Squeezed Light from Spin-Squeezed Atoms

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We propose to transfer quantum correlations from atoms to light by Raman scattering of a strong laser pulse on a spin-squeezed atomic sample. We prove that the emission is restricted to a single field mode which perfectly inherits the quantum correlations of the atomic system.

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Squeezed light and entangled beams of light can be used to probe matter and to study mechanical motion with better resolution than classical light [1]. Entangled photon sources can be used for lithography with a resolution below the optical wavelength [2,3], and the active field of quantum information profits from the development of nonclassical light sources [4–6]. Nonlinear crystals in optical parametric oscillators and laser diodes with suitable feedback have been the work horses in most experiments on nonclassical light. As a figure of merit for the degree of nonclassicality of these light sources, one may refer to the noise suppression observed in direct or homodyne photon detection measurements. Compared to classical sources, it has thus far been possible to reduce the noise (variance) by about 1 order of magnitude. In order to make a significant difference in practical applications, further noise reduction is really necessary, and it has been proposed that the large nonlinearity and low absorption in resonant Raman systems can lead to ideal four-wave mixing and substantial squeezing [7–9].

In this Letter, we show how collective quantum states of atoms may be transferred efficiently to a pulse of light. In particular, atoms can be entangled in such a way that the fluctuations in occupancies of different internal states are significantly suppressed. This phenomenon is referred to as spin squeezing, because a two level atom can be described formally as a spin $1/2$ particle, and the interest in spin-squeezed states is natural in connection with ultraprecise spectroscopy and atomic clocks [10]. Squeezing of spins was originally believed to be very complicated, but recent proposals based on quantum nondemolition measurements of atomic populations [11], on coherent interactions in Bose-Einstein condensates [12,13], and on interactions between laser excited atoms [14] suggest that really significant spin squeezing is achievable. The main purpose of the Letter is to demonstrate that the atomic quantum correlations can be perfectly transferred to the field. This result is readily obtained within a simplified model where both atoms and field are described by single harmonic oscillators [15], but we show that correlations in the atoms can be mapped perfectly on the field also in an *a priori* multimode situation.

The emission of light is treated by a simple generalization of the theory of stimulated Raman scattering [16–18]. This part of our proposal can be analyzed without specifying the model for spin squeezing. Our ensemble of two-state atoms is assumed to be strongly elongated and it is treated in a 1D approximation. It is illuminated by a strong laser field E_s propagating along the z axis of the system. This opens up a channel for an atom in the b state to go to the a state by absorbing a photon of frequency ω_s and emitting a photon of frequency $\omega_q \sim \omega_s + \omega_{ba}$. As a result, a field at this frequency builds up and propagates through the sample as described by the following coupled set of equations for the atomic and field operators:

$$\frac{\partial}{\partial t}[\hat{\psi}_b^\dagger(z,t)\hat{\psi}_a(z,t)] = -i\kappa_1^* E_s^*(z,t)[\hat{\psi}_a^\dagger(z,t)\hat{\psi}_a(z,t) - \hat{\psi}_b^\dagger(z,t)\hat{\psi}_b(z,t)]\hat{E}_q(z,t), \tag{1}$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\hat{E}_q(z,t) = -i\kappa_2\hat{\psi}_b^\dagger(z,t)\hat{\psi}_a(z,t)E_s(z,t), \tag{2}$$

where $\kappa_1 = \sum_i \mu_{ai}\mu_{bi}/(\hbar^2\Delta_i)$ and $\kappa_2 = 2\pi\hbar\omega_q\kappa_1/c$. μ_{ji} are dipole moments of the atomic transitions and Δ_i are the (large) detunings with respect to intermediate levels (see Fig. 1). $\hat{\psi}_a(z,t), \hat{\psi}_a^\dagger(z,t)$ and $\hat{\psi}_b(z,t), \hat{\psi}_b^\dagger(z,t)$ are annihilation and creation operators for atoms in states a and b; $\hat{\psi}_b^\dagger(z,t)\hat{\psi}_a(z,t)$ is the positive frequency part of the atomic dipole operator, taking into account the atomic density at position z in the ensemble. In Eq. (1) we have assumed that there is no dephasing of the ab coherence, and we have assumed the validity of the slowly varying envelope approximation for the emitted field in Eq. (2).

