Saturation of Bunch-Wave Interaction in an Active Medium

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We determine the set of equations which describe the dynamics of electrons in the presence of a wave propagating in an active medium. Simulation results indicate that, even when virtually all the energy is drained from the medium, electrons remain trapped by the accelerating wave. In spite of saturation, gradients of a few GV/m may become available.

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Acceleration of electrons by radiation at optical wavelength is one of the most promising alternatives for future electron acceleration. Generally speaking, the optical schemes may be divided into two main groups: In the case of *plasma* based schemes, a laser pulse is injected in plasma where it excites a space-charge wake that may in turn accelerate a trailing bunch of electrons [1–4]. Another group corresponds to the various *inverse radiation* processes such as inverse Cherenkov [5–7], inverse free electron laser [8,9], and inverse transition radiation [10,11]. In the case of an inverse radiation process, the laser pulse is injected at identical conditions as when the radiation is emitted by electrons propagating in the structure. In all these schemes, energy stored in an active medium is transformed into radiation in the cavity of the laser, and the output radiation is injected in a different structure for acceleration of the electrons.

Recently, it was suggested [12,13] to use energy stored in an active medium for acceleration of electrons. Specifically, it was demonstrated that a Cherenkov wake generated by a small trigger bunch may be amplified by the medium and the amplified wake, in turn, may accelerate a second bunch trailing behind; see Fig. 1. This concept was originally demonstrated in the framework of a *linear* theory. In this Letter, we formulate a set of equations that determine the *nonlinear* interaction of electrons and waves in an active medium.

In order to envision a typical system, consider a Nd:YAG rod with a small vacuum tunnel bored along the axis where the bunch of electrons is injected. An adjacent flashlamp generates the population inversion of the medium, and for the initial synchronization of the electrons with the radiation field, a relatively weak field is assumed to be generated by a trigger bunch. Both the flashlamp and the Nd:YAG rod are surrounded by a metallic resonator illustrated in the top-right frame of Fig. 1.

The analysis that follows focuses on a single mode that is assumed to interact with the electrons, and therefore this assumption needs justification since the typical transverse dimension of a rod is much larger than the wavelength, e.g., 6 mm diameter vs 1 μm wavelength. Single mode operation is a result of a double selection process: synchronism between the electromagnetic mode and the filtering linked to the long lifetime of the metastable state (>200 μs) that determines the width of the spectrum of amplification. With these assumptions in mind, we may rely on the theory of the beam-wave interaction in a *passive* electromagnetic structure that may be summarized by

$$\frac{d}{d\xi}\gamma_i = -\frac{1}{2}(\mathcal{E}e^{j\chi_i} + \text{c.c.}), \tag{1}$$

$$\frac{d}{d\xi}\chi_i = \Omega\left(\frac{1}{\beta_i} - \frac{1}{\beta_{\text{ph}}}\right), \tag{2}$$

$$\frac{d}{d\xi}\mathcal{E} = \alpha\langle e^{-j\chi_i}\rangle. \tag{3}$$

Here, $\xi \equiv z/d$ is the normalized coordinate, d is the interaction length, $\mathcal{E} \equiv eE_z d/mc^2$ is the normalized electric field that interacts with the electrons, $\Omega \equiv \omega d/c$ is the normalized angular frequency, and α is the normalized

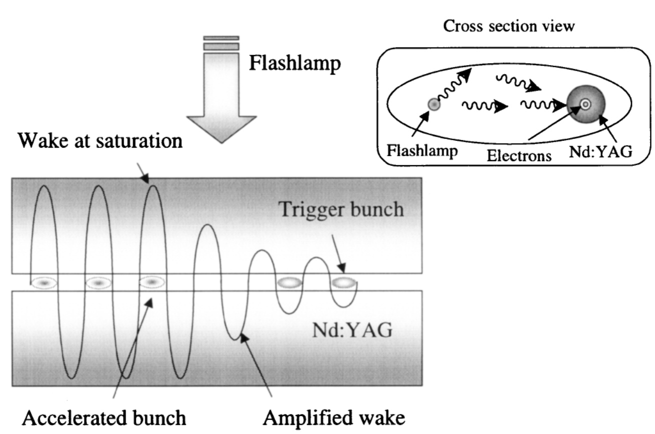

FIG. 1. Conceptual schematics of the system under consideration. A dielectric rod (Nd:YAG) excited by a flashlamp forms an active electromagnetic structure; both are surrounded by a metallic cavity that ensures efficient excitation of the medium and confinement of the electromagnetic mode as may be concluded from the cross section view (top right). The wake generated by the trigger bunch is amplified by the medium, and in parallel this wake accelerates another group of electrons trailing behind.

