## Q-Ball Candidates for Self-Interacting Dark Matter

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We show that nontopological solitons, known as Q-balls, are promising candidates for selfinteracting dark matter. They can satisfy the cross-section requirements for a broad range of masses. Unlike previously considered examples, Q-balls can stick together after collision, reducing the effective self-interaction rate to a negligible value after a few collisions per particle. This feature modifies predictions for halo formation. We also discuss the possibility that Q-balls have large interaction cross sections with ordinary matter.

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The standard cold dark matter model based on nonrelativistic, collisionless particles successfully predicts the formation of structure on large scales exceeding a megaparsec, but appears to make problematic predictions about structure on galactic and subgalactic scales. The dark matter density profile in the cores of galaxies, the number of satellites, the thickening of disks, the density of low mass objects, gravitational lens statistics, and the asphericity of cluster cores found in numerical simulations appear to be at variance with observations [1,2]. The difficulties suggest either that dark matter is not cold [3] or that dark matter is not collisionless [1]. In either case, the conventionally preferred candidates for dark matter, weakly interacting massive particles (WIMPs) or axions, would be ruled out.

In this paper, we propose nontopological solitons known as Q-balls as candidates for collisional dark matter. Q-balls occur in a wide range of particle physics models, can be produced copiously in the early Universe, and can be stable. As candidates for self-interacting dark matter, they possess particularly advantageous and interesting features, as shown in this paper. First, Q-balls can satisfy the requisite cross-section conditions for a much wider range of masses than ordinary pointlike particles. Second, depending on the detailed interactions of the fields from which they are generated, Q-balls can scatter inelastically, leading to modifications of halo evolution compared to elastically scattering collisional dark matter. A particularly interesting limit is where they collide and stick. In this case, the population evolves and the cross section and mass relations change as scattering proceeds. Depending on the phase space distribution of the dark matter particles, it is conceivable that the scattering becomes insignificant as time proceeds. The initial scatterings smooth out halo cores, but heat conduction ceases afterwards and gravothermal collapse is avoided. (Gravothermal collapse is the instability to core collapse due to the negative specific heat of gravity.) A third feature of Q-balls is that they can have significant interactions with ordinary matter (although this is not required). A large range of parameter space is ruled out by current experimental bounds, but PACS numbers: 95.35.+d, 98.80.Cq

significant unconstrained range remains, suggesting new directions for dark matter searches.

Basic properties of Q-balls.—Q-balls are the ground state configurations for fixed charge Q in theories with interacting scalar fields  $\phi$  that carry some conserved global U(1) charge [4–6]. If the field configuration is written in the form

$$\phi(x,t) = e^{i\omega t}\bar{\phi}(x),\tag{1}$$

its charge is

$$Q = \frac{1}{2i} \int \phi^* \, \overleftarrow{\partial}_t \phi \, d^3 x = \omega \int \, \overline{\phi}^2 \, d^3 x \,. \tag{2}$$

The form of  $\overline{\phi}(x)$  is determined minimizing the energy

$$E = \int d^3x \left[ \frac{1}{2} |\dot{\phi}|^2 + \frac{1}{2} |\nabla \phi|^2 + U(\phi) \right], \quad (3)$$

where the potential  $U(\phi)$  has a minimum at  $\phi = 0$  and is invariant under the global U(1) transformation  $\phi \rightarrow e^{i\theta}\phi$ . In the thin-wall limit  $\bar{\phi}(x) = \phi_0$  is nearly constant in the interior (r < R) and drops rapidly to zero for r > R. Using (2), one can write the energy (3) as

$$E \approx \frac{Q^2}{2V\phi_0^2} + VU(\phi_0), \qquad (4)$$

where V is the Q-ball volume. The minimum of energy in Eq. (4) with respect to V is  $E = \mu Q$ , where  $\mu = \sqrt{2U(\phi_0)/\phi_0^2}$ . In the thin-wall limit, the minimum of E with respect to the value  $\phi_0$  corresponds to

$$\mu \to \mu_0 = \min\left(\sqrt{\frac{2U(\phi)}{\phi^2}}\right).$$
 (5)