We restrict our analysis to the case where the atoms are almost entirely in the a state when the E_s field is applied. This implies that the population difference appearing in Eq. (1) can be replaced by the density of atoms $n(z)$. This density we represent as a c-number throughout the duration of the output coupling. This allows us to define a dipole operator "per atom" by $\hat{\psi}_b^\dagger(z,t)\hat{\psi}_a(z,t) = n(z)\hat{Q}(z,t)$ and we obtain the linear operator equations,

$$\frac{\partial}{\partial t}\hat{Q}(z,t) = -i\kappa_1^* E_s^*(z,t)\hat{E}_q(z,t), \quad (3)$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{t}\right)\hat{E}_q(z,t) = -i\kappa_2 n(z)E_s(z,t)\hat{Q}(z,t). \quad (4)$$

If we generalize the analysis in [16–18] to inhomogeneous media, we can solve Eqs. (3) and (4) analytically in terms of the input field $\hat{E}_q(0,t)$ at the entrance face of the sample at $z = 0$ and the initial position dependent atomic polarization $\hat{Q}(z,0)$. In particular, for the field operator we get

$$\hat{E}_q(z,\tau) = \hat{E}_q(0,\tau) - i\kappa_2 E_s(\tau)\int_0^z \psi_b^\dagger(z',0)\psi_a(z',0)J_0\left(2\sqrt{a(\tau)\int_{z'}^z n(z'')\,dz''}\right)dz', \quad (5)$$

where the new time coordinate is $\tau \equiv t - z/c$, $J_0(\cdot)$ is a Bessel function of the first kind, and $a(\tau) = \kappa_1^*\kappa_2\int_0^\tau |E_s(\tau')|^2\,d\tau'$.

The expression (5) must be evaluated at the position $z = L$ of a detector outside the atomic sample. In the absence of atoms, the field equals the incident quantum vacuum field $\hat{E}_q(0,\tau)$. The atomic sample is able to replace the vacuum with an entirely different field. In order to analyze the quantum properties of $\hat{E}_q$, it is convenient to imagine a time integrated homodyne detection at the detector. By choosing the temporal form of the strong local oscillator field in this detection we select a certain spatiotemporal mode of the field represented by the field operator,

$$\hat{a} = \sqrt{\frac{c}{2\pi\hbar\omega_q}}\int_0^\infty \mathcal{E}^*(\tau)\hat{E}_q(L,\tau)\,d\tau, \quad (6)$$

where $\int|\mathcal{E}|^2\,d\tau = 1$. In choosing $\mathcal{E}(\tau)$, we should seek to ensure that $\hat{a}$ is a mapping of the precise collective operator of the atomic sample that is known to be squeezed. Mathematically, it is easy to show that in order to probe $\int h(z')\hat{Q}(z',0)\,dz'$ we should choose $\mathcal{E}(\tau)$ as the normalized solution of Eqs. (3) and (4) with the initial condition $n(z')\hat{Q}(z',0)$ replaced by $h(z')$. Physically, this reflects that the question of mode matching coincides with the problem of identifying the classical field radiated by a classical dipole distribution. In particular, if the uniform integral of atomic operators $\hat{J}_- \equiv \int\psi_b^\dagger(z',0)\psi_a(z',0)\,dz'$ is squeezed, mode matching is accomplished by taking $\mathcal{E}(\tau)$ to be the (normalized) solution to the classical Raman scattering problem. In that case we get the mapping,

$$\hat{a} = \frac{1}{\sqrt{N}}J_-, \quad (7)$$

where N is the total number of atoms. Note that, in contrast to the cw output from optical parametric oscillators, we are dealing with a single mode of the quantized field. Such a single field pulse may be applied at the "dark" input port of a beam splitter to cause the splitting of a strong, mode-matched pulse into two twin pulses with perfectly matched photon statistics. To verify that the field is squeezed, one should repeat the entire process of spin squeezing and output coupling several times to check that it was not only "beginner's luck" that the split beams have the same photon number.

