interchain coupling, H', typically [10] generates interactions that are relevant in the RG sense at the decoupled fixed point. Starting from a microscopic viewpoint, one would be tempted to take H' to consist of a single particle hopping term which couples each A chain to its nearest neighbor B chain. This interaction is manifestly irrelevant, both because of the presence of a spin gap, and because of the mismatch in Fermi wave numbers, $k_F^{(A)} \neq k_F^{(B)}$. However, in the initial stages of renormalization, all imaginable local terms consistent with symmetry are generated, both relevant and irrelevant. We will therefore skip this initial step, and directly study the perturbative β functions for potentially relevant interchain couplings. Here we assume that H' contains only the most relevant interactions, between first and second neighbor chains, for the range of Luttinger exponents discussed above,

$$H' = \sum_n \int dx \Big\{ -t_{BB} \sum_\sigma [\Psi^\dagger_{2n-1,\sigma}\Psi_{2n+1,\sigma} + \mathrm{H.c.}] - J_{BB}\vec{S}_{2n-1}\cdot\vec{S}_{2n+1} - J_{AB}[\hat{\Delta}^\dagger_{2n}\hat{\Delta}_{2n+1} + \mathrm{H.c.}]$$
$$+ J'_{AB}[\hat{\Delta}^\dagger_{2n}(\Psi_{2n-1,\uparrow}\Psi_{2n+1,\downarrow} + \Psi_{2n+1,\uparrow}\Psi_{2n-1,\downarrow}) + \mathrm{H.c.}] - J_{AA}[\hat{\Delta}^\dagger_{2n}\hat{\Delta}_{2n+2} + \mathrm{H.c.}]\Big\}, \quad (2)$$

where $\hat{\Delta}_n \equiv \Psi_{n,\uparrow}\Psi_{n,\downarrow}$ is the singlet pair creation operator, $\vec{S}$ is the spin-density operator, and it is implicitly understood in the above expression that any piece of the interaction that is rapidly oscillating (with wave number $2k_F$) is to be omitted. Here we have neglected possibly relevant backscattering interactions which could potentially promote charge-density wave (CDW) formation; the basis for this is discussed in Ref. [10] and below.

The only nonstandard term in H' is the term proportional to J'_{AB} which removes a pair from an A chain, rotates it by 90°, and reinserts it across the two neighboring B chains; the sign of this term (after renormalization, away from the decoupled, α, fixed point) determines whether the superconducting order is d-like or s-like. Microscopic calculations [6] on $t - J$ and Hubbard ladders lead us to believe that most likely all the pair-tunneling terms $J_X > 0$. The d-like pairing tendency observed in these calculations implies $J'_{AB} > 0$, while the positivity of the remaining J_X implies an unfrustrated superconducting state; e.g., $J_{AB} < 0$ would imply " π-junctions" between neighboring chains.

We begin by considering the regime in which all of the coupling constants in H' are small. In this limit, the system is at the decoupled fixed point α. It is easy to determine the (perturbative) role of the various processes in H'. In cases in which a gap prohibits the operation of one of these interactions in lowest order, that interaction is manifestly irrelevant; i.e., its dimension is infinite. Otherwise, to leading order in powers of the interchain couplings, the perturbative β functions have the form

$$\frac{dg}{d\ln a} = (2 - D_g)g + \dots. \quad (3)$$

D_g is the scaling dimension of the perturbation with coupling constant g. For $D_g < 2$, the operator is perturbatively relevant, and otherwise it is irrelevant. The dimension of the various operators are listed in Table I.

TABLE I. Scaling dimensions of the interchain couplings at the decoupled fixed point with $\Delta_s^A > 0$ and $\Delta_c^A = 0$.

	$\Delta_s^B = 0$, $\Delta_c^B = 0$	$\Delta_s^B > 0$, $\Delta_c^B = 0$	$\Delta_s^B = 0$, $\Delta_c^B > 0$	$\Delta_s^B > 0$, $\Delta_c^B > 0$
t_{BB}	$(1/4)[K_c^{(B)} + 1/K_c^{(B)} + 2]$	∞	∞	∞
J_{BB}	$[K_c^{(B)} + 1]$	∞	1	∞
J_{AA}	$1/K_c^{(A)}$	$1/K_c^{(A)}$	$1/K_c^{(A)}$	$1/K_c^{(A)}$
J_{AB}	$(1/2)[1/K_c^{(A)} + 1/K_c^{(B)} + 1]$	$(1/2)[1/K_c^{(A)} + 1/K_c^{(B)}]$	∞	∞
J'_{AB}	$(1/4)[2 + 2/K_c^{(A)} + K_c^{(B)} + 1/K_c^{(B)}]$	∞	∞	∞

Forward scattering interactions between the charge currents and densities on neighboring chains are marginal.

We now return to the issue of the CDW couplings between chains; if relevant, these would lead to an ordered, insulating state. In Ref. [10], it was shown that forward scattering interactions between chains, whether direct or induced by dynamical fluctuations of the stripe geometry, strongly affect the scaling dimension of these operators, tending to make them less relevant. In particular, there is a finite regime of parameters, especially when the stripe fluctuations are significant, where the CDW couplings are irrelevant, and so can be neglected.

Since in all the cases considered here, the decoupled fixed point is perturbatively unstable in some way, our next task is to determine where the RG flows go.

