Comment on "Scaling of the Conductivity with Temperature and Uniaxial Stress in Si:B at the Metal-Insulator Transition"

Bogdanovich, Sarachik, and Bhatt [1] (BSB) recently employed uniaxial stress (S) to tune Si:B through the metal-insulator transition and fit the data to the finite-T scaling expression $\sigma(t,T) = AT^x f[(t - t_c)/T^y]$ where t is a control parameter (dopant density or stress). BSB obtain $x = 0.5$, $y = 0.31$, and find for $\sigma(t, T \to 0) \propto (t - t_c)^\mu$ the surprising result $\mu = x/y = 1.6$. The scaling exponent μ has been found to be 0.5 for unstressed Si:B, Si:P, Si:As [2] stressed Si:P [3] and Ge:Ga [4]. In addition, Itoh *et al.* [4] have found a good fit to the above scaling form with $x = 0.27$, $y = 0.54$, and $\mu = 0.50$ for the metallic side of the transition. This Comment suggests this large discrepancy is explained by stress inhomogeneity (SI) from sample bending.

For a column in static equilibrium $EId^2y/dz^2 = M(z) = -Py$, where E is Young's modulus, I is the cross section (area $A = ab$, $a < b$) moment-of-inertia, and $M(z)$ is the bending moment for an axial force P and lateral deflection y. For a column of length L with free or pinned ends [5] $y(z) = d_m \sin\pi z/L$ where the critical force $P_c = \pi^2 EI/L^2$. The deflection y depends on L/a, force eccentricity, load bearing surface defects, and the rigidity of the surfaces applying the forces. S varies linearly across the cross section at z between $P/A(1 - 6y/a)$ and $P/A(1 + 6y/a)$ with P/A the average stress $\underline{S}$.

For a stress distribution $f(S,z)$ $[\int f(S,z)\,dS = 1]$ one finds $f(S) = 1/2\underline{S}\lambda$ for $\underline{S}(1 - \lambda) < S < \underline{S}(1 + \lambda)$ $[\lambda = 6y/a]$ and zero outside this range. For a uniform stress $\sigma(S, T = 0) = \sigma_o(1 - S/S_c)^\beta$. For a nonuniform stress $\sigma(z) = \sigma_o \int (1 - S/S_c)^\beta f(S,z)\,dS$. Using this $f(S)$ for $1 - \underline{S}/S_c \gg \lambda\underline{S}/S_c$ one obtains

$$\langle\sigma(T = 0)\rangle = \sigma_o(1 - \underline{S}/S_c)^\beta[1 - O\{(\langle\lambda\rangle\underline{S}/S_c)/(1 - \underline{S}/S_c)\}^2)], \quad (1)$$

and the effect of SI is small. Experiment determines the resistance R between the voltage electrodes (z_1, z_2) as $R = A^{-1}\int_{z1}^{z2}\rho(z)\,dz$ $[\rho(z) = 1/\sigma(z)]$, hence $\langle\sigma\rangle = (z_2 - z_1)/\int_{z1}^{z2} dz/\sigma(z)$. In the limit $1 - \underline{S}/S_c \ll \lambda\underline{S}/S_c$ for $\underline{S} > S_c$

$$\langle\sigma(T = 0)\rangle = [\sigma_o S_c/2\underline{S}(1 + \beta)](z_2 - z_1)/\int[dz\,\lambda(z)/[1 - \underline{S}/S_c + \lambda(z)\underline{S}/S_c]^{\beta+1}. \quad (2)$$

