Multiparticle Quantum Superposition and Stimulated Entanglement by Parity Selective Amplification of Entangled States

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We report the creation of an entangled multiphoton quantum superposition by quantum injection of entangled 2-photon states into a parity selective parametric amplifier. The *information preserving* property of the state transformation suggests for these macrostates the name of large qubits. They are ideal objects for investigating the emergence of the classical world in complex quantum systems and have relevant new applications in quantum information.

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Since the golden years of quantum mechanics the interference of classically distinguishable quantum states, first introduced by the famous "Schrödinger Cat" apologue [1], has been the object of extensive theoretical studies and recognized as a major conceptual paradigm of physics [2]. In modern times the sciences of quantum information and quantum computation deal precisely with collective processes involving a multiplicity of interfering states, generally mutually entangled and rapidly dephased by decoherence. In many respects the experimental implementation of this intriguing classical-quantum condition represents today an unsolved problem in spite of recent successful studies carried out mostly with atoms [3]. A nearly decoherence-free all-optical scheme based on the process of the quantum injected optical parametric amplification (OPA) of a single photon in a quantum superposition state, i.e., a *qubit,* has been proposed [4].

As a relevant step forward in the realization of the quantum injection scheme, the present work reports a novel OPA system that transforms any input linear-polarization (π) entangled, 2-photon state (e-bit) into a quantum superposition of π-entangled, multiphonon states, indeed a Schrödinger Cat state (S-Cat) here referred to as a "large" (or "massive") qubit: M-qubit. In order to implement the new scheme we adopted the single nonlinear (NL) crystal configuration shown in Fig. 1. According to this scheme the exciting UV optical beam, the pump of the OPA interaction, was first injected at a low intensity level into the NL crystal with the wave vector (wv) $\mathbf{k}'_p$ oriented towards the left (L) side of the figure. By this process, commonly referred to as spontaneous parametric down conversion (SPDC) or L amplification, single photon pairs in π-entangled states were generated. The UV beam was then back reflected with the wv $\mathbf{k}_p$ oriented towards the right (R) direction, and tightly refocused into the NL crystal together with the two back-reflected SPDC generated photons. This provided the main amplification (R amplification) of the quantum-injected e-bit states and their final transformation into the multiphoton S-Cat states. Of course, since SPDC is an aleatory process it was necessary to *postselect* the amplified S-Cat states against the squeezed-vacuum (s-v) noise arising from the R amplification of the vacuum state in the absence of the injected particles. This difficult problem was solved by implementing at the output of our system an efficient parity-selective device, usually referred to as *nonlocal entangled interferometer* (NEIF) [5]. In the language of electrical engineering, NEIF conveyed on different output channels the s-v "noise" and the "signal," viz. the amplified e-bit state. This resulted in the final detection of the S-Cat macrostate with a large, virtually infinite signal-to-noise ratio S/N.

Let us now venture into a more detailed description of the apparatus. A NL beta-barium-borate (BBO) crystal slab cut for Type II phase matching and 1.5 mm thick, was excited in both left and right directions by an UV beam back reflected by a spherical mirror M_p with curvature radius $r_p = 30$ cm. A computer controlled mount allowed micrometric displacements of M_p along the axis $\mathbf{Z}$ parallel to the wv $\mathbf{k}_p$ of the UV beam. This one was generated by a Ti:Sa coherent MIRA mode-locked

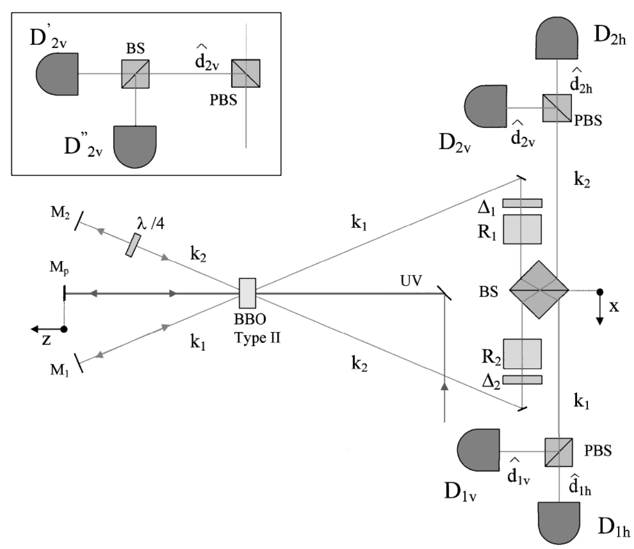

FIG. 1. Experimental apparatus. Inset: measurement apparatus to detect couples of photons simultaneously emitted over the output mode d_{2v}.