coupling coefficient between the electromagnetic mode and the beam [$\alpha \propto I$; I is the peak current in one period of the wave]; $\langle \ldots \rangle$ represents the ensemble average of all particles in one bunch. For more details, see Ref. [14].

Within the framework of a one-dimensional theory, Eq. (1) is the equation of motion, Eq. (2) determines the change in the phase (χ_i) of the ith particle relative to the interacting wave, and Eq. (3) determines the change in the amplitude of the interacting wave, $\mathcal{E}$, in terms of the particles' dynamics. It is important to point out that this set of equations tacitly ignores internal reflections and temporal variation of the various quantities due to the interaction. Both are justified by the fact that we shall consider here a one-pass process that occurs on the time scale it takes the bunch to traverse the interaction region.

Energy conservation linked with Eqs. (1)–(3) may be deduced by averaging the single particle equation of motion, (1), over the entire ensemble of particles and substituting the amplitude equation, (3), hence,

$$\frac{d}{d\xi}\left[\underbrace{\langle \gamma_i \rangle - 1}_{\text{kinetic energy}} + \underbrace{\frac{1}{2\alpha}|\mathcal{E}|^2}_{\text{EM energy}}\right] = 0. \tag{4}$$

The first term represents the average kinetic energy of the particles, whereas the second term is the electromagnetic energy *per electron*; both quantities are normalized with the rest mass energy of the electrons located in one period of the wave, $N_e mc^2$.

After introduction of the equations that describe the beam-wave interaction in a *passive* electromagnetic structure, we now extend the analysis to an *active* structure. For this purpose consider a unit volume, determined by the effective cross section of the system and one wavelength: In this volume the kinetic energy is $N_e mc^2(\langle \gamma \rangle - 1)$, the electromagnetic energy is $N_e mc^2(|\mathcal{E}|^2/2\alpha)$, and it is now assumed that the energy stored in N_{ex} excited atoms is $N_{ex}\hbar\omega$. Consequently, the normalized energy stored in excited atoms per electron of the microbunch is given by $N_{ex}\hbar\omega/N_e mc^2$ and energy conservation may be deduced to read

$$\frac{d}{d\xi}\left[\langle \gamma_i \rangle - 1 + \frac{1}{2\alpha}|\mathcal{E}|^2 + \frac{N_{ex}\hbar\omega}{N_e mc^2}\right] = 0. \tag{5}$$

This expression reflects the fact that, for acceleration purposes, the nominal density of excited atoms has to exceed the electron density by many orders of magnitude. For example, if the input electromagnetic power is negligible and it is necessary to accelerate 10^5 electrons of 300 MeV to an energy of 3 GeV, then the number of excited atoms in one period of the wave (1 μm) must be at least 3×10^{14}. Assuming a Nd:YAG slab of 6 mm diameter, the required density of excited atoms is of the order of 10^{19} cm^{-3}, which is within the range of parameters of available materials, e.g., 1.38×10^{20} cm^{-3} as quoted in Ref. [15].

Our next step is to determine the dynamics of excited atom density due to combined interaction of electrons, radiation, and medium. In the absence of the electron beam and based on phenomenological laser theory [16], the amplitude of the mode varies in space according to $\frac{d}{d\xi}\mathcal{E} = (\frac{1}{2}\sigma n_{ex} d)\mathcal{E}$, wherein σ is the transition cross section and n_{ex} is the effective *density* of excited atoms. The difference between the actual density of excited atoms and the effective one is roughly the ratio between the cross section of the active medium (A_{am}) and the effective area of the wave (A_w), i.e., $n_{ex} = N_{ex}/\lambda A_w = (N_{ex}/\lambda A_{am})F_f$, wherein the form factor is approximately given by $F_f = A_{am}/A_w$, and $N_{ex}/\lambda A_{am}$ is the density of the population inversion in the material.

