Bloch Oscillations and Mean-Field Effects of Bose-Einstein Condensates in 1D Optical Lattices

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We have loaded Bose-Einstein condensates into one-dimensional, off-resonant optical lattices and accelerated them by chirping the frequency difference between the two lattice beams. For small values of the lattice well depth, Bloch oscillations were observed. Reducing the potential depth further, Landau-Zener tunneling out of the lowest lattice band, leading to a breakdown of the oscillations, was also studied and used as a probe for the effective potential resulting from mean-field interactions as predicted by Choi and Niu [Phys. Rev. Lett. **82**, 2022 (1999)]. The effective potential was measured for various condensate densities and trap geometries, yielding good qualitative agreement with theoretical calculations.

The properties of ultracold atoms in periodic light-shift potentials in one, two, and three dimensions have been investigated extensively in the past ten years [1]. In near-resonant and, more recently, far-detuned optical lattices, a variety of phenomena have been studied, such as the magnetic properties of atoms in optical lattices, revivals of wave-packet oscillations, and Bloch oscillations in accelerated lattices [2]. While in most of the original optical lattice experiments the atomic clouds had temperatures in the micro-Kelvin range, corresponding to a few recoil energies of the atoms, samples with subrecoil energies are now routinely produced in Bose-Einstein condensation experiments. Many aspects of Bose-Einstein condensed atomic clouds (BECs) have been studied [3], ranging from collective excitations to superfluid properties and quantized vortices. So far, the majority of these experiments have been carried out essentially in harmonic-oscillator potentials provided by magnetic traps or optical dipole traps. The properties of BECs in periodic potentials constitute a vast new field of research (see, for instance, Refs. [4–12]). Several experiments in the pulsed standing wave regime [13,14] as well as studies of the tunneling of BECs out of the potential wells of a shallow optical lattice in the presence of gravity [15], the creation of squeezed states in condensates [16], and the search for superfluid dynamics [17] have taken the first steps in that direction. In this paper, we present the results of experiments on BECs of ^{87}Rb atoms in accelerated optical lattices. In particular, we demonstrate coherent acceleration and Bloch oscillations of BECs adiabatically loaded into optical lattices and the reduction of the effective potential seen by the condensates due to mean-field interactions. The latter was inferred from measurements of Landau-Zener tunneling when the lattice depth was further reduced and/or the acceleration increased. We loaded the condensate into optical lattices with different spatial periods, generating the periodic optical lattice either from two counterpropagating laser beams or two laser beams enclosing an angle θ different from 180 deg.

The properties of a Bose-Einstein condensate located in a periodic optical lattice with depth U_0 are described through the Gross-Pitaevskii equation valid for the single-particle wave function [18]. In agreement with the Bloch approach, the condensate excitation spectrum exhibits a band structure, and in the presence of an acceleration of the optical lattice Bloch oscillations of the condensate should occur [4,6]. We present experimental evidence for Bloch oscillations preserving the condensate wave function. The nonlinear interaction of the condensate may be described through a dimensionless parameter [6,10,11] $C = g/E_B$ corresponding to the ratio of the nonlinear interaction term $g = 4\pi n_0\hbar^2 a_s/M$ and the lattice Bloch energy $E_B = \hbar^2(2\pi)^2/Md^2$. The parameter C contains the peak condensate density n_0, the s-wave scattering length a_s, the atomic mass M, the lattice constant $d = \pi/\sin(\theta/2)k$, with k the laser wave number, and θ the angle between the two laser beams creating the 1D optical lattice. From this it follows that a small angle θ should result in a large interaction term C. In fact, creating a lattice with $\theta = 29$ deg allowed us to realize a value of C larger by a factor of more than 10 with respect to [15] using a comparable condensate density. In the following, the parameters d, E_B, and C always refer to the respective lattice geometries with angle θ.

The role of the nonlinear interaction term of the Gross-Pitaevskii equation may be described through an effective potential in a noninteracting gas model [6,10]. In the perturbative regime of [6] the effective potential is

$$U_{\text{eff}} = U_0/(1 + 4C), \qquad (1)$$

so that the potential seen by the condensate is $U(x) = U_{\text{eff}}\sin(2\pi x/d) + \text{const}$. We therefore expect that for large values of C, i.e., large mean-field effects, the effective optical lattice potential acting on the condensate should be significantly reduced.