laser emitting SHG pulses at a 76 Mhz rate with wavelength (wl) $\lambda_p = 397.5$ nm, duration $\delta_t = 180$ fs, average power 0.35 W.

Consider first the L amplification of the input vacuum state, viz. the SPDC process. This was excited by focusing the UV beam on the NL crystal by a lens with focal length $f_p = 1.5$ m. The measured rates of single and double photon pairs simultaneously generated by SPDC were 3.2×10^4 sec^{-1} and $\lesssim 10^2$ sec^{-1}, respectively. Consider the single pairs only. In general the two photons with equal wl $\lambda = 795$ nm, belonging to any pair are generated in a π-entangled state $|\Psi\rangle$ with an entanglement phase equal to the OPA *intrinsic phase* Ψ determined by the spatial orientation of the NL crystal [5]. The dynamical role of Ψ is further clarified by the quantum theory below. The generated SPDC state is: $|\Psi\rangle = 2^{-1/2}[|11.00\rangle + \exp(i\Psi)|00.11\rangle] = 2^{-1/2}[|\uparrow\rangle_1|\downarrow\rangle_2 + \exp(i\Psi)|\downarrow\rangle_1|\uparrow\rangle_2]$, where $|n_1 n_2 \cdot n_3 n_4\rangle \equiv |n_{1h}\rangle|n_{2v}\rangle|n_{1v}\rangle|n_{2h}\rangle$ are the Fock state occupancies of the relevant $\mathbf{k}$ modes, $\mathbf{k}_j$ $(j = 1,2)$ with horizontal (h) or vertical (v) π's. It is well known that in any $\mathbf{k}$-nondegenerate Type II OPA process the two optical j modes belonging to each couple $\{1h, 2v\}$ and $\{1v, 2h\}$ are parametrically correlated by two mutually independent amplifiers OPA$_A$ and OPA$_B$ [4].

In our experiment each SPDC entangled photon pair was selected by a couple of very narrow pinholes, back reflected, and refocused onto the NL crystal by two equal spherical mirrors M_j $(j = 1,2)$ with radii $r_\lambda = 30$ cm, and placed at an adjustable distance $1 \simeq 30$ cm from the NL crystal. Care was taken to precisely overlap all back reflected beams in the crystal within a common focal region, with diameter $\phi \approx 50$ μm. During the 2-photon back reflection and before reinjection of the pair into OPA, a $\lambda/4$ plate introduced a change of the entanglement phase of $|\Psi\rangle$: $\Psi \rightarrow \Phi$, i.e., the original pair state $|\Psi\rangle$ was transformed into $|\Phi\rangle = 2^{-1/2}[|\uparrow\rangle_1|\downarrow\rangle_2 + \exp(i\Phi)|\downarrow\rangle_1|\uparrow\rangle_2]$. In our case the adopted initial condition was $\Psi = 0$ and the phase shifting action of the $\lambda/4$ plate was chosen to transform before reinjection the originally L-generated even-parity triplet $|\Psi\rangle$ with $\Psi = 0$ into the odd-parity singlet: $|\Phi\rangle_s = 2^{-1/2}[|11.00\rangle - |00.11\rangle]$. The pair state $|\Phi\rangle_s$ was indeed the input state transformed by R amplification into a superposition of *odd-parity* multiphoton pure states, i.e., the entangled S-Cat macrostate: $|\Phi\rangle_{\mathrm{OUT}} = 2^{-1/2}[|\Psi_A\rangle - |\Psi_B\rangle]$.

