

Fate of Vector Dominance in Effective Field Theory

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We reveal the full phase structure of the effective field theory for QCD, based on hidden local symmetry (HLS) through the one-loop renormalization group equation including quadratic divergences. We then show that vector dominance (VD) is not a sacred discipline of the effective field theory but rather an accidental phenomenon peculiar to three-flavored QCD. In particular, the chiral symmetry restoration in the HLS model takes place in a wide phase boundary surface, on which the VD is nowhere realized. This suggests that VD may not be valid for chiral symmetry restoration in hot and/or dense QCD.

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Since Sakurai advocated vector dominance (VD) as well as vector meson universality [1], VD has been a widely accepted notion in describing vector meson phenomena in hadron physics. In fact, several models such as the gauged sigma model [2] are based on VD to introduce the photon field into the Lagrangian. Moreover, it is often taken for granted in analyzing the dilepton spectra to probe the phase of quark-gluon plasma for the hot and/or dense QCD [3].

As far as the well-established hadron physics for the $N_f = 3$ case is concerned, it in fact has been extremely successful in many processes such as the electromagnetic form factor of the pion [1] and the electromagnetic $\pi\gamma$ transition form factor (see, e.g., Ref. [4]), etc. However, there has been no theoretical justification for VD and as it stands might be no more than a mnemonic useful only for the three-flavored QCD at zero temperature/density. Actually, *VD is already violated* for the three-flavored QCD for the anomalous processes such as $\gamma \rightarrow 3\pi/\pi^0 \rightarrow 2\gamma$ [5–7] and the $\omega\pi$ transition form factor (see, e.g., Ref. [8]). This strongly suggests that VD may not be a sacred discipline of hadron physics but may largely be violated in the different parameter space than the ordinary three-flavored QCD (nonanomalous processes) such as in the large N_f QCD, N_f being number of massless flavors, and hot and/or dense QCD where the chiral symmetry restoration is expected to occur. It is rather crucial whether or not VD is still valid when probing such a chiral symmetry restoration through vector meson properties [9,10].

Here we emphasize that in the hidden local symmetry (HLS) model [6,11] *the vector mesons are formulated precisely as gauge bosons*; nevertheless, *VD as well as the universality is merely a dynamical consequence* characterized by the parameter choice $a = 2$.

In this paper we reveal the full phase structure of the effective field theory including the vector mesons, based on the one-loop renormalization group equation (RGE) of the HLS model. It turns out that *in view of the phase diagram VD is very accidentally realized and only for $N_f = 3$ QCD*. On the other hand, we find *a wide phase boundary surface of chiral symmetry restoration* in the HLS model, *on which the VD is nowhere realized*. Furthermore, *only a single point* of the phase boundary is shown to be *selected*

by QCD through the Wilsonian matching [12], which actually coincides with the *vector manifestation (VM)* [13] realized for large N_f QCD where *VD is badly violated with $a = 1$* .

Let us first describe the HLS model based on the $G_{\text{global}} \times H_{\text{local}}$ symmetry, where $G = \text{SU}(N_f)_L \times \text{SU}(N_f)_R$ is the global chiral symmetry and $H = \text{SU}(N_f)_V$ is the HLS. The basic quantities are the gauge bosons $\rho_\mu = \rho_\mu^a T_a$ of the HLS and two $\text{SU}(N_f)$ -matrix valued variables ξ_L and ξ_R . They are parametrized as $\xi_{L,R} = e^{i\sigma/F_\sigma} e^{\mp i\pi/F_\pi}$, where $\pi = \pi^a T_a$ denote the pseudo-scalar Nambu-Goldstone (NG) bosons associated with the spontaneous breaking of G and $\sigma = \sigma^a T_a$ the NG bosons absorbed into the HLS gauge bosons ρ_μ which are identified with the vector mesons. F_π and F_σ are relevant decay constants, and the parameter a is defined as $a \equiv F_\sigma^2/F_\pi^2$. ξ_L and ξ_R transform as $\xi_{L,R}(x) \rightarrow h(x)\xi_{L,R}(x)g_{L,R}^\dagger$, where $h(x) \in H_{\text{local}}$ and $g_{L,R} \in G_{\text{global}}$. The covariant derivatives of $\xi_{L,R}$ are defined by $D_\mu \xi_L = \partial_\mu \xi_L - ig\rho_\mu \xi_L + i\xi_L \mathcal{L}_\mu$, and similarly with replacement $L \leftrightarrow R$, $\mathcal{L}_\mu \leftrightarrow \mathcal{R}_\mu$, where g is the HLS gauge coupling, and \mathcal{L}_μ and \mathcal{R}_μ denote the external gauge fields gauging the G_{global} symmetry.