Our apparatus used to achieve Bose-Einstein condensation of ^{87}Rb is described in detail in [19]. Essentially, 5×10^7 atoms captured in a magneto-optical trap were transferred into a triaxial time-orbiting potential (TOP) trap [20]. Subsequently, the atoms were evaporatively cooled down to the transition temperature for Bose-Einstein condensation, and after further cooling we

obtained condensates of $\approx 10^4$ atoms without a discernible thermal component in a magnetic trap with frequencies around 15–30 Hz. In one set of experiments, the magnetic trap was then switched off and a horizontal 1D optical lattice was switched on, while in the other case the interaction between the condensate and the lattice took place inside the magnetic trap, which was subsequently switched off to allow time-of-flight imaging. The lattice direction was parallel to the strong axis of the trap, and the lattice beams were created by a 50 mW diode slave laser injected by a grating-stabilized master-laser blue-detuned by $\Delta \approx 28$–35 GHz from the ^{87}Rb resonance line. After passage through an optical fiber, the laser light was split and passed through two acousto-optic modulators (AOMs) that were separately controlled by two phase-locked RF function generators operating at frequencies around 80 MHz, with a frequency difference δ. The first-order output beams of the AOMs generated the optical lattice, and an acceleration of the lattice was effected by applying a linear ramp to δ.

For the values of the detuning and laser intensity used in our experiment, the spontaneous photon scattering rate (≈ 10 s^{-1}) was negligible during the interaction times of a few milliseconds. In our experimental setup, we realized a counterpropagating lattice geometry with $\theta = 180$ deg and an angle geometry with $\theta = 29$ deg, leading to lattice constants d of 0.39 and 1.56 μm, respectively [21]. Inserting a peak condensate density of $n_0 \approx 10^{14}$ cm^{-3} (typical of trap frequencies >100 Hz) in the Thomas-Fermi limit into the expression for the interaction parameter C leads to $C = 0.06$ for the counterpropagating configuration and $C = 0.25$ for $\theta = 29$ deg. In order to determine the lattice depth at low condensate densities, we measured the Rabi frequency on a first-order Bragg resonance [13]. Typically, for well-aligned lattice beams we measured lattice depths up to 20% lower than the theoretically expected value inferred from the laser intensity and detuning. These discrepancies are within the calibration error of our laser power measurements.

In order to accelerate the condensate, we adiabatically loaded it into the lattice by switching one of the lattice beams on suddenly and ramping the intensity of the other beam from 0 to its final value in 200 μs [22]. Thereafter, the linear increase of the detuning δ provided a constant acceleration $a = \frac{\lambda}{2\sin(\theta/2)} \frac{d\delta}{dt}$ of the optical lattice, leading to a final lattice velocity $v_{\text{lat}} = \frac{\lambda}{2\sin(\theta/2)} \delta_f$, where δ_f is the final detuning between the beams. After a few milliseconds of acceleration, the lattice beams were switched off and the condensate was imaged after another 10–15 ms of free fall. As the lattice can transfer momentum to the atoms in the condensate only in units of the Bloch momentum $p_B = \hbar(2\pi/d)$, the acceleration of the condensate showed up as diffraction peaks corresponding to higher momentum classes as time increased. Since for our magnetic trap parameters the initial momentum spread of the condensate (which is transferred into a spread of the lattice quasi-

momentum during an adiabatic switch-on) was much less than a recoil momentum of the optical lattice, the different momentum classes $|p = \pm np_B\rangle$ (where $n = 0, 1, 2, \ldots$) occupied by the condensate wave function could be resolved directly after the time of flight [see, for instance, the peaks corresponding to $n = 0$ and $n = 1$ in Fig. 1(a)]. As described in [23], the acceleration process within a periodic potential can also be viewed as a succession of adiabatic rapid passages between momentum states $|\pm np_B\rangle$. We observed a momentum transfer of up to $6p_B$ without a detectable reduction of the phase-space density of the condensate. We verified that in the process of the acceleration and tunneling, the condensed fraction was not reduced for low condensate densities. Our investigation did not, however, test the evolution of the condensate phase, but on the basis of the Bragg scattering experiments of [13] we assume that the interaction times of our experiment should not destroy the condensate phase.

The average velocity of the condensate was derived from the occupation numbers of the different momentum states. Figure 2 shows the results of the acceleration of a condensate in the counterpropagating lattice with $U_{\text{eff}} = 0.29E_B$ and $a = 9.81$ m s^{-2}. In the rest frame of the lattice [Fig. 2(b)], one clearly sees Bloch oscillations of the condensate velocity corresponding to a Bloch-period $\tau_B = \frac{h}{M_{\text{Rb}}ad} = 1.2$ ms. The shape of these oscillations agrees well with the theoretical curve calculated from the lowest energy band of the lattice.