Let us analyze the main R amplification process in more detail. According to the previous analysis, the amplifiers OPA$_A$ and OPA$_B$ induce unitary transformations, respectively, on two pairs of time dependent field operators: $\hat{a}_1(t) \equiv \hat{a}(t)_{1h}$, $\hat{a}_2(t) \equiv \hat{a}(t)_{2v}$, and $\hat{b}_1(t) \equiv \hat{a}(t)_{1v}$, $\hat{b}_2(t) \equiv \hat{a}(t)_{2h}$ for which, for any $i, j = 1,2$ is $[\hat{a}_i, \hat{a}_j^\dagger] = [\hat{b}_i, \hat{b}_j^\dagger] = \delta_{ij}$ and $[\hat{a}_i, \hat{b}_j^\dagger] = 0$, being $\hat{a}_i \equiv \hat{a}_i(0)$, $\hat{b}_i \equiv \hat{b}_i(0)$ the fields at the initial interaction time $t = 0$. The interaction Hamiltonian is $H_I = i\hbar\chi[\hat{A}^\dagger + e^{i\Psi}\hat{B}^\dagger] +$ H.c., where $\hat{A}^\dagger \equiv \hat{a}_1(t)^\dagger \hat{a}_2(t)^\dagger$, $\hat{B}^\dagger \equiv \hat{b}_1(t)^\dagger \hat{b}_2(t)^\dagger$, and $g \equiv \chi t$ is a real number expressing the amplification gain proportional to the product of the 2nd-order NL susceptibility of the crystal and of the pump field here assumed "classical" and undepleted by the interaction. The quantum dynamics of OPA$_A$ and OPA$_B$ is expressed by the commuting unitary squeeze operators: $U_A(t) = \exp[g(\hat{A}^\dagger - \hat{A})]$ and $U_B(t) = \exp[g(e^{i\Psi}\hat{B}^\dagger - e^{-i\Psi}\hat{B})]$, implying the following Bogoliubov transformations for the field operators: $\hat{a}_i(t) = C\hat{a}_i + S\hat{a}_j{}^\dagger$; $\hat{b}_i(t) = C\hat{b}_i + \tilde{S}\hat{b}_j{}^\dagger$ with $i \neq j$. Here: $S \equiv \sinh g$, $\tilde{S} \equiv e^{i\Psi}S$, $C \equiv \cosh g$, and $\Gamma \equiv S/C$. Assume the OPA intrinsic phase $\Psi = 0$. By use of the evolution operator $U_{AB}(t) = U_A(t)U_B(t)$ the quantum injection of the input state $|\Phi\rangle$ leads to the Schrödinger-Cat macrostate [1,2,4]

$$|\Phi\rangle_{\mathrm{OUT}} = U_{AB}(t)|\Phi\rangle = 2^{-1/2}[|\Psi_A\rangle + e^{i\Phi}|\Psi_B\rangle], \quad (1)$$

i.e., to the quantum superposition of the multiparticle, normalized, and mutually orthogonal entangled states

$$\begin{aligned} |\Psi_A\rangle &= C^{-2}\sum_{n,m:0}^{\infty}(S^{-2}n - 1)\Gamma^{\xi}|nn \cdot mm\rangle; \\ |\Psi_B\rangle &= C^{-2}\sum_{n,m:0}^{\infty}(S^{-2}m - 1)\Gamma^{\xi}|nn \cdot mm\rangle, \end{aligned} \quad (2)$$

with $\xi \equiv n + m + 1$. Note that the phase Φ of the input state, and then its parity is reproduced into the output macrostate and determines the superposition character of the S-Cat. This is but one aspect of a very general *information preserving* property of the quantum-injected OPA transformation by which any input state $|\Phi\rangle = \tilde{\alpha}|\uparrow\rangle_1|\downarrow\rangle_2 + \tilde{\beta}|\downarrow\rangle_1|\uparrow\rangle_2$ with $\tilde{\alpha}$, $\tilde{\beta}$ complex and $|\tilde{\alpha}|^2 + |\tilde{\beta}|^2$ is transformed into $|\Phi\rangle_{\mathrm{OUT}} = \tilde{\alpha}|\Psi_A\rangle + \tilde{\beta}|\Psi_B\rangle$. This justifies the term "massive qubit" and will lead to relevant applications in quantum information. A quantum analysis will also show that $|\Phi\rangle_{\mathrm{OUT}}$ is very robust against decohering photon losses [4,5].

