Generation of an Axial Magnetic Field from Photon Spin

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In circularly polarized light the spins of the photons are aligned. When a short intense pulse of circularly polarized laser light is absorbed by a plasma, a torque is delivered initially to the electron species, resulting primarily in an opposing torque from an induced azimuthal electric field. This electric field, in general, has a curl and leads to the generation of an axial magnetic field. It also is the main means for transferring angular momentum to the ions. The time-dependent magnetic field has a magnitude proportional to the transverse gradient of the absorbed intensity but inversely proportional to the electron density, in contrast to earlier theories of the inverse Faraday effect.

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Photons have spin $\hbar$, and in circularly polarized light these spins are aligned, so that a beam of intensity of finite radius has an angular momentum. The light beam can be represented as a number density n_ν of photons of energy $\hbar\omega$ moving in the z direction with the speed of light c. Thus, in vacuum, the intensity I is simply $n_\nu \hbar\omega c$, while the spin density is $\pm n_\nu \hbar$ or $\pm I/(\omega c)$ for right-hand $(+)$ and left-hand $(-)$ polarization and is independent of the energy of each photon. The local density of angular momentum is related to spin density in an analogous way as diamagnetic velocity is related to the density of magnetic moments. Thus, for a quasiaxisymmetric beam of circularly polarized light, the mean axial component of angular momentum $\overline{M}_z$ is given by

$$\overline{M}_z = -\frac{1}{\omega c}\frac{r}{2}\frac{\partial I}{\partial r}, \tag{1}$$

and is usually concentrated near the edge of the beam. The total angular momentum of the beam represents the total spin, however, as can be seen from the integral,

$$\int_0^{r_0} \overline{M}_z 2\pi r\, dr = -\frac{2\pi}{\omega c}\left\{\left[\frac{r^2 I}{2}\right]_{r=0}^{r_0} - \int_0^{r_0} Ir\, dr\right\} = \frac{1}{\omega c}\int_0^{r_0} I 2\pi r\, dr, \tag{2}$$

where I is 0 at $r \geq r_0$, the beam radius.

The angular momentum of light is discussed by Heitler [1], and he makes reference to a measurement by Beth [2] of the mechanical angular momentum transferred to a screen that absorbs circularly polarized light. In a recent article, Padgett and Allen [3] distinguish further between orbital angular momentum associated with helical wave fronts and photon spin, but this effect will not be included in this paper.

When the absorbing medium is a plasma, the angular momentum of the absorbed photons is transferred primarily to the electron species, i.e., the electrons experience a torque. If this were simply translated into a change of angular momentum of the electrons, it would constitute a very large azimuthal current and associated axial magnetic field. The resulting induced azimuthal electric field E_θ opposing this generation of magnetic field would, however, especially in the typical case considered here of a short- ($\sim$1 ps) laser pulse of intensity $>10^{18}$ W cm^{-2} propagating through an underdense plasma, be a much larger term. Indeed, to a good approximation, the torque from the absorbed laser light is instead largely balanced by an equal and opposite torque associated with E_θ, and the inertia of the electrons plays only a small role. There is a reversed torque by E_θ on the ion species, and it is through this and collisions with electrons that the ions also acquire angular momentum, which after the laser pulse is over will represent most of the absorbed angular momentum.

The equation for the mean rate of change of angular momentum of the electrons per unit volume can be written as

$$n_e m_e r \frac{dv_{e\theta}}{dt} = -n_e e r E_\theta - n_e e r (v_{ez} B_r - v_{er} B_z) + \frac{\alpha_{ab}\overline{M}_z c}{L} - n_e m_e \nu_{ei} r v_{e\theta}, \tag{3}$$

where α_{ab} is the fraction of the laser intensity absorbed over an axial distance L; n_e, v_e, and ν_{ei} are the electron number density, velocity, and collision frequency with the ions, and $\overline{M}_z$ is the density of angular momentum in the z direction averaged over a wavelength. The magnetic field terms are not important in the early stage. It will be verified later that the inertial and collisional terms are small so that Eq. (3) is essentially

$$rE_\theta \simeq -\frac{\alpha_{ab} r}{n_e e \omega L}\frac{\partial I}{\partial r}, \tag{4}$$

while Faraday's law gives

$$\frac{\partial B_z}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta). \tag{5}$$

Thus, the axial magnetic field B_z can be expressed as

$$B_z = -\frac{2}{\omega L e r_0^2}\int \frac{\alpha_{ab} I_0}{n_e}\, dt, \tag{6}$$

assuming that B_z is uniform within the beam radius, i.e., I is $I_0(1 - r^2/r_0^2)$, a parabolic intensity profile.