The HLS Lagrangian is given by [6,11]

$$\mathcal{L} = F_\pi^2 \text{tr}[\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp}^\mu] + F_\sigma^2 \text{tr}[\hat{\alpha}_{\parallel\mu} \hat{\alpha}_{\parallel}^\mu] + \mathcal{L}_{\text{kin}}(\rho_\mu), \quad (1)$$

where $\mathcal{L}_{\text{kin}}(\rho_\mu)$ denotes the kinetic term of ρ_μ and

$$\hat{\alpha}_{\parallel}^\mu = (D_\mu \xi_R \cdot \xi_R^\dagger \mp D_\mu \xi_L \cdot \xi_L^\dagger)/(2i). \quad (2)$$

By taking the unitary gauge, $\xi_L^\dagger = \xi_R$ ($\sigma = 0$), the Lagrangian in Eq. (1) gives the following tree level relations for the vector meson mass m_ρ , the ρ - γ transition strength g_ρ , the $\rho\pi\pi$ coupling constant $g_{\rho\pi\pi}$, and the direct $\gamma\pi\pi$ coupling constant $g_{\gamma\pi\pi}$: [6,11]

$$\begin{aligned} m_\rho^2 &= ag^2 F_\pi^2, & g_{\rho\pi\pi} &= \frac{1}{2}ag, \\ g_\rho &= agF_\pi^2, & g_{\gamma\pi\pi} &= \left(1 - \frac{a}{2}\right)e, \end{aligned} \quad (3)$$

where e is the electromagnetic coupling constant.

Expressions for $g_{\rho\pi\pi}$ and g_ρ in Eq. (3) lead to the celebrated Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [14] (version I):

$$g_\rho = 2F_\pi^2 g_{\rho\pi\pi}, \quad (4)$$

independently of the parameter a . This is the low energy theorem of the HLS [15], which was proved at one-loop [16], and then at any loop order [17]. On the other hand, making a dynamical assumption of a parameter choice $a = 2$, the following outstanding phenomenological facts are reproduced from Eq. (3) [6,11]: (i) $g_{\rho\pi\pi} = g$ (universality of the ρ coupling) [1]; (ii) $m_\rho^2 = 2g_{\rho\pi\pi}^2 F_\pi^2$ (KSRF II) [14]; (iii) $g_{\gamma\pi\pi} = 0$ (vector dominance of the electromagnetic form factor of the π) [1]. Thus, even though the vector mesons are gauge bosons in the HLS model, VD as well as the universality is not an automatic consequence but rather a dynamical one of a parameter choice of $a = 2$.

Actually, due to quantum corrections, the parameters change their values by the energy scale, which are determined by the RGE's. Accordingly, values of the parameters F_π , a , and g cannot be freely chosen, although they are independent at tree level. Thus, we first study the RG flows of the parameters and the phase structure of the HLS to classify the parameter space. Here we stress that, thanks to the gauge symmetry in the HLS model, it is possible to perform a systematic loop expansion including the vector mesons in addition to the pseudoscalar mesons [7,12,16,18,19] in a way to extend the chiral perturbation theory [20]. There the loop expansion corresponds to the derivative expansion, so that the one-loop calculation of the RGE is reliable in the low energy region.

As shown in Refs. [12,21] it is important to include quadratic divergences in calculating the quantum corrections. Because of quadratic divergences in the HLS dynamics, it follows that even if the bare theory defined by the cutoff Λ is written as if it were in the broken phase characterized by $F_\pi^2(\Lambda) > 0$, the quantum theory can be in the symmetric phase characterized by $F_\pi^2(0) = 0$ [21]. The one-loop RGE's for F_π , a , and g including quadratic divergences are given by [12,21]

$$\begin{aligned} \mu \frac{dF_\pi^2}{d\mu} &= C[3a^2 g^2 F_\pi^2 + 2(2-a)\mu^2], \\ \mu \frac{da}{d\mu} &= -C(a-1) \\ &\quad \times \left[3a(a+1)g^2 - (3a-1)\frac{\mu^2}{F_\pi^2} \right], \\ \mu \frac{dg^2}{d\mu} &= -C \frac{87-a^2}{6} g^4, \end{aligned} \quad (5)$$

where $C = N_f/[2(4\pi)^2]$ and μ is the renormalization scale. It is convenient to use the following quantities:

$$X(\mu) \equiv C\mu^2/F_\pi^2(\mu), \quad G(\mu) \equiv Cg^2(\mu). \quad (6)$$

Then, the RGE's in Eq. (5) are rewritten as

$$\begin{aligned} \mu \frac{dX}{d\mu} &= (2-3a^2G)X - 2(2-a)X^2, \\ \mu \frac{da}{d\mu} &= -(a-1)[3a(a+1)G - (3a-1)X], \\ \mu \frac{dG}{d\mu} &= -\frac{87-a^2}{6} G^2. \end{aligned} \quad (7)$$