We may now inspect the superposition status of the S-Cat by investigating the Wigner function of $|\Phi\rangle_{\mathrm{OUT}}$. We first evaluate the characteristic function of the set of complex variables $(\eta, \eta^*, \xi, \xi^*) \equiv \{\eta, \xi\} : \chi_S\{\eta, \xi\} = \langle\Phi|D[\eta(t)]D[\xi(t)]|\Phi\rangle$ given in terms of the displacement operators $D[\eta(t)] \equiv \exp[\eta(t)\hat{a}(0)^\dagger - \eta^*(t)\hat{a}(0)]$ and $D[\xi(t)] \equiv \exp[\xi(t)\hat{b}(0)^\dagger - \xi^*(t)\hat{b}(0)]$ where $\eta(t) \equiv (\eta C - \eta^* S)$; $\xi(t) \equiv (\xi C - \xi^* S)$. The Wigner function $W\{\alpha, \beta\}$ of the phase-space variables $(\alpha, \alpha^*, \beta, \beta^*) \equiv \{\alpha, \beta\}$ is the 4th-dimensional Fourier transform of $\chi_S\{\eta, \xi\}$. We could evaluate analytically in closed form either $\chi_S\{\eta, \xi\}$ and $W\{\alpha, \beta\}$ and obtain

$$\begin{aligned} W\{\alpha, \beta\} = \overline{W}\{\alpha\}\overline{W}\{\beta\}[1 - \tilde{\Delta}\{\alpha\} - \tilde{\Delta}\{\beta\} \\ + |e^{i\Phi}\Delta\{\alpha\} + \Delta\{\beta\}|^2], \end{aligned} \quad (3)$$

where $\Delta\{\alpha\} \equiv \frac{1}{2}[|\gamma_{A+}|^2 - |\gamma_{A-}|^2 - i\,\mathrm{Re}(\gamma_{A+}\gamma_{A-}^*)]$, $\tilde{\Delta}\{\alpha\} \equiv |\gamma_{A+}|^2 + |\gamma_{A-}|^2$ are given in terms of squeezed variables $\gamma_{A+} \equiv (\alpha_1 + \alpha_2^*)e^{-g}$; $\gamma_{A-} \equiv i(\alpha_1 - \alpha_2^*)e^{+g}$. Analogous expressions for B, β arise from the substitutions: $A \rightarrow B$, $\alpha \rightarrow \beta$. The Gaussians $\overline{W}\{\alpha\} \equiv$

$\pi^{-2}\exp(-\tilde{\Delta}\{\alpha\})$, $\overline{W}\{\beta\} \equiv \pi^{-2}\exp(-\{\tilde{\Delta}\beta\})$ definite positive over the four-dimensional spaces $\{\alpha\}$ and $\{\beta\}$ represent the effect of the squeezed vacuum in the absence of any quantum injection. Inspection of Eq. (3) shows that precisely the superposition character implied by the entangled nature of the injected state $|\Phi\rangle$ determines, through the modulus square term, the Φ-dependent dynamical quantum interference of the *macroscopic* devices OPA_A and OPA_B, the ones that otherwise act as *uncoupled,* mutually independent objects.