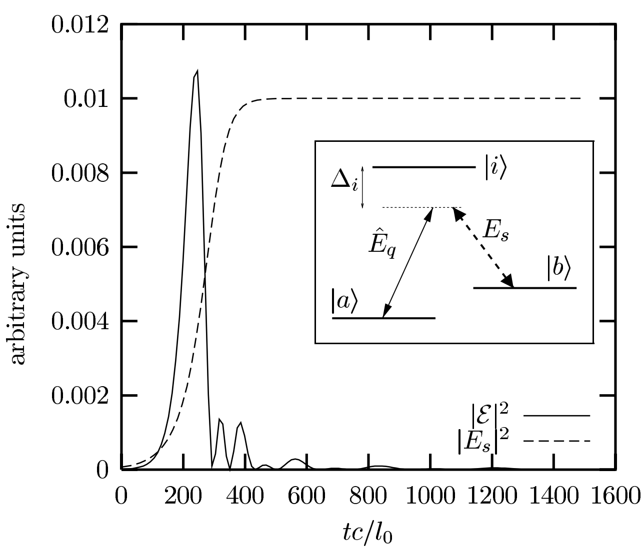

FIG. 1. The shape of the E_s pulse and the mode function $\mathcal{E}$ of the emitted $\hat{E}_q$ pulse (not to scale). The mode function is found by diagonalizing the first-order correlation function of the field, and it coincides with the signal expected when the single atom operator $\hat{Q}$ is uniform over the sample. The inset shows the atomic level scheme of the proposal.

It is a natural concern, whether a 3D analysis will preserve the possibility to couple to only a single spatiotemporal field mode. The issue has been addressed in the classical case, where a diagonalization of the first-order coherence function for the field indeed shows that a single mode dominates the output, provided the Fresnel number of the (active part of the) atomic sample is of the order of unity [17]. Choosing the experimental parameters accordingly, we thus expect this result to apply also for the quantum field in three dimensions.

We recall that several schemes now exist for the generation of nonclassical states of atomic spins. The above analytical treatment is general and independent of the method of spin squeezing, and, as shown by Eq. (7) (valid only if the majority of atoms occupy the state a), the atomic state is transferred perfectly to the light field. Mean values and variances for the field observables are therefore explicitly known, and, e.g., the orders of magnitude squeezing derived in [12,13] apply to the emitted light pulse. It may be useful, however, to relate the emitted quantum field directly to the dynamical variables in the spin squeezing process. This may provide further insight in the origin of squeezing; it may provide a useful tool for the application of the squeezed light as input to another quantum system [19,20]; and it may be useful to analyze processes where the spin squeezing and the light emission occurs simultaneously.

As an example, consider spin squeezing by collisional interactions in a two-component Bose-Einstein condensate [12,13]: First, a Bose-Einstein condensate is formed in only one of the internal states a. By a short resonant Raman pulse, all atoms are transferred to an equal superposition of a and b. We assume that the interaction strengths between the atoms are not all equal, $g_{aa} = g_{bb} \neq g_{ab}$, so that the interaction terms in the fully quantized interaction Hamiltonian,

$$\hat{H} = \int d^3r \left\{ \sum_{i=a,b} \left[\hat{\psi}_i^\dagger(\vec{r}) \hat{h}_i \hat{\psi}(\vec{r}) + \frac{g_{ii}}{2} \hat{\psi}_i^\dagger(\vec{r}) \hat{\psi}_i^\dagger(\vec{r}) \hat{\psi}_i(\vec{r}) \hat{\psi}_i(\vec{r}) \right] + g_{ab} \hat{\psi}_a^\dagger(\vec{r}) \hat{\psi}_b^\dagger(\vec{r}) \hat{\psi}_b(\vec{r}) \hat{\psi}_a(\vec{r}) \right\}, \tag{8}$$

cause a Kerr-like phase evolution of amplitudes on states with different numbers of atoms in the two internal states. This effectively implements the spin squeezing proposal by Kitagawa and Ueda [21].