It should be noticed that the RGE's in Eq. (7) are valid above the ρ mass scale m_ρ , where m_ρ is defined by the on-shell condition $m_\rho^2 = a(m_\rho)g^2(m_\rho)F_\pi^2(m_\rho)$. In terms of X , a , and G , the on-shell condition becomes $a(m_\rho)G(m_\rho) = X(m_\rho)$. Then the region where the RGE's in Eq. (7) are valid is specified by the condition $a(\mu)G(\mu) \leq X(\mu)$.

Seeking the parameters for which all right-hand sides of three RGE's in Eq. (7) vanish simultaneously, we obtain three fixed points and one fixed line in the physical region and one fixed point in the unphysical region (i.e., $a < 0$ and $X < 0$). Those in the physical region (labeled by $i = 1, \dots, 4$) are given by

$$\begin{aligned} (X_i^*, a_i^*, G_i^*) &= (0, \text{any}, 0), (1, 1, 0), \left(\frac{3}{5}, \frac{1}{3}, 0\right), \\ &\left(\frac{2(2+45\sqrt{87})}{4097}, \sqrt{87}, \frac{2(11919-176\sqrt{87})}{1069317}\right). \end{aligned} \quad (8)$$

Note that $G = 0$ is a fixed point of the RGE for G , and $a = 1$ is the one for a . Hence, RG flows on the $G = 0$ plane and the $a = 1$ plane are confined in the respective planes.

Let us first study the phase structure of the HLS for $G = 0$ (see Fig. 1) in which case m_ρ vanishes and the RGE's (7) are valid all the way down to the low energy limit, $\mu \geq m_\rho = 0$. There are one fixed line and two fixed points [first three in Eq. (8)]. Generally, the phase boundary is specified by $F_\pi^2(0) = 0$, namely, governed by the infrared fixed point such that $X(0) \neq 0$ [see Eq. (6)]. Such a fixed point is the point $(X_2^*, a_2^*, G_2^*) = (1, 1, 0)$, which is nothing but the VM point [13]. Then the phase boundary is given by the RG flows entering (X_2^*, a_2^*, G_2^*) . Since $a = 1/3$ is a fixed point of the RGE for a in Eq. (7), the RG flows for $a < 1/3$ cannot enter (X_2^*, a_2^*, G_2^*) . Hence there is no phase boundary specified by $F_\pi^2(0) = 0$ in the $a < 1/3$ region. Instead, $F_\pi^2(0)$ vanishes even though $F_\pi^2(0) \neq 0$, namely, $a(0) = X(0) = 0$. Then the phase boundary for $a < 1/3$ is given by the RG flow entering the point $(X, a, G) = (0, 0, 0)$. In Fig. 1 the phase boundary is drawn by the dashed line, which divides the phases into the symmetric phase [22] (upper side; cross-hatched area) and the broken one (lower side).

In the case of $G > 0$, on the other hand, the ρ becomes massive ($m_\rho \neq 0$), and thus decouples at the m_ρ scale. Below the m_ρ scale a and G no longer run, while F_π still runs by the π loop effect. Thus, to study the phase

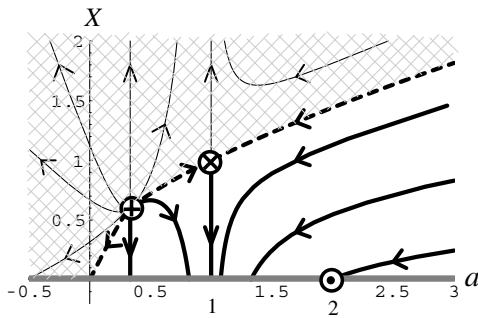


FIG. 1. Phase diagram on the $G = 0$ plane. Arrows on the flows are written from the ultraviolet to the infrared. Gray line denotes the fixed line $(X_1^*, a_1^*, G_1^*) = (0, \text{any}, 0)$. Points indicated by \oplus and \otimes (VM point) denote the fixed points $(3/5, 1/3, 0)$ and $(1, 1, 0)$, respectively. Dashed lines divide the broken phase (lower side) and the symmetric phase (upper side; cross-hatched area): Flows drawn by thick lines are in the broken phase, while those by thin lines are in the symmetric phase. The point indicated by \odot , $(X, a, G) = (0, 2, 0)$, correspond to the VD, $a(0) = 2$.

