Ultrasonics without a Source: Thermal Fluctuation Correlations at MHz Frequencies

Richard L. Weaver and Oleg I. Lobkis
Theoretical & Applied Mechanics, University of Illinois, Urbana, Illinois 61801
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Noise generated in an ultrasonic receiver circuit consisting of transducer and amplifier is usually ignored, or treated as a nuisance. Here it is argued that acoustic thermal fluctuations, with displacement amplitudes of 3 fm, contain substantial ultrasonic information. It is shown that the noise autocorrelation function is the waveform that would be obtained in a direct pulse/echo measurement. That thesis is demonstrated in experiments in which direct measurements are compared to correlation functions. The thermal nature of the elastodynamic noise that generates these correlations is confirmed by an absolute measurement of their strength, essentially a measurement of the sample temperature.

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Diffuse wave fields are widely understood as revealing no detail about the medium in which they propagate. Waves which have scattered from random heterogeneities or reflected from generic surfaces are used to monitor global features such as the presence of time-varying properties [1], eigenstatistics [2], and internal friction and the density and strength of scatterers [3]. They are relevant in mesoscopic electronics [4], seismology [5], nondestructive testing [6], quantum chaos [7], and room acoustics [8]. But detailed information about individual scatterers is understood to be lost in the many reflections and interferences. This is particularly true with diffuse waves associated with thermal fluctuations, the sources of which are spatially distributed and random.

This cannot be the whole story, however. The diffuse field amplitude present at a particular position and time must be correlated with that present at neighboring positions and time. The Bessel function like spatial correlations of a narrow-band diffuse field are well known [9,10]; temporal correlations of narrow-band processes have a similar origin, being defined essentially by the width of the process. In addition to these relatively trivial correlations, there must be deeper correlations as well. Waves which visit one position will revisit at a later time, a time which corresponds to the round trip travel time of an echo. Recent ultrasonic work [11] in which deterministic diffuse fields were generated and detected observed autocorrelation times that corresponded to known round-trip travel times. Furthermore, and in accord with theoretical arguments, the detected waveforms' cross-correlation function was found to resemble the directly obtained pitch-catch signal between transducers. Differences were ascribed to the deterministic nature of the source, and to imperfectly compensated dissipation.

The work reported here avoids that difficulty by employing the diffuse field offered by room temperature thermal fluctuations. At frequencies below kT/h, $\approx$6 THz, thermal fluctuations have a flat spectrum, with each natural mode of vibration being randomly excited in a Gaussian process with equal mean energy kT. The amplitude (and phase) of each mode fluctuates, with coherence time equal to the absorption time of an ultrasonic excitation of that mode [12]. This field is truly diffuse, it has a flat spectrum; it does not dissipate. Except for its low amplitude, it is ideal for the present purposes: examination of the relationship between the correlations of a diffuse field and direct responses.

The field-field correlation function of acoustic thermal fluctuations is equal to the acoustic Green's function. Similarly, the autocorrelation of the noise in an ultrasonic receiver circuit is the impulse response of that receiver. This may be argued in two distinct ways. One analysis begins with the modal expansion for the material displacement, $\mathbf{U}$, in an elastodynamic field [13]

$$\mathbf{U}(\mathbf{x}, t) = \mathrm{Re} \sum_{n=1}^{\infty} a_n \mathbf{u}_n(\mathbf{x}) \exp\{i\omega_n t\}, \qquad (1)$$

where the ω_n are the natural frequencies of the body, the $\mathbf{u}_n$ are the fixed real vector-valued vibration modes of the finite solid body, and the a_n are the modal amplitudes. The modes are normalized: $\int \rho(\mathbf{x})\mathbf{u}_n(\mathbf{x}) \cdot \mathbf{u}_n(\mathbf{x})\, d^3\mathbf{x} = \delta_{nm}$ where ρ is the mass density. If the field is thermal, the a_n are uncorrelated slowly fluctuating random complex numbers with mean square proportional to the Boltzmann factor kT; $\langle a_n a_m^* \rangle = [2kT/\omega_n^2]\delta_{nm}$. If $\mathbf{U}$ is detected by a passive linear transducer with receiver function $\mathbf{R}$ then the voltage waveform produced is the temporal and spatial convolution of $\mathbf{U}$ with the receiver function, $V(t) = \int \mathbf{R}(\mathbf{x}, t - t') \cdot \mathbf{U}(\mathbf{x}, t')\, d^2x\, dt' + N(t)$. N is the receiver circuit's electronic noise. The integral is over all past time $t' < t$, and over the area of the transducer face. Neglecting N [14], the time derivative of V's autocorrelation, $P \equiv d\langle V(t)V(t + \tau)\rangle/d\tau$, is