The two fluid fixed point.—It should be clear from the table that, under most conditions, all AB couplings are perturbatively irrelevant at point α of Fig. 1. Specifically, if we set $K_c^{(B)} = 1/2$, this is true so long as $K_c^{(A)} < 1$. (The phase diagrams in Figs. 1a, 1c, and 1d assume this condition is satisfied.) Even if $K_c^{(A)} > 1$ (as in Fig. 1b), the AB couplings are typically more weakly relevant than the couplings between like chains. We are therefore led to consider the RG flows in the limit in which all such couplings are set equal to zero, so that we have two interpenetrating, and decoupled, but genuinely two-dimensional systems. This limit is represented by the left-hand edge of the phase diagrams in Fig. 1.

In general, there are many possible higher dimensional fixed points to which the system could flow. Because the system can lower its kinetic energy by allowing pairs to move between chains [8], J_{AA} is typically relevant technically so long as $K_c^{(A)} > 1/2$. We will consider exclusively the state in which the A subsystem has true superconducting long-range order and a full spin gap. Because t_{BB} always has lower scaling dimension than J_{BB} so long as $\Delta_c^B = 0$, the most likely situation is that system B forms a highly anisotropic Fermi liquid. The corresponding fixed point is labeled β in Figs. 1a–1c. However, either if $\Delta_c^B > 0$ or if the residual interactions are sufficiently strong to drive an instability of the nearly nested Fermi surface, system B could order antiferromagnetically. The fixed point with coexisting superconductivity in A and antiferromagnetism in B is indicated by β' in Fig. 1d.

The operators which couple A and B have entirely different scaling dimensions at the two-fluid fixed points, β and β', than at α. The two-fluid fixed point β is unstable, due to the ordinary proximity effect. Specifically, in establishing the relevance of the operators J_{AB} and J'_{AB}, the pair creation operator on A can be replaced by their constant expectation value, while the pairing fields on B operate on a Fermi liquid state. For $J'_{AB} > 0$, this state is d-wave-like; the quasiparticle spectrum is

$$E^2(\vec{k}) = [v_F(k_x - k_F^B) - 2t_{BB}\cos(k_yL)]^2 + \langle\hat{\Delta}_A\rangle^2[J_{AB} - 2J'_{AB}\cos(k_yL)]^2\,, \qquad (4)$$

where x and y refer, respectively, to the directions parallel and perpendicular to the chains, v_F, t_{BB}, J_{AB}, and J'_{AB} are now to be interpreted as renormalized parameters at β, and L is the spacing between chains. There remains the quantitative issue, which depends on the microscopic details, of the relative magnitudes of these couplings. If $|J_{AB}| < 2|J'_{AB}|$, the quasiparticle spectrum is gapless. This is a stable phase of matter, the nodal superconductor, described by the fixed point γ in Fig. 1. We have assumed that this inequality is satisfied in Figs. 1a and 1b, where the RG flows run directly from β to γ. However, if $|J_{AB}| > 2|J'_{AB}|$, the flows run from β to the fully gapped superconducting state, signified by the stable fixed point δ in the figure, as in Fig. 1c.

By contrast, at the fixed point β', there are no finite dimensional operators which couple the two fluids since the antiferromagnet has a charge gap and the superconductor has a spin gap. This fixed point describes a stable phase with two coexisting order parameters, and a complete gap in the quasiparticle spectrum. Evidence of a phase with coexisting magnetic and superconducting order, presumably rendered glassy by quenched disorder, has been presented [11,12]; it is a prediction of this study that the ordered state has a fully gapped quasiparticle spectrum.

Fully coupled fixed points.—In addition to the stable fixed points γ and δ, described above, there are a number of other unstable fixed points whose existence is dictated by the topology of the phase diagram (assuming that the transitions are second order).

The phase transition between the nodal and the gapped superconductors is governed by the fixed point C_1 in the figure. The universal properties of this transition can be studied [13] in the weakly interacting limit. This transition, as approached from the nodal phase, is triggered by deforming the gap function (or band structure) such that two nodal points approach each other, and at the critical point, coalesce. As a consequence, at the single nodal point, the quasiparticle velocity in one direction vanishes. Beyond the critical point, a gap opens in the spectrum. By naive power counting, four fermion interactions are irrelevant at this fixed point, so this is all there is to it. Even when J_{AB} is perturbatively irrelevant at α, strong pair-tunneling will certainly produce superconductivity in B by the proximity effect. Moreover, the resulting state will be the fully gapped, ordered superconductor. Thus, there must exist an unstable critical point, C_2, similar to the smectic metal to superconductor critical point of Ref. [10].

The critical point C_3 in Fig. 1d is similar to one mentioned by Vojta *et al.* [13], for a transition, within a nodal superconducting phase, to a state with broken translational symmetry with an ordering vector which at C_3 spans the nodal points. Conversely, the robustness of the nodal superconducting phase embodied in Figs. 1a, 1b, and 1d, mirrors the asymptotic decoupling of the nodal quasiparticles from fluctuations associated with such a transition when the ordering vector does not span the nodes. The (as yet not analyzed) multicritical point C_4 is required by a minimal consistent construction of the RG flows.

Finally, it is clear that at the fixed point γ the nodal quasiparticles are well defined elementary excitations of the system at arbitrarily low energy and long wavelength. This is consistent with a widely held belief that such excitations must exist, even though the high temperature superconductors are far from the BCS limit. However, what is also clear from the circuitous flows that lead to this fixed point in Figs. 1a and 1b is that these quasiparticles can, under appropriate circumstances, be much less robust than the superconducting state itself. Thus, even well below T_c, where superconducting order is well established, the nodal quasiparticles can still be ill-defined objects, and only become sharp at very low temperatures. It is possible that this observation reconciles the strong evidence [14] from thermal conductivity of well-defined nodal quasiparticles at $T \ll T_c$ with the evidence from photoemission [15] of their nonexistence down to temperatures of order $T_c/2$.

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