As in our earlier work on this problem [13], we use the *positive* P method to describe the squeezing process. This means that averages of all normal ordered operator products can be calculated as ensemble averages by the substitution of atomic field operators $\hat{\boldsymbol{\psi}} \equiv (\hat{\psi}_a, \hat{\psi}_a^\dagger, \hat{\psi}_b, \hat{\psi}_b^\dagger)$ by pairs of two-component "wave functions," $\boldsymbol{\psi} = (\psi_{a1}, \psi_{a2}, \psi_{b1}, \psi_{b2})$. The dynamics of the wave functions is given by four coupled and noisy "Gross-Pitaevski" equations, see details in Ref. [13]. Starting from a coherent initial state of all atoms in an equal superposition of internal states a and b, we numerically simulate solutions of these equations to obtain an ensemble of $\boldsymbol{\psi}$'s describing exactly (up to sampling errors) the quantum correlations of the system.

Once a sizable spin squeezing is obtained, we want to transfer this special quantum state to a light pulse, and, to be able to use the results of the above analysis, we first bring the internal state of the atoms close to the a state; i.e., with a new resonant Raman pulse, we rotate the collective spin close to the north pole of the Bloch sphere. This rotation is applied to the individual sets of "wave function" realizations of the simulation ($i = 1, 2$):

$$\begin{pmatrix} \psi_{ai} \\ \psi_{bi} \end{pmatrix} \rightarrow \begin{pmatrix} \psi'_{ai} \\ \psi'_{bi} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \psi_{ai} \\ \psi_{bi} \end{pmatrix}, \tag{9}$$

with $\theta \cong \pi/2$.

The atom field interaction in the coupled equations (1) and (2) leads to a natural positive P representation of the field $\hat{E}_q(z,t)$: Replace, in Eq. (5), $\hat{E}_q(z,t)$ by a c-number field $E_{q1}(z,t)$ and $\psi_b^\dagger(z',0)\psi_a(z',0)$ by $\psi_{b2}(z',0) \times \psi_{a1}(z',0)$, and make a similar replacement in the Hermitian conjugate equation where $\psi_a^\dagger(z',0)\psi_b(z',0)$ is replaced by $\psi_{a2}(z',0)\psi_{b1}(z',0)$ to yield the c-number field $E_{q2}(z,t)$. From our original ensemble of quadruples of wave functions $\boldsymbol{\psi}$ describing the atoms just before we turn on E_s, we obtain in this way an ensemble of pairs $[E_{q1}(z,\tau), E_{q2}(z,\tau)]$ describing the generated light field $\hat{E}_q(z,\tau)$.

As an example, we have investigated 2000 atoms of mass m and with 1D interaction strengths $(g_{aa}, g_{ab}, g_{bb}) = (1.0, 0.5, 1.0) \times 5 \times 10^{-3}\hbar\Omega l_0$, where Ω is the frequency of the harmonic trap and $l_0 = \sqrt{\hbar/m\Omega}$ is the associated characteristic length. The atoms are spin squeezed by collisional interactions for a time $t = 3.0\ \Omega^{-1}$. They are subsequently driven towards the state a, and hereafter they are illuminated with the E_s light which builds up a maximum strength of $E_{\max} = 10^2\sqrt{2\pi\hbar\omega/l_0}$ in a time of roughly $t_{\text{rise}} = 100 l_0/c$. The matter-light coupling is chosen to be $\kappa_1 = 10^{-3}c/2\pi\hbar\omega$.

As we know the atoms to be spin squeezed, we also know one mode of the $\hat{E}_q$ field which will be squeezed: the one corresponding to the simple uniform integral of $\int \psi_b^\dagger(z',0)\psi_a(z',0)\,dz'$ as described in the discussion after Eq. (6). Even with the simplifications leading to Eqs. (3) and (4), the atomic state could in principle radiate into many other modes, and, to check whether such other modes are present in this case, we calculate the first-order correlation function of the field $\langle \hat{E}_q^\dagger(L,\tau')\hat{E}_q(L,\tau)\rangle = \overline{E_{q2}(L,\tau_2)E_{q1}(L,\tau_1)}$ ($\overline{\cdots}$ indicates averaging over the positive P ensemble). When it is diagonalized, we find that almost all population is in fact in the expected mode $\mathcal{E}(\tau)$ which is plotted in Fig. 1. In other scenarios, it might be difficult to calculate beforehand exactly which collective atomic operator is squeezed, and then an analysis such as this would be necessary in order to pick the local oscillator field for the homodyne detection.