structure for $G > 0$ we need the RGE for F_π for $\mu < m_\rho$ (denoted by $F_\pi^{(\pi)}$). This is given by $d[F_\pi^{(\pi)}]^2/d\mu^2 = 2C$ [21], which is readily solved as

$$[F_\pi^{(\pi)}(\mu)]^2 = [F_\pi^{(\pi)}(m_\rho)]^2 - 2C(m_\rho^2 - \mu^2). \quad (9)$$

Then the quadratic divergence [second term in Eq. (9)] of the π loop can give rise to chiral symmetry restoration $F_\pi^{(\pi)}(0) = 0$ [21]. Thus, the phase boundary is specified by the condition $[F_\pi^{(\pi)}(m_\rho)]^2 = 2Cm_\rho^2$. Note that the relation between $[F_\pi^{(\pi)}(m_\rho)]^2$ and $F_\pi^2(m_\rho)$, including the finite renormalization effect, is given by [12]

$$[F_\pi^{(\pi)}(m_\rho)]^2 = F_\pi^2(m_\rho) + Ca(m_\rho)m_\rho^2, \quad (10)$$

which is converted into the condition for $X(m_\rho)$ and $a(m_\rho)$. Combination of this with the on-shell condition specifies the phase boundary in the full (X, a, G) space, which is given by the collection of the RG flows entering points on the line specified by

$$\begin{aligned} 2 - a(m_\rho) &= 1/X(m_\rho), \\ a(m_\rho)G(m_\rho) &= X(m_\rho). \end{aligned} \quad (11)$$

Such a surface can be imagined from Figs. 1 and 2.

We now study the $a = 1$ plane (see Fig. 2). The flows stop at the on shell of ρ ($G = X$; dot-dashed line in Fig. 2) and should be switched over to RGE of $F_\pi^{(\pi)}(\mu)$ as mentioned above. From Eq. (11) with $a = 1$, the flow entering $(X, G) = (1, 1)$ (dashed line) is the phase boundary which distinguishes the broken phase (lower side) from the symmetric one (upper side; cross-hatched area).

For $a < 1$, RG flows approach to the fixed point $(X_3^*, a_3^*, G_3^*) = (3/5, 1/3, 0)$ in the idealized high energy limit ($\mu \rightarrow \infty$).

For $a > 1$, RG flows in the broken phase approach to $(X_4^*, a_4^*, G_4^*) \approx (0.2, 9.3, 0.02)$, which is precisely the fixed

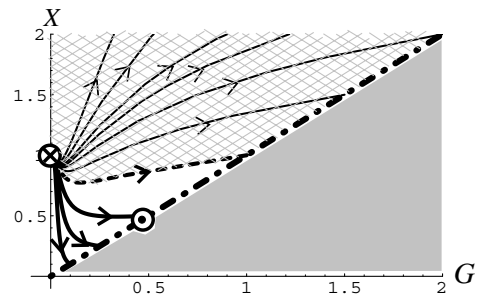


FIG. 2. Phase diagram on the $a = 1$ plane. Arrows on the flows are written from the ultraviolet to the infrared. Point indicated by \otimes denotes the VM fixed point $(X_2^*, a_2^*, G_2^*) = (1, 1, 0)$. Flows drawn by thick lines are in the broken phase, while those by thin lines are in the symmetric phase (cross-hatched area). Dot-dashed line corresponds to the on-shell condition $G = X$. In the shaded area the RGE's (7) are not valid since ρ has already decoupled. Point indicated by \odot , $(1/2, 1, 1/2)$, corresponds to the VD, $a(0) = 2$ [see Eq. (13)].

point that the RG flow of the $N_f = 3$ QCD belongs to. To see how the RG flow of $N_f = 3$ QCD approaches to this fixed point, we show the μ dependence of $X(\mu)$ in Fig. 3 where values of the parameters at $\mu = m_\rho$ are set to be $(X(m_\rho), a(m_\rho), G(m_\rho)) \approx (0.46, 1.22, 0.38)$ through Wilsonian matching with the underlying QCD [12]. The values of X close to $1/2$ in the physical region ($m_\rho \leq \mu \leq \Lambda$) are very unstable against RGE flow, and, hence, $X \sim 1/2$ is realized in a very accidental way.