When the experiment was repeated in the angle configuration of the lattice, larger values of the lattice depth in units of the Bloch energy E_B could be realized. Because
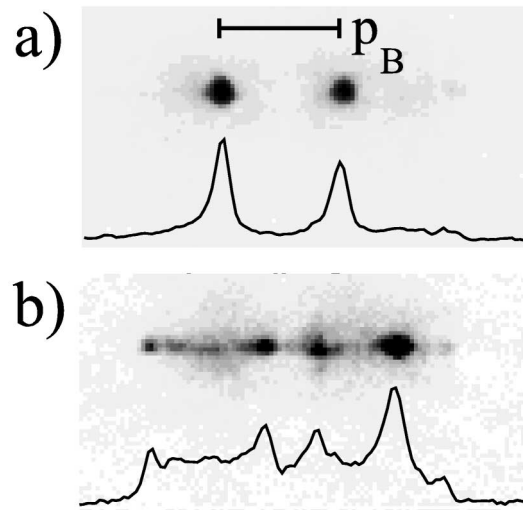

FIG. 1. Condensate acceleration in the counterpropagating optical potential. Shown here are typical time-of-flight images for moderate, $n_0 < 10^{14}$ cm^{-3}, in (a) and high condensate density, $n_0 > 2 \times 10^{14}$ cm^{-3}, in (b). The two-peaked coherent structure in (a) is easily interpretable within the periodic potential diffraction picture giving rise to two momentum classes, whereas the more complicated pattern in (b) cannot be described by this simple model.
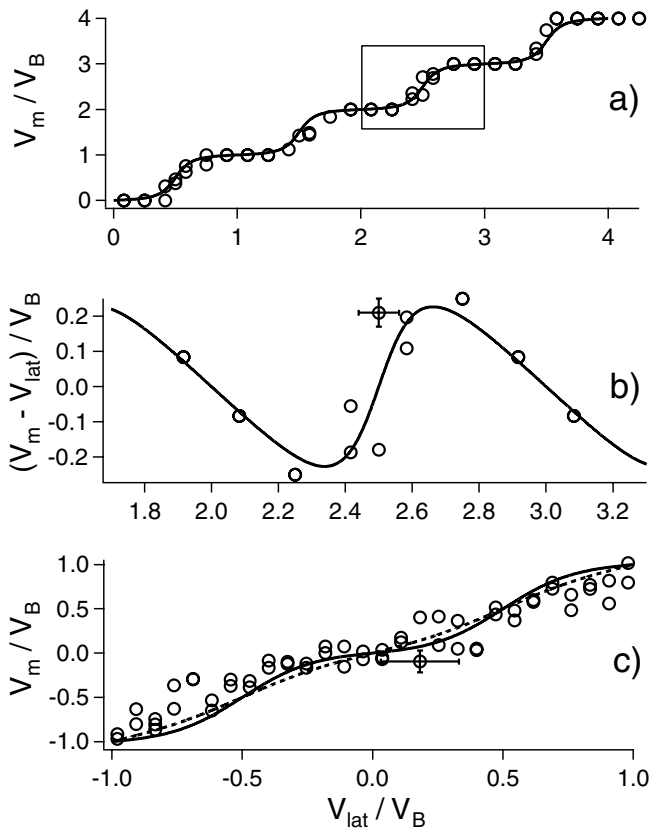
FIG. 2.   Bloch oscillations of the condensate mean velocity v_m in an optical lattice. (a) Acceleration in the counterpropagating lattice with $d = 0.39$ μm, $U_0 \approx 0.29E_B$, and $a = 9.81$ m s^{-2}. Solid line: theory. (b) Bloch oscillations in the rest frame of the lattice, along with the theoretical prediction (solid line) derived from the shape of the lowest Bloch band. (c) Acceleration in a lattice with $d = 1.56$ μm, $U_0 \approx 1.38E_B$, and $a = 0.94$ m s^{-2}. In this case, the Bloch oscillations are much less pronounced. Dashed and solid lines: theory for $U_0 = 1.38E_B$ and $U_{eff} \approx 0.88E_B$.