Having confirmed that only one mode is populated, we now turn to the quantum character of the field. It depends on how we choose θ in Eq. (9): For $\theta = \pi/2$, it will approximate squeezed vacuum; for θ slightly different from $\pi/2$, it will approximate a squeezed coherent state. As described in [13] the spin squeezing ellipse is at an angle to the coordinate axes. This carries over to the light field which is squeezed in an appropriate quadrature component $X_\phi = (\hat{a}e^{i\phi} + \hat{a}^\dagger e^{-i\phi})/\sqrt{2}$. With the above parameters, we find from the positive P simulations that the minimum variance is 0.04 ± 0.01 corresponding to a reduction by a factor of more than 12 from the standard quantum limit. As expected from Eq. (7), this coincides with

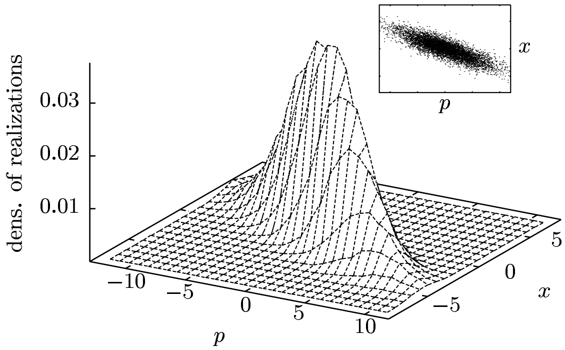

FIG. 2. Histogram illustrating the prediction for the squeezing of light. The quadrature components of the output field are represented by pairs of numbers $\mathrm{Re}(a_1 + a_2)/\sqrt{2}$, $\mathrm{Re}(ia_1 - ia_2)/\sqrt{2}$, the distribution of which forms a squeezed ellipsoid shape in phase space. The actual amount of squeezing cannot be directly determined from this plot; it requires a computation of mean values and variances for which the pairs of complex numbers a_1 and a_2 provide mean values of normally ordered field operators. 10^5 realizations contributed to the histogram and in the inset is shown a scatter plot of 6800 representative points.

the degree of atomic spin squeezing. Much higher factors of spin squeezing and thus of squeezing in light fields can be achieved with more atoms in the sample [12] but they would be difficult to treat with our positive P method.

To illustrate the positive P results in Fig. 2, we show both a histogram and a scatter plot obtained from the positive P ensemble of pairs (a_1, a_2) representing $(\hat{a}, \hat{a}^\dagger)$ for the field mode. On average, a_1 and a_2 are complex conjugate quantities, so that the expectation values of Hermitian field operators $\langle X_\phi \rangle = \overline{a_1 e^{i\phi} + a_2 e^{-i\phi}}$ are real for all ϕ. To represent our simulated results we have made a histogram for the values of $x = X_0$ and $p = X_{\pi/2}$, indicated by the *real part* of $(a_1 + a_2)/\sqrt{2}$ and $(ia_1 - ia_2)/\sqrt{2}$.

The perfect output coupling of atomic correlations is the main result of the paper, and we imagine that many other atomic quantum states may be taken as a starting point for nonclassical light generation, e.g., EPR-correlated separated atomic ensembles, and that the mechanism may be used also for reliable interspecies teleportation. It was recently demonstrated experimentally that a light pulse can be brought to a complete halt in an atomic sample and that it can be subsequently released [22,23]. It has also been predicted that the quantum state of the light is preserved in the process [24]. Combining these ideas with our mapping of spin squeezed and other entangled atomic states on light may lead to powerful manipulation of both known and unknown quantum states of light. Some schemes for atomic state preparation, e.g., by quantum nondemolition measurements, may even be compatible with the transmission or production of very slowly propagating light, so that an effective cw source of nonclassical light may be realized.

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