$$P(\tau) = kT \sum_{n=1}^{\infty} |\mathbf{R}(\omega_n) \cdot \mathbf{u}_n|^2 \sin\{\omega_n \tau\}/\omega_n . \qquad (2)$$

Equation (2) resembles the modal expansion for the echo received in the transducer at times t after it is excited by an impulse. An ω dependence in R indicates a Fourier transform. The impulse/echo waveform is the convolution of the Green's function of the sample with the source and receiver functions. At times after the ring times of the transducers it is

$$E(t) = \mathrm{Im} \sum_{n=1}^{\infty} \mathbf{R}(\omega_n) \cdot \mathbf{u}_n \mathbf{u}_n \cdot \mathbf{T}(\omega_n) \exp\{i\omega_n t\}/\omega_n \,, \tag{3}$$

where $\mathbf{T}$ is the transducer's source function. The echo E is identical to P, if $\mathbf{T} = \mathbf{R}^*$. $\mathbf{T} \approx \mathbf{R}$ follows from the usual reciprocal properties of piezoelectric transducers. $\mathbf{R} \approx \mathbf{R}^*$ follows if the transducer has little phase, i.e., if it has a sharp response in the time domain. In the event that $\mathbf{T} \neq \mathbf{R}^*$, one may nevertheless conclude that the correlation P will be similar to E, albeit somewhat distorted. In the event of an ideal passive receiver, for example, optical interferometric detection of displacements in direction $\mathbf{n}$ at a point $\mathbf{r}$ for which $\mathbf{R} = \mathbf{n}\delta(\mathbf{x} - \mathbf{r})\delta(t - t')$, and on confining attention to positive τ, Eq. (2) reduces to the $\mathbf{nn}$ component of the elastodynamic Green's function, $G_{\mathbf{nn}}(\mathbf{r}, \mathbf{r}, \tau)$.

The above argument treats the receiver as passive, and does not consider acoustic reradiation by the noise. Its conclusion nevertheless also follows from an alternative argument in terms of the impedance Z presented by the receiver, which makes no assumption of passivity. Z contains electronic contributions from the amplifier and transducer, and mechanical contributions from the transducer and sample. Such an impedance will be composed of smooth and rapidly varying parts: $Z(\omega) = \overline{Z}(\omega) + z(\omega)$, where $\overline{Z}(\omega)$ is smooth in frequency and contains the short time difference information; it is mostly electronic. The fine frequency features are exclusively in $z(\omega)$ and are related through the properties of the Fourier transform to long time scales that are primarily acoustic echoes.

Consider the simplified source/receiver circuit in Fig. 1. A charge Q on the pulser's capacitor C is released by suddenly closing the switch; it drains, largely through the pulser resistance R and loads the transducer with an excitation pulse $V(t) = X(t)$. At early times while the potential across the diode bridge is large so it presents no resistance, $V = X$ is given by the inverse Fourier transform of $QRZ(\omega)/[i\omega CZ(\omega)R + R + Z(\omega)]$. For the usual case of large Z, $X(\omega)$ is approximately $QR/[1 + i\omega RC]$ and $X(t)$ is exponential with a decay time RC (typically $\ll \mu$sec). This is observed in practice. During this excitation period, the current through the transducer and amplifier is given by $I(\omega) = X(\omega)/\overline{Z(\omega)}$.

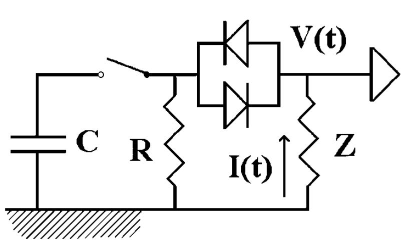

FIG. 1. Simplified circuit diagram for the pulser, transducer, and amplifier.