When electrons are present in the active electromagnetic structure, the change in amplitude of the interacting mode may be assumed to be given by the superposition of both contributions, namely,

$$\frac{d}{d\xi}\mathcal{E} = \alpha\langle e^{-j\chi_i}\rangle + \left(\frac{1}{2}\sigma n_{ex} d\right)\mathcal{E}. \tag{6}$$

Here, it is tacitly assumed that the decay times (T_1 and T_2) associated with the active medium are much longer than the laser-pulse duration, which in turn is much shorter than the time it takes the bunch to traverse the interaction region. In addition, it is assumed that the operating frequency is virtually identical to the resonant frequency of the medium. Further, we assume that, in the presence of the active medium, the dynamics of the particles is determined only by the longitudinal component of the electric field; in other words, Eq. (1) is valid also in an active medium. Moreover, it is assumed that the active medium does not affect directly the relative phase of the ith particle and therefore Eq. (2) remains unchanged.

Although the presence of the dielectric medium may contribute a Cherenkov-type force [17–20], this is ignored in the framework of the present analysis. Consequently, averaging Eq. (1) and substituting Eq. (6), we obtain

$$\frac{d}{d\xi}\left[\langle \gamma_i \rangle - 1 + \frac{1}{2\alpha}|\mathcal{E}|^2\right] = \left(\frac{1}{2\alpha}|\mathcal{E}|^2\right)(\sigma n_{ex} d). \tag{7}$$

This last expression together with the expression for energy conservation in (5) determine the dynamics of the population inversion, N_{ex}, namely,

$$\frac{d}{d\xi}N_{ex} = -\left(\frac{1}{2\alpha}|\mathcal{E}|^2\right)\left(\frac{\sigma}{A_{cm}}\frac{d}{\lambda}F_f\frac{mc^2}{\hbar\omega}N_e\right)N_{ex}. \tag{8}$$

It is convenient to summarize the equations that determine the bunch-wave interaction in an active electromagnetic structure in the following form:

$$\frac{d}{d\xi}\gamma_i = -\frac{1}{2}(\mathcal{E}e^{j\chi_i} + \text{c.c.}), \tag{9}$$

$$\frac{d}{d\xi}\chi_i = \Omega\left(\frac{1}{\beta_i} - \frac{1}{\beta_{\text{ph}}}\right), \tag{10}$$

$$\frac{d}{d\xi}\mathcal{E} = \alpha\langle e^{-j\chi_i}\rangle + \frac{1}{2}\nu N\mathcal{E}, \tag{11}$$

$$\frac{d}{d\xi}N = -\nu\left(\frac{1}{2\alpha}|\mathcal{E}|^2\right)N, \tag{12}$$

wherein $N \equiv N_{\text{ex}}\hbar\omega/N_{\text{e}}mc^2$ is the normalized population inversion (excited atoms) and $\nu \equiv [(\sigma d)/(A_{\text{am}}\lambda)] \times [(mc^2)/(\hbar\omega)]N_{\text{e}}F_{\text{f}}$ is the normalized transition cross section. Equations (9)–(12) describe the interaction of a bunch of electrons with an electromagnetic wave propagating in an active medium. This is the main result of this Letter. In the remainder, we shall present simulation results of this set of equations.

Consider an optical system (Nd:YAG) operating at 1.06 μm and a 2 mm long macrobunch of 10^7 electrons. Their kinetic energy is 30 GeV with a standard deviation of 0.5%; the interaction length is assumed to be $d = 1$ m, and $\mathcal{E}$ at the input equals 5.0 corresponding to 2.5 MW of power required to "synchronize" the bunch and the wave. Two different bunch configurations will be considered: In one case the macrobunch is uniform and in the second case, the bunch consists of 2 mm/1 μm $\sim$ 2000 microbunches, each one being initially 6 nm (2°) long. Since the peak current is 2 orders of magnitude larger in the latter case, the coupling coefficient α is 0.6 for a uniform bunch and 107 for the case of a microbunch train. Finally, the energy assumed to be stored in the medium corresponds to two different (linear) gains, 5 and 50 dB/cm.