Depending on the potential,  $\mu_0$  in Eq. (5) can be finite (type I) or infinite (type II). The mass of a type I *Q*-ball is  $M(Q) = \mu Q$ . For large *Q*-balls  $(Q \to \infty)$ ,  $\mu \to \mu_0$ and  $\phi \to \phi_0$  ("thin-wall" limit) [4–6]. For smaller values of *Q*,  $\mu$  can be computed in a "thick-wall" approximation [7]. For Q < 10 radiative corrections become important [8]. In any case,  $\mu$  is less than the mass of the  $\phi$  particle as a consequence of condition (5). The radius of the type I Q-ball is

$$R_Q \approx \left(\frac{3}{4\pi}\right)^{1/3} \frac{Q^{1/3}}{(\mu \phi_0^2)^{1/3}} \,. \tag{6}$$

Type II *Q*-balls occur if the scalar potential grows slower than the second power of  $\phi$ . Then the *Q*-ball never reaches the thin-wall regime, even if *Q* is large. The value of  $\phi$ inside the soliton extends to a value as large as the gradient terms allow, and the mass of a *Q*-ball is proportional to  $Q^p$ , p < 1. In particular, if the scalar potential has a flat plateau  $U(\phi) \sim m_{\text{flat}}^4$  at large  $\phi$ , then the mass of a *Q*-ball is [9]  $M(Q) \sim m_{\text{flat}}Q^{3/4}$  and the size is  $R_Q \sim m_{\text{flat}}^{-1}Q^{1/4}$ .

*Q*-balls as self-interacting dark matter.—As in generic examples of self-interacting dark matter, *Q*-ball scattering in regions of high density facilitates heat exchange in dark matter cores that smooths out their distribution and, also, enhances the stripping of dark matter from satellites that accelerates their tidal destruction. Both effects serve to resolve the problems of cold, collisionless dark matter. For these purposes, the basic requirement is that the ratio of self-interaction cross section  $\sigma_{DD}$  to particle mass *M* must be in the range [1]

$$S = \frac{\sigma_{DD}}{M} = 8 \times 10^{-25} - 1 \times 10^{-23} \text{ cm}^2 \text{ GeV}^{-1}$$
$$= 0.5 - 6 \text{ cm}^2 \text{g}^{-1}. \tag{7}$$

For point particles whose dominant scattering is s wave, Hui has shown that unitarity implies a cross section bounded above by  $\sigma_{DD} \sim 1/(Mv_{\rm rel})^2$ , where  $v_{\rm rel} \approx 300$  km/s is the typical velocity of the dark matter particles. In this case, the maximal mass for a point particle is  $M \approx 10$  GeV [10]. Q-balls are extended objects which can evade this bound. Higher partial waves contribute to their scattering such that their cross section is essentially geometric (except in the limit of very small coupling),  $\sigma_{DD} = \pi R_Q^2$ . Then, S for type I Q-balls is

$$\frac{\sigma_{DD}}{M} \approx \left(\frac{9\pi}{16}\right)^{1/3} \left(\frac{M(Q)}{\phi_0}\right)^{4/3} \frac{Q^{4/3}}{M(Q)^3}, \qquad (8)$$

and for type II *Q*-balls is

$$\frac{\sigma_{DD}}{M} \sim m_{\rm flat}^{-3} Q^{-1/4} \sim \frac{Q^2}{M(Q)^3} \,. \tag{9}$$

Note that both expressions for S can greatly exceed the unitarity bound for large  $Q > 10^5$ . Hence, it is possible to have Q-ball candidates that satisfy the requirements on S for a range of masses much greater than 10 GeV (the unitarity limit).

If *Q*-balls are of type I and no restriction is placed on the relative magnitude of  $\phi_0$  and  $\mu$ , the mass of the  $\phi$  particle can range from below a keV to well beyond the electroweak scale. If the mass of  $\phi > M_Z$ , such a scalar field could make extremely heavy, strongly interacting *Q*-balls (cf. Ref. [11]). Naturalness arguments, while not rigorous, suggest potentials in which  $\phi_0 \sim \mu$ . In this case, for type I *Q*-balls  $[M(Q) \sim \mu Q]$ , a satisfactory choice of parameters is in the range around  $\mu \sim \phi_0 \sim 20$  MeV,  $Q \sim 10-10^3$ . For type II *Q*-balls [with  $M(Q) \sim m_{\rm flat}Q^{3/4}$ ], the analogous relations are  $m_{\rm flat} \sim 20$  MeV and  $Q \sim 10^4-10^5$ . Note that, if the global U(1) symmetry of the *Q*-balls is associated with baryon number, as in most examples considered previously [12], empirical constraints specific to baryonic processes do not permit the requisite large cross sections. However, there is no problem with more general U(1) symmetries.