At later times, $t \gg RC$, the diode bridge presents an infinite resistance, and the total current through Z is zero. V is the convolution of Z with I, and I has support only for early times, so

$$V(t) = E(t) = \int_0^{\infty} Z(t - \tau) I(\tau)\, d\tau$$
$$= \int_0^{\infty} z(t - \tau) I(\tau)\, d\tau, \qquad t \gg RC \,. \tag{4}$$

On substituting $I(\omega) = X(\omega)/\overline{Z(\omega)}$ the echo E is found to be

$$E(\omega) = X(\omega) z(\omega)/\overline{Z}(\omega) \approx i\omega QRC_T z(\omega) \,. \tag{5}$$

The latter approximation follows from taking $\overline{Z}$ to be capacitive $\overline{Z}(\omega) = 1/i\omega C_T$ (as for most transducers and amplifiers and confirmed in separate measurements), and taking frequencies to be small; $\omega RC \ll 1$. Thus the pulse-echo signal E is (at late times $\gg RC$) the time derivative of the transducer/amplifier impedance. To the extent that $\overline{Z}$ may not be purely capacitive, E will be a distorted version of $\dot{z}$.

When the pulser is disconnected, thermal noise in the receiver circuit has a spectral power density $S_V(\omega)$ (the Fourier transform of V's autocorrelation function) given by the fluctuation-dissipation theorem [15].

$$S_V(\omega) = 2kT\, \mathrm{Re}[Z(\omega)]$$
$$= kT[\overline{Z(\omega)} + \overline{Z^*(\omega)} + z(\omega) + z^*(\omega)] \,. \tag{6}$$

Z is causal, the Fourier transform of a function without support at negative times. The inverse Fourier transform of the above is, at late times such that $\overline{Z}$ may be neglected and Z^* vanishes, $z(t)$. $P(\omega)$ is $i\omega$ times $S_V(\omega)$; therefore except for a constant factor kT/QRC_T, P is identical to E.

Both of these arguments suggest that the time derivative of the autocorrelation of the thermal noise in an ultrasonic receiver circuit will be, except perhaps at short times, identical to the directly obtained pulse-echo signal E. To confirm and illustrate these ideas, we have autocorrelated the noise from a sensitive piezoelectric acoustic emission transducer (Physical Acoustics model WD, diameter 15 mm with a useful frequency range to 1 MHz, impedance with cables ~325 pf) connected to low-noise ultrasonic preamplifiers (Panametrics models 5660B, input impedance 600 kΩ). The transducer was attached with oil couplant to the center of one of the parallel faces of a cylindrical aluminum body of diameter 17.8 cm, and thickness 10.2 cm. The noise signal was amplified by 100 dB, low-pass filtered at 4 MHz, digitized at 12.5 Msamples/sec, and passed to a PC. The sample, transducer, and amplifiers were isolated in a Faraday shield, a grounded foil-covered box. The autocorrelation was performed digitally and averaged by repeating the capture and calculation many times, until the procedure appeared to have converged. The result was compared to a conventional pulse-echo signal obtained by exciting the transducer with a 20 nsec 50 V pulse. Each signal was then digitally filtered to pass frequencies between 0.1 and 0.9 MHz, a process that removed high frequency electronic noise and ambient low frequency room vibrations. The two waveforms so obtained are shown in Fig. 2.

The waveforms are almost identical. Each shows the expected features. The first and second reflections from the opposite side are clear, as is the arrival of a Rayleigh wave reflected from the edge of the sample face. Differences may be attributed to the nonzero phase and imperfectly reciprocal transducer, $\mathbf{R}^* \neq \mathbf{T}$. Difference may also be due to contamination from long-lived correlations of nonacoustic noise N. The waveforms differ more at early times (not shown) where the nonacoustic noise is expected to contribute more substantially. That noise is weak enough and confined to early enough times that it does not overly contaminate the interesting features of the acoustic part.