Turn now the attention to the parity-selective interferometer acting on the output beams $\mathbf{k}_j$ emerging from the OPA amplifier [6]. Note first that NEIF reproduces exactly the Bell-state measurement configuration at the Alice's site of the first quantum state teleportation (QST) experiment [7]. Indeed the parity selectivity we deal with here is formally related to the *complete* Bell-state selectivity within that QST scheme. Consider the field emitted by the NL crystal after R amplification: Fig. 1. The beams associated with modes $\mathbf{k}_j$ $(j = 1, 2)$ were generally phase shifted $\Delta_j = (\psi_{jh} - \psi_{jv})$ by two equal birefringent plates Δ_j and π rotated by two equal Fresnel-Rhomb π rotators $R_j(\theta)$ by angles θ_j respect to directions taken at 45° with the horizontal. The beams were then linearly superimposed by a symmetrical, i.e., 50/50 beam splitter (BS) and coupled by two polarizing beam splitters (PBS) to 4 equal EGG SPCM-AQR-14 Si-avalanche detectors D_{1h}, D_{1v}, D_{2h}, D_{2v} with quantum efficiencies $QE \approx 0.35$. These ones measured the (h) and (v) π polarizations on the output modes associated with the fields $\hat{a}_{1h}$, $\hat{a}_{1v}$, $\hat{a}_{2h}$, $\hat{a}_{2v}$, respectively. A computer controlled mount allowed displacements of BS along the axis $\mathbf{X}$. Consider the probability that a single injected π-entangled photon couple generates a double coincidence event involving D_{1h} and D_{1v}: $P_2 \equiv (D_{1h}D_{1v}) \equiv \langle\Phi|\hat{N}_{1h}\hat{N}_{1v}|\Phi\rangle = (D_{2h}D_{2v})$, where $\hat{N}_{1h} \equiv \hat{a}^{\dagger}_{1h}\hat{a}_{1h}$. By accounting for the full set of transformations induced by the overall optical system on $|\Phi\rangle$ we get $P_2 = \frac{1}{2}\Im[1 - \cos\tilde{\Delta}] + S^2\{1 + \aleph + (1 - \aleph)\cos\Phi + \Im[5 + 3\cos\Delta + \cos\tilde{\Delta} - \cos\Phi] + \frac{1}{2}\cos(2\theta_1)\cos(2\theta_2)\} + O(S^4)$ with $\Delta \equiv (\Delta_1 - \Delta_2)$, $\tilde{\Delta} \equiv (\Delta - \Phi)$, $\Im \equiv \frac{1}{2}\sin^2(\theta_1 + \theta_2)$, $\aleph \equiv \frac{1}{4}[\cos^2(2\theta_1) + \cos^2(2\theta_2)]$. This shows that the phase Φ of the input state indeed critically determines the value of P_2, e.g., by setting $\Delta = 0$, $(\theta_1 + \theta_2) = \frac{1}{2}\pi$ it reaches its maximum value $P_2 \simeq \frac{1}{2}$ for any input odd-parity singlet state, $\Phi = \pm\pi$, while it is ≈zero for any input even-parity triplet, $\Phi = 0$. This phase selectivity property is inverted by adoption of the complementary coincidences $(D_{1h}D_{2v}) = (D_{2h}D_{1v})$ as shown by the data given in Fig. 2 as a function of the BS position $\mathbf{X}$: there an input triplet leads to a resonance peak and a singlet to a dip [6]. Consider now the s-v noise which is generated by R amplification of the vacuum state in the absence of the injected photon couples. This kind of noise is formally represented by a thermal distribution of entangled Fock states whose parity is determined *only* by the OPA intrinsic phase Ψ, e.g., if $\Psi = 0$ as in our case, the s-v noise is represented by an infinite sum of even-parity states irrespective of any optical device inserted in the apparatus [4]. This is the key mechanism of the parity selectivity of NEIF. Let us apply the above concepts to our experiment involving the parameters: $\Psi = 0$, $\Phi = \pi$, $\Delta = 0$, $\theta_1 = \theta_2 = \frac{1}{4}\pi$. According to these values the R-amplified e-bit, i.e., the "signal" consists of the superposition of the odd-parity states given by Eq. (1) with $\Phi = \pm\pi$. Correspondingly, the maximum value of the "signal" $(D_{1h}D_{1v}) = (D_{2h}D_{2v}) \simeq \frac{1}{2}$ is detected by a coincidence apparatus which is insensitive to the s-v "noise" since: $(D_{1h}D_{1v})_{\text{vac}} = (D_{2h}D_{2v})_{\text{vac}} = \frac{1}{2}S^2(1 + S^2)(1 - \cos\Delta)(1 - 2\Im) + S^4 \approx 0$. This may lead to a large, virtually infinite S/N ratio for S-Cat state post selection. Of course, the attainment of the last condition implies at least a 100% visibility V of the interference patterns shown in Fig. 2. In practice $V \simeq 0.93$ was obtained.