*Q-ball production in the early Universe.*—Several mechanisms could lead to a formation of *Q*-balls in the early Universe. First, they can be produced in the course of a phase transition [13]. Second, solitosynthesis, a process of gradual charge accretion similar to nucleosynthesis, can take place [14–16]. Finally, *Q*-balls can emerge from fragmentation of a scalar condensate [12] formed at the end of inflation.

Solitosynthesis occurs through an accretion of charge. It requires some universal asymmetry  $\eta_Q$  of the global charge Q, similar to baryon asymmetry of the Universe. When the temperature drops below some critical value  $T_c \sim (\mu/|\ln \eta_Q|)$  [15], a Q-ball minimizes both the energy and the free energy of the system, and a rapid coalescence of global charge into Q-balls occurs [14,15]. The number of Q-balls and their mass density in the Universe depends on the value of Q-asymmetry,  $\eta_Q$ , and is largely unconstrained.

Fragmentation of a coherent scalar condensate can lead to a copious production of O-balls [12]. At the end of inflation, scalar fields develop large expectation values along those directions in the potential that have small masses or flat plateaus [17]. The subsequent rolling of the condensate can encounter an instability, as a result of which the scalar condensate can break up into Q-balls [12]. This process has been studied both analytically [12,18] and numerically [19,20] and was shown to produce a sharply peaked distribution of sizes of Q-balls. There is also some evidence that Q-balls and anti-Q-balls can form from the same condensate while the overall charge asymmetry  $\eta_0$  [20] is small or zero. The number density of Q-balls formed in this way depends on the shape of the potential at large  $\phi$  and the horizon size at the time of formation. The only strict constraint is that the separation between Q-balls should be of the order of their size at the time of formation. For us this translates into a redshift at which Q-balls are formed.

*Q-ball scattering and sticking.*—Whereas previous studies of collisional dark matter assumed that they scatter elastically, *Q*-balls can either merge or split after a collision depending on whether energy can be dissipated [21]. The merger of *Q*-balls requires the kinetic energy to dissipate quickly on the time scale of the collision. In the absence of additional interactions, emission of  $\phi$ 

particles is the only channel for such dissipation. There are many modes of oscillations of an excited *Q*-ball: volume, surface, etc. Some of these modes can produce  $\phi$  particles very efficiently, especially if they are enchanced by parametric resonance. Additional interactions of  $\phi$  with other light states can enhance the dissipation and increase the probability of merging. For type-II *Q*-balls, there is a strong energetic bias toward merging as opposed to fragmentation. The subject of *Q*-ball collisions, clearly, deserves further studies.

Merger or sticking together of dark matter can lead to novel dynamics of the halo compared to the standard case of elastic self-interactions. As perturbations begin to grow, the density of Q-balls is too low for there to be significant scattering. As the halo density profile becomes steeper and denser, Q-ball collision and merger takes place. Mergers replace two particles with charge Q with a single particle of charge 2Q. According to Eqs. (8) and (9), the ratio  $S \equiv \sigma_{DD}/M$  decreases as Q increases, and the mean velocity  $v_{rel}$  decreases by two. Both effects decrease the interaction rate. Another important factor is the phase space distribution. If the collisions occur primarily among particles with circular orbits, interactions will simply cause particles to dissipate kinetic energy and the density to grow, the opposite of what we are aiming for. However, based on the orbital eccentricities observed in simulations [22,23], a more likely possibility is that the collisions occur primarily between fast moving particles with highly eccentric orbits with large radii and slow-moving particles in the denser cuspy core. Then, kinetic energy is effectively pumped into the cusp particles and the distribution is flattened. For an appropriate range of cross sections, this effect would cause the interaction rate to decrease. If this scenario is correct, as we hope to test by computer simulations, then, the interactions might shut off altogether after a few collisions per particle. Consider, then, the evolution of small halos. For collisionless dark matter, there are many such halos and they are extremely cuspy and dense. For elastically scattering dark matter, collisions smooth the core distribution but, then, gravothermal collapse causes the central density to rise again. In the case of merging Q-balls, it is conceivable that the cuspy core is smoothed, the density is reduced by a finite factor, and then the profile remains unchanged.