Similar autocorrelations have recently been constructed from signals that are not thermal in origin, but rather due to diffuse insonification by a distant ultrasonic piezoelectric source [11]. Confirmation that the present waveform $P(\tau)$ is indeed due to thermal fluctuations is carried out by evaluation of P's amplitude in the diffuse regime, $\tau >$ msec, i.e., at times τ long compared to transit times across the sample. The measurements were repeated in a sample of irregular shape, but size similar to that of the cylinder used above. The irregular shape accelerates the formation of a diffuse field and aids in the application of statistical arguments for understanding the waveforms [15]. The earliest times were deleted from the waveform P as potentially contaminated by the noise N, or imperfectly diffuse. The remainder was narrow bandpass filtered [by a digital filter B centered on frequency ω_b, with bandwidth $b = \int |B(\omega)|^2 \, d\omega$], squared and time averaged. According to Eq. (2), the root mean square so constructed is proportional to kT, and to the square of the transducer sensitivity $\mathbf{R}$.

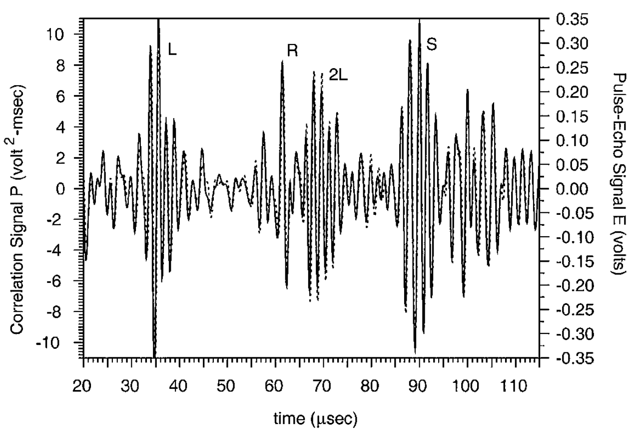

FIG. 2. Comparison of the noise autocorrelation function P (solid line) and the direct pulse/echo signal E (dotted line). They are substantially the same. The arrivals are not crisp, due to the long ring time of the transducer that was designed for sensitivity rather than resolution. Nevertheless, arrivals can be identified. The strong first arrival at 34 μsec is the longitudinal (L) ray reflected from the bottom. The arrival at 68 μsec (2L) is a longitudinal wave that has reflected from the bottom twice. The strong arrival (S) at 90 μsec is consistent with a shear wave that has reflected from a far bottom corner. The Rayleigh wave (R) arrival at about 60 μsec is also apparent. The autocorrelation was assembled from an average over 500 noise waveforms, each of 2.56 msec length, each digitized at 12.5 Msamples/sec.

The square of the transducer sensitivity also enters into an expression for the mean square response to a specified source. The transducer was therefore calibrated by evaluating its response to a broken glass capillary. This ultrasonic source applies a normal surface step force of magnitude $F_0 = 10$ N, and rise time 200 nsec [16]. Such a source generates a waveform $W(t)$ whose amplitude depends on transducer sensitivity. Arguments [15] for the statistics of the modes permit derivation of an expression for the late time value of the mean square of W, an expression that depends on transducer sensitivity in precisely the same way as does the root mean square of P. It also depends on the mean square amplitudes of normalized modes at the position of the broken capillary source. This is set to $p/3M$, where M is the mass of the sample, and p is a dimensionless quantity that represents the degree to which surface motions participate in a diffuse field [5,17]. In aluminum, $p = 2.33$. The result is an expression in which the only unknown is transducer sensitivity. On taking a ratio with the root mean square P, one constructs an estimate for temperature in which the unknown transducer sensitivity does not appear.

$$kT = \frac{\mathrm{rms}_B\{P\}}{\mathrm{ms}_B\{W\}} \frac{\sqrt{Db}\, F_0^2 p}{6M\omega_b^3}. \tag{7}$$

D is the sample's modal density at the frequency ω_b.

In order to confirm the thermal phonon origin of the waveform P, the temperature as determined from (7) was compared with the known sample temperature. Figure 3 shows the late time (after 1 msec, before 9 msec) power spectra of the autocorrelation signal P and the directly obtained pulse-echo signal E obtained in the irregular block. As in Fig. 2, they are almost identical. Differences are ascribed to an imperfectly reciprocal transducer or to the presence of two factors of the antialiasing filter in P and only one such factor in E. The figure also shows the power spectrum of the response, W, to the broken glass capillary.