Figure 2 illustrates the spatial growth of the accelerating gradient for all four cases. Several conclusions are evident: (i) In all four cases the gradient saturates. (ii) For the case of a microbunch train gradients up to 3 GV/m may become available; this is probably more than breakdown constraint may allow. (iii) For a single macrobunch the gradient saturates already at 200 MV/m, implying that saturation is affected by the current density. (iv) The location where saturation occursvadjust depends on

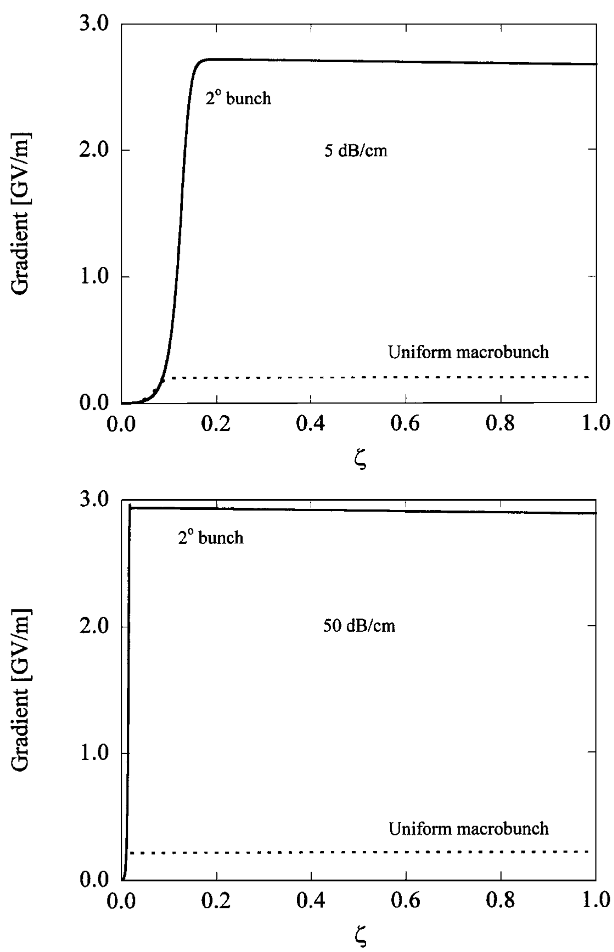

FIG. 2. Spatial growth of the accelerating gradient. The current density of the bunch affects significantly the saturation level.