In the case of splitting, binding energy can be converted into kinetic energy. Since the binding energy can exceed the gravitational binding to the halo, splitting can lead to conversion of two similar-size Q-balls into one large Q-ball that remains gravitationally bound to the halo and one small fragment that escapes. It is possible to imagine that both merger and splitting play a role. Suppose  $S \equiv \sigma_{DD}/M$  is initially large. In a small young halo with a dense core, collision and merger transforms the population into large Q-balls with small S. Large Q-balls are more likely to split and have energy escape. The halo structure will be influenced by both effects.

f  $\phi$  gests that  $m_{\phi}$  must be close to 0.02 GeV. But, if  $\phi$  has and a mass below 50 MeV, it cannot have strong interactions with matter because of a combination of collider bounds, in particular because a neutral pion could decay into  $\phi \bar{\phi}$ pairs. Likewise, a light  $\phi$  field cannot have weak interactions because it would violate the precision measurements at LEP of the Z width. Q-balls can interact strongly with ordinary matter if  $m_{\phi} > 50$  MeV. Alternatively, there is the possibility of a significant cross section with ordinary matter if the interactions are mediated by some new physics. In a number of grand unified and string inspired models, light fields

actions are mediated by some new physics. In a number of grand unified and string-inspired models, light fields are accompanied by additional interactions. For example, an interesting possibility is that an additional gauge U(1)', unrelated to either the global  $U_Q(1)$  or the standard model gauge group, is spontaneously broken at some high scale [24]. We will use this model to illustrate possible interactions that the field  $\phi$  can have with matter, which can ultimately make Q-balls detectable. Let us suppose that  $\phi$  interactions with matter are mediated by some vector boson with mass  $M_{Z'}$ . Then the cross section for Q-ball interactions with a nucleon is, roughly,

What kinds of interaction are possible between Q-balls

and baryons?—If the field  $\phi$  has only gravitational interactions with matter, Q-balls cannot be detected directly.

Condition (7) for  $\phi_0 \sim \mu$  (the naturalness condition) sug-

$$\sigma_{Qp} \sim F \, \frac{g^2 Q^2}{M_{Z'}^2},\tag{10}$$

where g is some coupling constant and F is a form factor. The  $Q^2$  dependence occurs because of the coherent scattering of a nucleus off the  $\phi$  quanta in the condensate, and the form factor F accounts for a fraction of Q matter that scatters coherently. If the size of the Q-ball is smaller than that of a nucleus, F is of the order of 1. If the Q-ball is much larger than the nucleus,  $F \sim (R_n/R_Q)^3$ , where  $R_n$ is the size of the nucleus.

The resulting dark matter (*Q*-ball)/proton cross sections,  $\sigma_p$ , are large enough to be detected for a broad range of parameters. Figure 1 summarizes the current limits on  $\sigma_{Op}$ and M based on existing searches [2]. Superposed are the predictions for *Q*-balls. A large range of parameters is already ruled out, but there remain unexplored regimes. One consists of Q-balls with large cross sections and masses larger than a TeV. Since the local dark matter density is  $0.4 \text{ GeV/cm}^3$  and the mean velocity is 300 km/s, the flux is of the order of  $10^5$  cm<sup>-2</sup> s<sup>-1</sup>. Another possibility is relatively light O-balls with masses about 1 GeV and a weak cross section, below  $10^{-31}$  cm<sup>2</sup>. The flux of these particles would be high, of the order of  $10^{10}$  cm<sup>-2</sup> s<sup>-1</sup>. Different strategies would have to be adopted to search in the two regimes, but both are feasible, as will be discussed in a future paper.

To summarize, the self-interactions of Q-balls are characterized by a large cross section due to their extended geometry, a property that, along with the rest of their



FIG. 1. Empirical constraints (shades of gray) on *Q*-ball/proton cross section and mass assuming  $\phi$  interacts through an intermediate boson Z' with  $M_{Z'} \approx 1$  TeV and  $g \approx 0.1$ . Experiments are described in Ref. [25]. The most likely range for *Q*-balls is enclosed by hatched boundary (either type I with  $\phi_0/\mu$  within a few orders of magnitude of unity or type II). White regions are currently unconstrained experimentally. If no restriction is placed on  $\phi_0/\mu_0$ , the predicted range for *Q*-balls expands to the dot-dashed boundary, including even more untested territory.

features, makes them interesting candidates for selfinteracting cold dark matter. If *Q*-balls scatter inelastically and merge, scattering may cease after the profile is flattened. It is conceivable that *Q*-balls have significant interactions with ordinary matter, either strong interactions or interactions mediated by a heavy Z' boson. In this case, the *Q*-balls can be detected in near-future experiments.

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