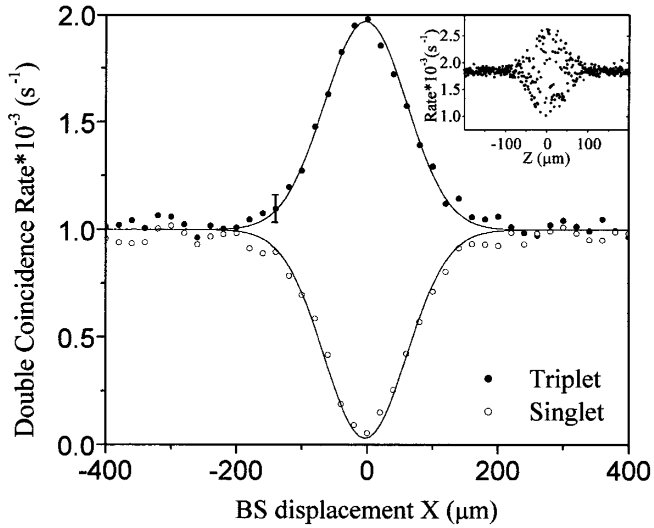

FIG. 2. Double coincidence rate $R_2(\mathbf{X}) = (D_{h1}D_{2v})$ vs the position $\mathbf{X}$ of the beam splitter BS expressing the parity selectivity of the NEIF system. Inset: Oscillatory interference pattern due to the indistinguishability of the L/R emission directions of the SPDC photon couples as a function of the coordinate $\mathbf{Z}$ of the UV mirror M_p.

A large 1st-order quantum phenomenon was found when the position $\mathbf{Z}$ of the mirror M_p was adjusted to realize the exact time superposition of the back-reflected UV pump pulse (wl λ_p) and of the back-reflected SPDC pulses (wl λ). A sinusoidal interference pattern with periodicity $= \lambda$ and $V \gtrsim 0.4$ was revealed within either single-detector and double-coincidence measurements, (inset of Fig. 2). This striking effect is due to the *in principle* indistinguishability, for any detector's frame, of the two possible directions over which the detected entangled photon pair was originally emitted: that one could have been R generated by the back-reflected UV pulse or L generated by SPDC and then back reflected. Apart from its relevance *per se* and its novelty because of its first realization with an entangled state, this effect was helpful to identify the

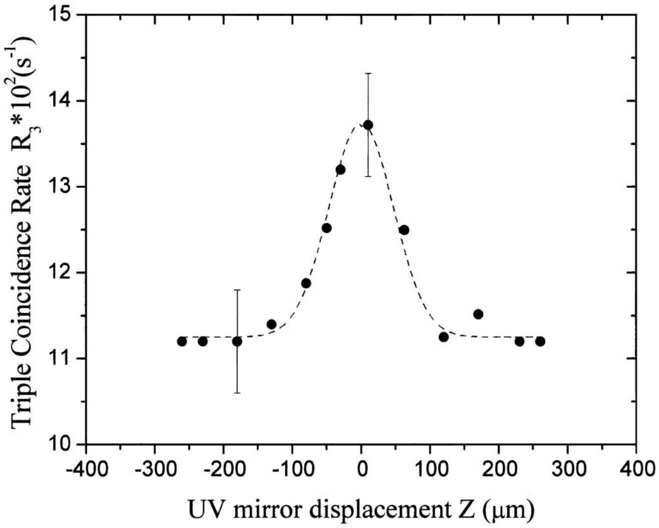

FIG. 3. Attenuated triple coincidence rate $R_3(\mathbf{Z}) = (D'_{2v}D''_{2v}D_{2h})$ vs the coordinate $\mathbf{Z}$ of M_p showing the OPA amplification of a quantum injected singlet entangled state resulting in the generation of a Schrödinger Cat odd-parity macrostate. Each point was determined by averaging over $\approx 10^3$ detection samples. The $\approx 10^{-4}$ attenuation was due to the detector QE's and to imperfections of output mode selection.

condition of maximum R amplification, i.e., $\mathbf{Z} = 0$ in both Figs. 2 and 3.