Let us now discuss the VD which is characterized by $a(0) = 2$. Since F_σ^2 does not run for $\mu < m_\rho$ while F_π^2 does, we have [12]

$$a(\mu) \equiv \begin{cases} F_\sigma^2(\mu)/F_\pi^2(\mu) & (\mu > m_\rho) \\ F_\sigma^2(m_\rho)/[F_\pi^{(\pi)}(\mu)]^2 & (\mu < m_\rho). \end{cases} \quad (12)$$

Then, by using Eqs. (9) and (10), $a(0)$ is given by

$$a(0) = a(m_\rho)/[1 + a(m_\rho)X(m_\rho) - 2X(m_\rho)]. \quad (13)$$

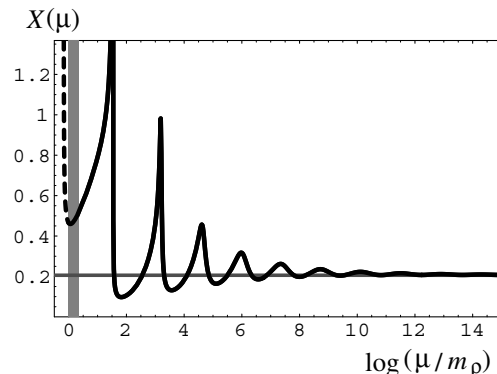


FIG. 3. Scale dependence of $X(\mu)$ in QCD with $N_f = 3$. Shaded area denotes the physical region, $m_\rho \leq \mu \leq \Lambda$. Flow shown by the dashed line is obtained by extending it to the (unphysical) infrared region by taking literally the RGE's in Eq. (7). In an idealized high energy limit the flow approaches to the fixed point $X_4^* = 2(2 + 45\sqrt{87})/4097 \approx 0.2$.

This implies that the VD [$a(0) = 2$] is realized only for $(X(m_\rho), a(m_\rho)) = (1/2, \text{any})$ or $(\text{any}, 2)$.

In $N_f = 3$ QCD, the parameters at m_ρ scale, $(X(m_\rho), a(m_\rho), G(m_\rho)) \simeq (0.46, 1.22, 0.38)$, happen to be near such a VD point. However, the RG flow actually belongs to the fixed point (X_4^*, a_4^*, G_4^*) which is far away from the VD value. Thus, the VD in $N_f = 3$ QCD is accidentally realized by $X(m_\rho) \sim 1/2$ which is very unstable against the RG flow (see Fig. 3). For $G = 0$ (Fig. 1) the VD holds only if the parameters are (accidentally) chosen to be on the RG flow entering $(X, a, G) = (0, 2, 0)$ (indicated by \ominus) which is an end point of the line $(X(m_\rho), a(m_\rho)) = (\text{any}, 2)$. For $a = 1$ (Fig. 2), on the other hand, the VD point $(X, a, G) = (1/2, 1, 1/2)$ (indicated by \odot) lies on the line $(X(m_\rho), a(m_\rho)) = (1/2, \text{any})$.

Then, phase diagrams in Figs. 1 and 2 and their extensions to the entire parameter space (including Fig. 3) show that neither $X(m_\rho) = 1/2$ nor $a(m_\rho) = 2$ is a special point in the parameter space of the HLS. Thus, we conclude that the VD as well as the universality can be satisfied only accidentally. Therefore, when we change the parameter of QCD, the VD is generally violated. In particular, neither $X(m_\rho) = 1/2$ nor $a(m_\rho) = 2$ is satisfied on the phase boundary surface characterized by Eq. (11) where the chiral restoration takes place in the HLS model. Therefore, VD is nowhere realized on the chiral restoration surface.

Moreover, when the HLS is matched with QCD, only the point $(X_2^*, a_2^*, G_2^*) = (1, 1, 0)$, the VM point, on the phase boundary is selected, since the axial vector and vector current correlators in HLS can be matched with those in QCD only at that point [13]. Therefore, QCD predicts $a(0) = 1$, i.e., a large violation of the VD at chiral restoration. Actually, for the chiral restoration in the large N_f QCD [23,24], the VM can in fact take place [13], and thus the VD is badly violated.

Finally, we suggest that, if the VM takes place in other chiral restoration such as the one in the hot and/or dense QCD, the VD should be largely violated near the critical point.

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