At $\underline{S} = S_c$ (2) yields $\langle\sigma\rangle = [\sigma_o/2(1 + \beta)]g(\lambda)$ with $g(\lambda) = (z_2 - z_1)/\int dz\,\lambda^{-\beta}$ $[z_2 - z_1 = L/8 = 1$ mm for the BSB case]. $\sigma(z) \to 0$ for $\underline{S} = S_c/[1 - \lambda(z)]$ giving an apparent critical stress $S_c^*(z) = S_c/[1 - \lambda(z)]$ so that $\sigma(z) \propto [1 - \underline{S}/S_c^*(z)]^{\beta+1}$. SI shifts S_c to $S_c^*(z)$ and raises the scaling exponent to $\beta + 1$ *independent of z*. The BSB data in Fig. 1 show $\sigma(T = 0)$ vs $\underline{S}$. BSB infer $S_c^* = 613$ bar and $\mu = 1.6 = \beta + 1$ suggesting $\beta \sim 0.6$.

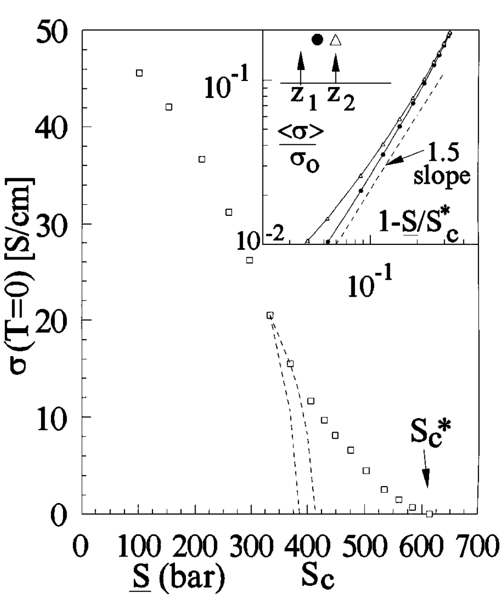

FIG. 1. $\sigma(\underline{S}, T = 0)$ vs $\underline{S}$ for Si:B. S_c is the actual critical point (CP). S_c^* is the apparent CP resulting from SI. The inset shows $\langle\sigma(T = 0)\rangle/\sigma_o$ vs $1 - \underline{S}/S_c^*$ for $\lambda_m = 0.35$.

The data for $\underline{S} < 333$ bar is a good fit to $\beta \sim 0.5$ with $S_c \sim 400 \pm 15$ bar corresponding to $\langle\lambda\rangle \sim \frac{1}{3}$. $S_c^*/S_c = 1.53$ yields $\lambda_m \sin\pi z_1/L = 0.347$ and $\lambda_m = 0.354$ for $z_1/L = 0.4375$. The inset shows $\langle\sigma\rangle/\sigma_o$ vs $1 - \underline{S}/S_c^*$ for two positions of (z_1, z_2) relative to d_m from numerical integration of (2). It confirms SI from bending gives $1.5 < \mu < 1.6$ for $\beta = \frac{1}{2}$, weakly dependent on the geometry for $\langle\sigma\rangle/\sigma_o > 0.02$ $[\langle\sigma\rangle > 1\ S/\text{cm}]$. For $\underline{S} > S_c^*$ the entire sample is insulating and $\langle\sigma(\underline{S}, T)\rangle$ is a complex average of variable range hopping and activated conduction. The linear SI from bending produces a crossover from the true $\mu \sim \frac{1}{2}$ to an apparent $\mu \sim 1.6$.

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[1] S. Bogdanovich, M. P. Sarachik, and R. N. Bhatt, Phys. Rev. Lett. **82**, 137 (1999).
[2] T. G. Castner, Phys. Rev. B **52**, 12 434 (1995).
[3] M. A. Paalanen *et al.,* Phys. Rev. Lett. **48**, 1284 (1982); G. A. Thomas *et al.,* Phys. Rev. B **27**, 3897 (1983).
[4] K. M. Itoh *et al.,* Phys. Rev. Lett. **77**, 4058 (1996); Bull. Am. Phys. Soc. **44**, 240 (1999).
[5] E. R. Maurer and M. O. Withey, *Strength of Materials* (Wiley, New York, 1946), pp. 284–287.

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