The construction (7) is shown in Fig. 4 where the strength of the source of the correlation P is evaluated in successive 40 kHz bands. The spectra of W and P are those shown in Fig. 3. The agreement between the recovered temperature and the known room temperature of 293 K is good. The recovered temperature rises a bit at high frequency, due to our error in assuming a constant F_0 at all frequencies; the capillary source has a finite rise time and lacks high frequency components.

That thermal phonons can be detected in the ultrasonic regime and used to construct ultrasonic waveforms is astonishing. A simple calculation establishes that the thermal displacement amplitudes whose statistics are responsible

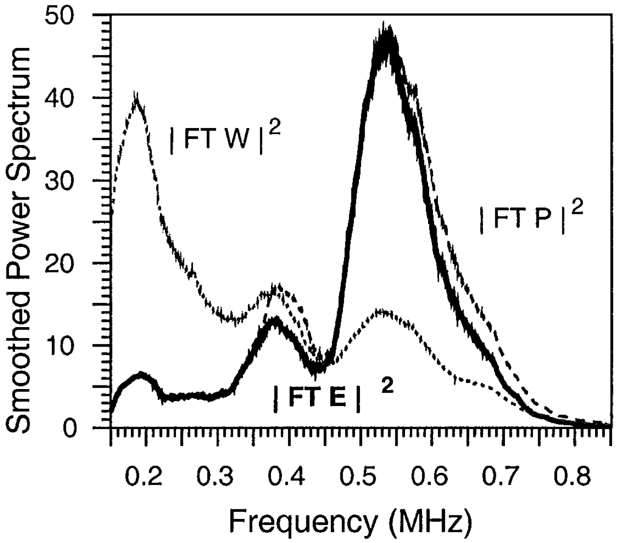

FIG. 3. The power spectra (smoothed square of Fourier transform) of the correlation signal P (dashed line), the direct pulse/echo signal E (bold line), and the signal W detected after breaking a glass capillary (dotted line), are compared. Units are arbitrary and the spectra have been scaled to similar magnitude. Each was constructed from data in the interval [$1 < t < 9$ msec] in which there is little contamination from nonacoustic noise and little absorption.

for the waveform P are about $[2pkT\Delta f/9\pi\rho c^3_{\text{shear}}]^{1/2} \approx 3$ fm in a band of width $\Delta f = 1$ MHz. That ultrasonic waveforms are recovered from such small mechanical motions is testimony to the great sensitivity of modern piezoelectric devices. We entertain the possibility that these observations will have applications. Thermal fluctuations, with spectra ranging well above the conventional ultrasonic regime that normally concludes at a few GHz [18], might prove useful for constructing very high frequency ultrasonic waveforms and noninvasively probing micron sized features [19] and material properties such as dispersion [20] and attenuation and stress in modern materials.

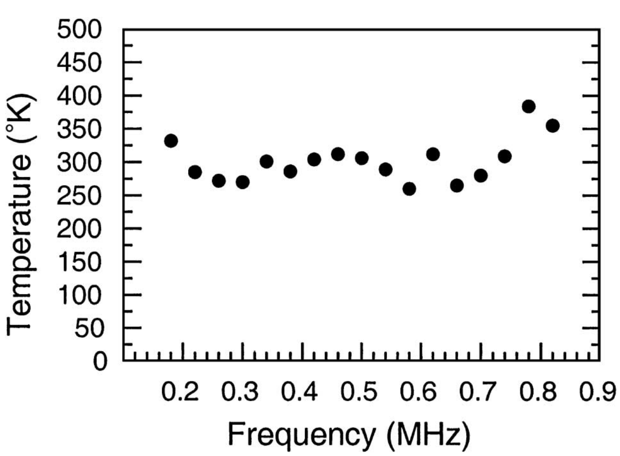

FIG. 4. The sample temperature as recovered using Eq. (7). That the temperature recovered is equal to the known room temperature is a confirmation of the thermal phonon origin of the transducer noise.

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