of the reduced Bloch velocity v_B in this geometry, the acceleration process was extremely sensitive to any initial velocity of the condensate, which in our TOP trap is intrinsically given by the micromotion [20] at the frequency of the bias field. For the trap parameters used in our experiments, the velocity amplitude of the micromotion could be of the same order of magnitude as v_B and the condensates could, therefore, have quasimomenta close to the edge of the Brillouin zone. In order to counteract this, we performed the acceleration experiments inside the magnetic trap, eliminating the velocity of the condensate relative to the lattice by phase modulating one of the lattice beams at the same frequency and in phase with the rotating bias field of the TOP trap. In this way, in the rest frame of the lattice the micromotion was compensated. Nevertheless, a residual sloshing of the condensate with amplitudes <3 μm could not be ruled out, so that the uncertainty in the initial velocity of the condensate was still around 0.5 mm s^{-1}, corresponding to $\approx 0.15v_B$ in this geometry. Figure 2(c) shows the results of a mea-

surement of the acceleration of the condensate as a function of the final lattice velocity for a theoretical lattice depth of $U_0 \approx 1.38E_B$ together with the theoretical curves for the same potential and the (assumed) effective potential $U_{eff} \approx 0.65U_0 \approx 0.88E_B$ as calculated from the condensate density using the perturbative expression of Eq. (1). As expected from the band-structure calculations, the Bloch oscillations are much less pronounced in this geometry.

In order to measure more accurately the variation of the effective potential U_{eff} with the interaction parameter C, we studied Landau-Zener tunneling out of the lowest Bloch band for small lattice depths in both geometries. To this end, the acceleration of the lattice was increased in such a way that when the condensate crossed the edge of the Brillouin zone, an appreciable fraction of the atoms tunneled across the band gap into the first excited band (and, therefore, effectively to the continuum, as the gaps between higher bands are negligible for the shallow potentials used here). According to Landau-Zener theory, this fraction is [23]

$$r = \exp\left(-\frac{\pi U_{eff}^2}{8\hbar p_B a}\right),\qquad(2)$$

giving a velocity $v_m = (1 - r)v_B$ of the condensate at the end of the acceleration process for a final velocity v_B of the lattice [24]. We verified that this formula correctly described the tunneling of the condensate in our experiment by varying both the potential depth and acceleration.

Thereafter, we studied the variation of the final mean velocity v_m as a function of the condensate density for the two lattice geometries. The density was varied by changing the mean frequency of the magnetic trap (from ≈ 25 Hz to ≈ 100 Hz). From the mean velocity the effective potential was then calculated using the Landau-Zener formula given above. Figure 3 shows the ratio U_{eff}/U_0 as a function of the peak density n_0 for the counterpropagating geometry and the angle geometry. As expected, the reduction of the effective potential is much larger in the angle geometry. The theoretical predictions of Eq. (1) are also shown in the figure, with the potential U_0 calculated taking into account losses at the cell windows and imperfections of the polarizations of the lattice beams. The residual combined error due to uncertainties in the absolute intensity measurements, the position of the beam axes relative to the position of the BEC, and a small initial velocity due to sloshing of the BEC in the magnetic trap, as well as a systematic error due to the difference in position of about 300 μm of the condensate between the weakest and the strongest trap used, was estimated at about 20%. Within these experimental uncertainties, qualitative agreement with theory was good [25]. Although we could realize peak densities up to 4×10^{14} cm^{-3} by using larger trap frequencies, data points for $n_0 > 10^{14}$ cm^{-3} in the counterpropagating lattice and $n_0 > 5 \times 10^{13}$ cm^{-3} in the angle geometry were
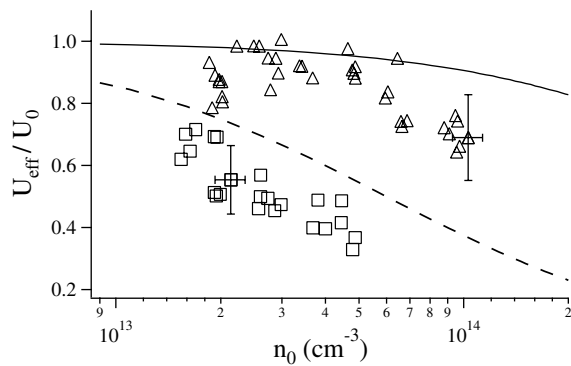
FIG. 3.   Dependence of the effective potential U_{eff} on the peak density n_0 for the two lattice geometries. The experimental results for the counterpropagating (triangles) and angle geometries (squares) are plotted together with the theoretical predictions (solid and dashed lines, respectively). Parameters in these experiments were $a = 23.4$ m s^{-2} and $U_0 = 0.28 E_B$ for the counterpropagating lattice and $a = 3.23$ m s^{-2} and $U_0 = 0.71 E_B$ for the angle geometry.

not included in the graph as the resulting diffraction patterns were not easily interpretable within the simple model described above [see Fig. 1(b)].