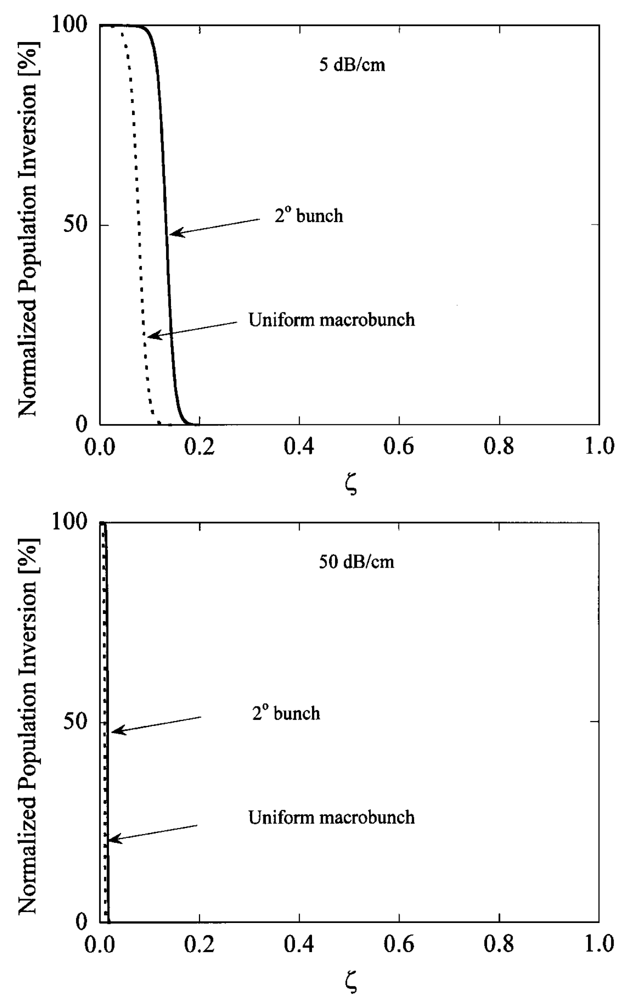

FIG. 3. Spatial dependence of the normalized population due to the interaction with the wave and the electrons.

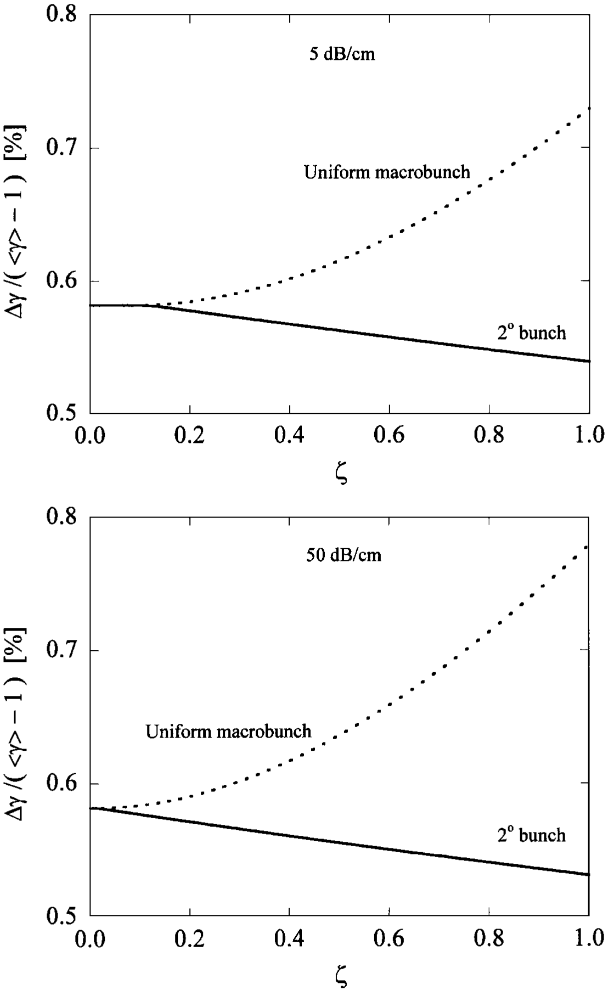

FIG. 4. Energy spread of the electrons along the interaction region.

the (linear) spatial growth rate. (v) In the case of the microbunch train, both frames show a small gradual reduction of the gradient, which is indicative of a beam loading effect.

Saturation is even more clearly revealed by Fig. 3, illustrating the (normalized) population inversion in all four cases. It shows that energy is drained from the medium within a distance controlled by the linear spatial growth, namely, the initial energy stored in the medium. In the case of the microbunch train, the current density is higher and so is the saturation level; therefore, given the spatial growth rate, the saturation is reached in a longer distance in comparison with a single macrobunch.

As the electromagnetic wave is amplified by the medium, it accelerates the microbunch and, since saturation is reached, there is no significant difference between the electrons' final energy in the case of 5 or 50 dB/cm. In either one of the cases, the kinetic energy of the microbunch train increases linearly in space. However, the energy spread of the electrons, as illustrated in Fig. 4, reveals a significant increase in the case of the macrobunch, whereas in the case of the microbunch train, the normalized energy spread actually decreases due to the increase in the energy of the electrons. In other words, saturation of the wave-medium interaction does not seem to degrade the energy spread of the electrons.

In conclusion, we have determined the set of equations that govern the dynamics of electrons in the presence of a wave interacting with an active medium. Simulation results indicate that, in a nonlinear regime, namely, when virtually all the energy is drained from the medium, electrons remain trapped by the accelerating wave and their energy spread is not deteriorated.

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