The inverse dependence of B_z on n_e should be noted, as this contrasts with earlier theories of the inverse Faraday effect. This will be discussed later. Of course as n_e goes to zero, α_{ab} also tends to zero faster and there is no singularity. Of interest in this formula is that, if this axial magnetic field is measured by Faraday rotation of a probing laser beam, the angle of rotation is proportional to $n_e BL$, which is a direct factor of Eq. (6). An experimental measurement should therefore be quite robust provided the absorption is measured. In Najmudin *et al.* [4], a straightforward substitution of the experimental parameters $t = 10^{-12}$ s, $I_0 = 7.3 \times 10^{22}$ W/m^2, $r_0 = 10^{-5}$ m, $\omega = 1.79 \times 10^{15}$ s^{-1}, $n_e = 2.1 \times 10^{25}$ m^{-3}, $L = 10^{-3}$ m and assuming $\alpha_{ab} = 1$ gives $B_z = 240$ T (2.4 MG) compared to the experimental measurement for this density of 4 MG. However, the laser power of 2.2×10^{13} W greatly exceeds that for self-focusing [5,6]. This effect will increase the intensity I and reduce the beam radius r_0, leading to a very sensitive increase in B_z in Eq. (6). Indeed the critical value of laser power for self-focusing, calculated assuming only weak relativistic effects, is 8.5×10^{11} W compared to the laser power of 2×10^{13} W in the experiment. A radial expansion of the heated plasma in the focal spot of the laser will also occur, lowering the electron density. This can be estimated as follows, assuming that the electron pressure in the focal spot increases as $\frac{2}{3}\alpha_{ab} It/L$. The ions as well as the electrons will have time to expand radially as can be seen from integrating

$$\frac{n_e m_i}{Z}\frac{dv_r}{dt} = -\frac{\partial}{\partial r}\left(\frac{2}{3}\frac{\alpha_{ab} It}{L}\right), \tag{7}$$

to give the time for cavitation t_c,

$$t_c = \left(\frac{9 n_e m_i r_0^2 L}{2Z\alpha_{ab} I}\right)^{1/3}, \tag{8}$$

which for $Z = 2$, $m_I = 4m_p$ (helium), and the above parameters gives 10^{-12} s. Ponderomotive forces will also cause plasma expansion and later also the axial magnetic field pressure itself. There could also be a pinching azimuthal magnetic field set up by absorption of photon momentum and ponderomotive forces, not considered here. That quasineutrality is a good approximation (i.e., electron cavitation with no ion motion is a poor assumption on these time scales) can be shown by calculating $\delta n/n = \varepsilon_0 T/(e r_0^2 n_e)$ for an electron temperature T_e of 1 MeV and r_0 and n_e as above. Then $\delta n/n$ is 0.03. The other assumptions in the model, namely, that the electron inertia and collisions are small, can be tested by comparing their values to the source term in Eq. (3); their relative values are 0.03 and 5×10^{-8}, respectively, the latter for a classical collision frequency at $T_e = 1$ MeV. If an anomalous collision frequency is triggered, this argument may be modified.

Generally, there is reasonable agreement within the uncertainties of the experiment and the simplifications of the model. Of importance is the trend for higher magnetic fields at higher intensities and lower electron densities, which is seen in the experiment.

It should be noted that, though the photon spin is $\hbar$, the angular momentum density in the circular polarized laser beam is $-\frac{1}{2} r\partial I/\partial r/\omega c$ and is a classical expression, which should be described by the electric and magnetic fields of a laser beam propagating *in vacuo*. It is instructive to consider first a plane-polarized beam of circular cross section πr_0^2. If the electromagnetic wave is represented solely by

$$E_x = E_0(r)\cos(\omega t - kz), \tag{9}$$

$$B_y = B_0(r)\cos(\omega t - kz), \tag{10}$$

with these fields shown in Fig. 1, there is an obvious problem of how to satisfy $\nabla \cdot \mathbf{B} = 0$ and $\nabla \cdot \mathbf{E} = 0$ for fields that appear to originate and end at the edges of the beam. It is clear that axial components of both fields of the form and magnitude,

$$E_z \sim \mp\frac{E_0}{kr_0}\sin(\omega t - kz), \qquad x > 0/x < 0, \tag{11}$$

$$B_z \sim \mp\frac{B_0}{kr_0}\sin(\omega t - kz), \qquad x > 0/x < 0, \tag{12}$$

must exist to satisfy $\partial E_x/\partial x = -\partial E_z/\partial z$ and $\partial B_x/\partial x = -\partial B_z/\partial z$. At the same time, Faraday's and Ampere's laws $\partial E_x/\partial y = \partial B_z/\partial t$ and $\partial B_y/\partial x = \mu_0\varepsilon_0\partial E_z/\partial t$ are satisfied with this ordering, the precise functional form depending on the beam profile. The field lines are thus propagating as closed loops as illustrated in Fig. 2.