In summary, we have investigated the coherent acceleration and Bloch oscillations of Bose-Einstein condensates adiabatically loaded into a 1D optical lattice. Through Landau-Zener tunneling out of the lowest Bloch band, we have studied the dependence of the effective potential on the interaction parameter C. The results obtained are in good qualitative agreement with the available theories and extend the experimental work on ultracold atoms in optical lattices into the domain of Bose-Einstein condensates. In order to improve the theoretical description of our experiment, the finite extent of the condensate leading to the occupation of only a few lattice sites and the three-dimensional nature of the condensate evolution as well as the role of the interaction term in the adiabaticity criterion for switching on the lattice will have to be taken into account.

[1] For reviews see P. S. Jessen and I. H. Deutsch, Adv. At. Mol. Opt. Phys. **37**, 95 (1996); G. Grynberg and C. Triché, in *Coherent and Collective Interactions of Particles and Radiation Beams,* edited by A. Aspect, W. Barletta, and R. Bonifacio (IOS Press, Amsterdam, 1996); D. R. Meacher, Contemp. Phys. **39**, 329 (1998).

[2] M. Raizen, C. Salomon, and Q. Niu, Phys. Today **50**, No. 7, 30 (1997).

[3] See reviews by W. Ketterle *et al.* and by E. Cornell *et al.,* in *Bose-Einstein Condensation in Atomic Gases,* edited by M. Inguscio, S. Stringari, and C. Wieman (IOS Press, Amsterdam, 1999).

[4] K. Berg-Sørensen and K. Mølmer, Phys. Rev. A **58**, 1480 (1998).

[5] D. Jaksch *et al.,* Phys. Rev. Lett. **81**, 3108 (1998).

[6] D. Choi and Q. Niu, Phys. Rev. Lett. **82**, 2022 (1999).

[7] J. Javanainen, Phys. Rev. A **60**, 4902 (1999).

[8] A. Brunello *et al.,* Phys. Rev. Lett. **85**, 4422 (2000).

[9] O. Zobay and B. M. Garraway, Phys. Rev. A **61**, 033603 (2000).

[10] M. L. Chiofalo *et al.,* Eur. Phys. J. D **11**, 371 (2000).

[11] A. Trombettoni and A. Smerzi, Phys. Rev. Lett. **86**, 2353 (2001).

[12] S. Pötting *et al.,* e-print cond-mat/0102206, 2001.

[13] M. Kozuma *et al.,* Phys. Rev. Lett. **82**, 871 (1999); J. E. Simsarian *et al., ibid.* **85**, 2040 (2000); Yu. B. Ovchinnikov *et al., ibid.* **83**, 284 (1999).

[14] J. Stenger *et al.,* Phys. Rev. Lett. **82**, 4569 (1999); **84**, 2283(E) (2000).

[15] B. P. Anderson and M. A. Kasevich, Science **282**, 1686 (1998).

[16] C. Orzel *et al.,* Science **231**, 2386 (2001).

[17] S. Burger *et al.,* Phys. Rev. Lett. **86**, 4447 (2001).

[18] A single mode description leading to the Gross-Pitaveskii equation is justified for our lattice depths well below the tight binding limit.

[19] J. H. Müller *et al.,* J. Phys. B **33**, 4095 (2000).

[20] J. H. Müller *et al.,* Phys. Rev. Lett. **85**, 4454 (2000).

[21] A large lattice constant also means that only a few potential wells are occupied by the condensate.

[22] The adiabaticity criterion of Ref. [23] refers to interband transitions due to the rate of change of the band energies of noninteracting particles. This criterion was extended in [6] to the condensate on the basis of the band energies for the effective potential. However, other criteria such as collective excitations of the condensate [4] or the perturbation of the global condensate phase [[16] and C. Williams (private communication)], may impose more stringent conditions but have not been considered here. In the Landau-Zener tunneling experiments, we extended the ramping time to 3 ms.

[23] M. Ben-Dahan *et al.,* Phys. Rev. Lett. **76**, 4508 (1996); E. Peik *et al.,* Phys. Rev. A **55**, 2989 (1997).

[24] These expressions are exact only if the final velocity of the lattice is an integer multiple of v_B, as in that case the mean velocity of the condensate is independent of the band structure.

[25] The theoretical analysis of Ref. [6] assumes a uniform condensate density, while in our experiment there is a spatial dependence of the density due to the harmonic trapping potential. In calculating the theoretical values for U_{eff}/U_0 through the parameter C, we used the peak density of the condensate.