The main R amplification was investigated by a measurement apparatus similar to the one just considered. Precisely, we measured the rate $R_3(\mathbf{Z}) \propto P_3(\mathbf{Z}) \equiv (D'_{2v}D''_{2v}D_{2h})$ by a *triple coincidence* scheme involving D'_{2v}, D''_{2v} coupled to the same output field $\hat{d}_{2v}$ by a 50/50 BS (Fig. 1, inset). This was done in order to discriminate the amplified contribution to $P_2 \propto S^2$ against the dominant contribution due to the nonamplified input pairs. In addition, the five-coincidence rate $R_5(\mathbf{Z}) \propto P_5(\mathbf{Z}) = (D'_{2v}D''_{2v}D_{2h}D_{1v}D_{1h})$ was also measured. The resonant-like shape of the rate function $R_3(\mathbf{Z})$ shown in Fig. 3, and the similar one for $R_5(\mathbf{Z})$, express the evidence of the generation of the multiparticle macrostate $|\Phi\rangle_{\mathrm{OUT}} = 2^{-1/2}[|\Psi_A\rangle - |\Psi_B\rangle]$ arising from the OPA amplification of the injected 2-photon singlet $|\Phi\rangle_s$. By the peak values of $R_3(\mathbf{Z} \approx 0)$ and $R_5(\mathbf{Z} \approx 0)$ it was estimated that our OPA system generated odd-parity entangled 4-photon states $|\Phi_2\rangle = 2^{-1/2}[|22.00\rangle - |00.22\rangle] \equiv 2^{-1/2}[|\uparrow\uparrow\rangle_1|\downarrow\downarrow\rangle_2 - |\downarrow\downarrow\rangle_1|\uparrow\uparrow\rangle_2]$ with a rate $\Re_4 = 5 \times 10^3\ \mathrm{sec}^{-1}$ and odd-parity 6-photon states $|\Phi_3\rangle$ with $\Re_6 \sim 1.90 \times 10^3\ \mathrm{sec}^{-1}$. States associated with a number of particles $2N \geq 8$ were also generated at lower rates, in agreement with the statistical distributions expressed by Eq. (2) and with the value of the gain evaluated on the basis of the NL properties of BBO: $g \approx 0.31$.

This highly nontrivial result could be linearly scaled by adoption of a more efficient NL crystal and of a more powerful UV source. At least a factor 17 increase of the value of g could be attained by the adoption of a standard Ti:Sa regenerative amplifier coherent REGA9000 operating with $\delta_t = 150$ fsec pulses, a 270 kHz rep rate, and an average UV output power ≈ 0.30 W. In this case $\Gamma \approx 1$, and the average number of photon pairs QED stimulated by any single injected e-bit will be very large $\tilde{N} \gg 1$, as implied by the explicit expressions of $|\Psi_A\rangle$ and $|\Psi_B\rangle$, Eq. (2).

In conclusion, we have reported the successful creation of a multiphoton entangled "Schrödinger Cat" state. This result is expected to open a new field of investigations on the persistence of the validity of several crucial laws of quantum mechanics for systems of increasing complexity, in a virtually decoherence-free environment [4,8]. An example is the study of the violation of Bell-type inequalities in the multiparticle regime: indeed, a new perspective in the basic endeavor on quantum nonlocality [9]. In the framework of quantum information the generation of M-*qubits* carrying the information of corresponding qubits/e-bits will make possible the practical realization of the universal multiple-qubit logic gates. For instance, a multiparticle M-*qubit* acting as control qubit will greatly ease the implementation of the typical quantum nondemolition function of the XOR or Fredkin gates [8]. Other basic applications in quantum information, e.g., the realization of the NOT gate, are now in progress [5,8].

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