When a second but orthogonal plane-polarized wave, 90° out of phase, is added, with fields

$$E_y = E_0(r)\sin(\omega t - kz), \tag{13}$$

$$B_x = -B_0(r)\sin(\omega t - kz), \tag{14}$$

together with axial fields,

$$E_z \sim \pm\frac{E_0}{kr_0}\cos(\omega t - kz), \qquad y > 0/y < 0, \tag{15}$$

$$B_z \sim \mp\frac{B_0}{kr_0}\cos(\omega t - kz), \qquad y > 0/y < 0, \tag{16}$$

a rotating component of the Poynting vector is obtained. This is because the E_z field of the first plane wave given by Eq. (11) is exactly in phase with the B_x field [Eq. (14)] of the second wave, and leads to a $\pm y$ component of the Poynting vector above and below the y axis, respectively. Similarly, there is a contribution in the $\pm x$ direction to the

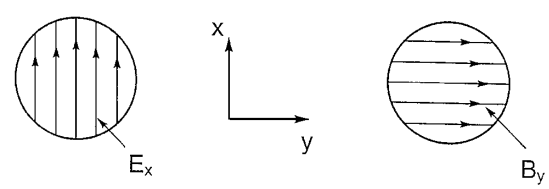

FIG. 1. Electric and magnetic fields in the cross-sectional plane for a plane-polarized wave.

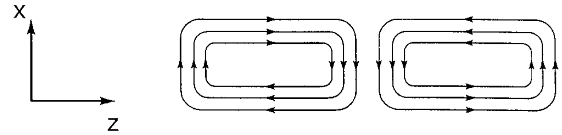

FIG. 2. Electric field in the x-z plane for a plane-polarized wave.

left and the right of the x axis as shown in Fig. 3 at $t = 0$, $kz = \pi/2$. Similar arguments can be made at $t = 0$, $z = 0$ for the axial components of the second wave interacting with the transverse components of the first wave. In total, there is a local density of angular momentum [7] $\mathbf{M}$ given by

$$\begin{aligned}\mathbf{r} \times \mathbf{p} &= \varepsilon_0 \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) \\ &= \varepsilon_0[(\mathbf{r} \cdot \mathbf{B})\mathbf{E} - (\mathbf{r} \cdot \mathbf{E})\mathbf{B}] = \mathbf{M}, \end{aligned} \tag{17}$$

the component of interest being axial.

We thus find that the mean azimuthal component of the Poynting vector is $\sim I/(kr_0)$ or rather $-\frac{1}{2}\partial I/\partial r/k$, and the mean angular momentum density $\overline{M}_z$ is $-\frac{1}{2}r\partial I/\partial r/\omega c$ as stated earlier. There is thus complete equivalence between the photon spin picture and classical fields provided the longitudinal field components are included. We note in Fig. 3 that there is a rotating quadrupole structure in the x-y plane. It is through the absorption of this angular Poynting flux, rather analogous to the ponderomotive force in the longitudinal direction, that electrons and ions acquire a torque. (Reflection of circularly polarized light leads to no deposition of angular momentum but, in contrast, doubles the axial linear momentum deposited.)

The theory of axial magnetic field generation resulting in Eq. (6) contrasts markedly with existing theories of the inverse Faraday effect. The earliest theories, e.g., Deschamps *et al.* [8] and Steiger and Woods [9], noted that the quiver trajectory of electrons in a circularly polarized laser light are circles, each of radius $eE_0/m_e\omega^2$, and claimed that therefore these lead to a magnetization per unit volume $\mathbf{M}_v$ given by

$$\mathbf{M}_v = -\frac{n_e e^3 E_0^2}{2m_e^2\omega^3}\,\hat{\mathbf{z}}, \tag{18}$$

the magnetization current density being $\nabla \times \mathbf{M}_v$. Associated with this, it is further postulated there is an axial magnetic field $\mathbf{B}$ in the $-z$ direction for right-hand circularly polarized light propagating in the $+z$ direction, and is given by

$$\begin{aligned}\mathbf{B} = +\mu_0\mathbf{M}_v &= -\frac{1}{2}\,a^2\,\frac{\omega_p^2}{\omega^2}\,\frac{m_e\omega}{e}\,\hat{\mathbf{z}} \\ &= -\frac{1}{2}\,\frac{\omega_p^2}{\omega^2}\,\frac{I}{c}\,\frac{e}{m_e\omega}\,\hat{\mathbf{z}},\end{aligned} \tag{19}$$

where a is $eE_o/(m_e\omega c)$. Note that in this formula the magnetic field is instantly on (or off) with the laser field (i.e., the E_θ field is ignored) and is proportional to n_e (through ω_p^2) in contrast to Eq. (6).

This formula was modified by Sheng and Meyer-ter-Vehn [10] to include the inhomogeneity of both the electron density and the laser beam together with the change in dispersion arising from the existence of the axial magnetic field. Another variation was found by Bychenkov, Demin, and Tikhonchuk [11] for a laser beam of short length. Lehner [12,13] added in another magnetic field source due to the ponderomotive force proportional to $\nabla(E^2)$ or $\nabla\gamma$ and assumed that a conduction electron current is free to move in this direction. (Presumably, such a force would be in the axial and radial directions in this model and would lead rather to an azimuthal component of magnetic field.) Gorbunov and Ramazashvili [14] assume in the individual electron motion that the canonical angular momentum is conserved (which essentially requires azimuthal symmetry), and, expanding in powers of the normalized quiver momentum, impose the constraint that this parameter has a zero divergence. As a result, an axial magnetic field arises only in the second order and is proportional to $n^{1/2}I^2$ in contrast to Eq. (19). Horovitz *et al.* [15] compare their experimental measurements with the formula in Ref. [10] and find that their results are a factor of 50 higher; in a later paper [16] with a wider range of intensity, better agreement was found using Lehner's model [12] where the magnetic field scales as $n^{1/2}I^{1/2}$. At intensities above 10^{13} W cm^{-2}, the experiment gave higher values than this theory. The angular momentum of the light was considered [15] but the authors assumed that the electrons acquired a fraction of the instantaneous angular momentum density in the beam, regardless of whether there is absorption, and by comparison with the formula in Ref. [10] determined this fraction.

Berezhiani *et al.* [17] criticize the earlier works (e.g., Refs. [9,11]) and develop a theory involving conservation of canonical angular momentum (i.e., assuming azimuthal symmetry) but including the development of parallel or

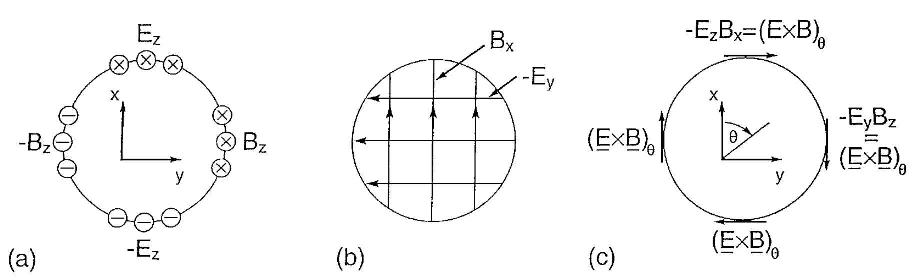

FIG. 3. The beam cross section at $t = 0$, $kz = \pi/2$ for (a) beam 1, (b) beam 2, and (c) the combined azimuthal Poynting flux.

axial electric fields. Even though these are small, they find a significant reduction in the generated magnetic field which now has a value independent of the electron density.

The main inconsistency in these papers is to consider that the circular orbits of electrons produce an axial magnetic field. Aside from the complications of parallel field components at the edge of the laser beam, the orbits merely arise from a linear superposition of two plane-polarized beams 90° out of phase. Each beam satisfies Maxwell's equations in the plasma with no axial magnetic field being generated. It is incorrect to use the currents associated with the quiver velocity twice. Besides, at any instant in time in any x-y cross section, the electrons' quiver velocity is unidirectional and not circulating. Similarly, the integral $\oint \mathbf{E} \cdot \mathbf{dl}$ in the x-y plane within the region of uniform intensity is zero at any instant.

In addition, the previous theories neglect the induced E_θ azimuthal electric field which would oppose the instantaneous creation or decay of the axial magnetic field. Even without this, the postulated circulatory currents would cancel everywhere except at the edge of the beam (for a uniform case), rather like diamagnetism. But, as has been shown, the physics is indeed in the edge of the beam, but due instead to a rotating Poynting flux associated with a rotating quadrupole structure of fields involving parallel components.

In conclusion, a mechanism has been found to generate an axial magnetic field through the deposition of the spin of the photons during the absorption of circularly polarized light [18]. Fields in the megagauss range can be obtained with intense short-pulse laser beams ($\sim 10^{19}$ W cm^{-2}) as has been found experimentally. The model is being further developed to include the diffusion of this magnetic field and its saturation, not least by plasma expansion (cavitation) from the heated region and ponderomotive forces.

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