Clock Synchronization with Dispersion Cancellation

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The dispersion cancellation feature of pulses which are entangled in frequency is employed to synchronize clocks of distant parties. The proposed protocol is insensitive to the pulse distortion caused by transit through a dispersive medium. Since there is cancellation to all orders, also the effects of slowly fluctuating dispersive media are compensated. The experimental setup can be realized with currently available technology, at least for a proof of principle.

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If two distant persons want to synchronize their clocks, one of them (say Alice) sends a pulse, which is bounced back by the other (Bob). Alice measures the pulse creation and travel time and Bob the time at which the bounce occurs. By exchanging their measurement results, each of them may know the time of the other relative to his own clock. This is the idea underlying the Einstein synchronization scheme [1]. Of course, if the medium between Alice and Bob is dispersive, then the pulses they exchange get distorted and the measurements of the bounce time at Bob's side and of the arrival time at Alice's side acquire an error, which must be summed to the intrinsic error in such kind of measurements. In the actual technological problem of synchronizing clocks of distant parties [2], it seems that the main cause of errors is given by the (possibly fluctuating) dispersion effects of the medium through which the pulses travel. Franson [3] proposed a scheme capable of suppressing the dispersion effects to all orders, given a suitably tailored dispersive media. Steinberg, Kwiat, and Chiao [4,5] proposed and experimentally implemented a scheme capable of suppressing to first order the effects of dispersion of arbitrary media. In their scheme the time-resolved coherence properties of frequency-entangled pulses to first order do not acquire any spread when traveling through a dispersive medium. However, their scheme is not suitable for clock synchronization since the dispersion cancellation is present only if the time of arrival is *not* determined accurately [5,6] and since only the interferometer arms path length difference is recovered.

Here a modified version of the Steinberg, Kwiat, and Chiao interferometer is presented, which allows the synchronization of the clocks of Alice and Bob without being bothered by the pulse distortion as it travels through the intervening medium. This scheme is an application of the proposal [7] to employ frequency-entangled pulses to achieve an accuracy increase in clock synchronization. The synchronization protocol employed is quite different from the Einstein clock synchronization scheme: no time measurements are needed, and the relative distance or the transit time between Alice and Bob or any dispersive property of the medium play no role in the protocol. As in the Einstein protocol, the only (rather reasonable) hypothesis is that the pulse time of travel is the same both ways. This allows one to also employ the scheme in the presence of fluctuations in the medium under the requirement that the fluctuations have a time scale longer than the pulse travel time.

The setup is realizable with currently available technology, since it employs conventional parametric down converter crystals as frequency-entanglement source.

In Sect. I of this paper the time synchronization protocol that the proposed scheme employs is described. In Sect. II the experimental setup and its features are presented. In Sect. III the equations of motion of the system are solved and the hypotheses needed to have dispersion cancellation are given.

I. Clock synchronization protocol.—In this section, the protocol underlying the proposed experimental setup is described and compared to the Einstein clock synchronization. It is a classical protocol [8]: the quantum mechanical features of the setup that will be introduced in the following sections are employed only to achieve enhanced accuracy and dispersion cancellation.

Consider the following scenario for the sake of illustrating the method: a conveyor belt (i.e., a physical system in which the transit time from A to B is the same as from B to A) connects Alice and Bob (as in Fig. 1). Alice pours a quantity of sand which is proportional to the time shown on her clock on both sides A and A' of the conveyor belt, i.e., on the side traveling towards Bob and on the side traveling towards the point M. Bob, at one end of the conveyor belt (point B), scoops away a quantity of sand proportional to twice the time shown on his clock. If the proportionality constant of Alice and Bob are the same, then the quantity

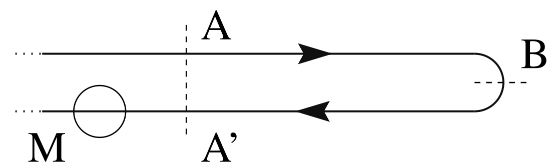

FIG. 1. Schematic of the "conveyor belt synchronization" scheme.

of sand at point M will be constant in time, after an initial transient when they start to act on the system. Thus, it is sufficient to measure such a quantity in order to recover the exact time difference between Alice's clock and Bob's clock. The precision with which this time difference may be recovered depends only on the precision with which it is possible to measure the quantity of sand at M.

The main advantage of this scheme over the Einstein clock synchronization procedure is that no time of arrival measurements need to be performed. In fact, the time of arrival measurement of a pulse has an intrinsic unavoidable error dependent on its bandwidth. The conveyor belt synchronization strikingly allows time synchronization without any timing measurements. On the other hand, while Einstein's protocol measures the distance between Alice and Bob (if the pulse speed is known), this procedure does not allow the recovery of any information on this distance, unless Bob stops scooping the sand away.

A practical realization of this classical scheme is readily implemented. An intense continuous polarized beam which travels from Alice to Bob and back is employed as conveyor belt. Alice, on her side, rotates at positions A and A' the beam polarization by an amount proportional to the time shown on her clock. Bob rotates the polarization at position B by an amount proportional to twice his time, but with opposite direction. The shift in polarization measured at position M allows Alice and Bob to determine the difference in the time of their clocks up to a rotation period. In the next section, a scheme is introduced that exploits quantum properties to enhance the accuracy and cancel the effects of dispersion.

II. Experimental setup.—In this section, an experimental scheme and procedure to implement the protocol described in the previous section is described. The experimental setup is sketched in Fig. 2. It is based on the Hong, Ou, and Mandel (HOM) interferometer [9], which uses the frequency-entangled output beam generated by a parametric down converter crystal. After propagating through different optical paths, the signal (S) and idler (I) beams are interfered at a beam splitter. By measuring the photon coincidence rate P_c at the output ports 1 and 2 of the beam splitter, one may acquire very precise information on the path length difference in the two arms. The precision limit is given by the inverse of the bandwidth of the down-converted state (twin beams) and is, to a large extent, independent of the precision of the photon time of arrival measurement. Steinberg, Kwiat, and Chiao [4,5] showed that in the presence of dispersive media in the interferometer, it is possible to cancel the effects of such dispersion using detectors with a wide integration window. Here this idea is pursued further. By using the scheme proposed here, Alice and Bob may check whether their clocks are synchronized, and may keep them synchronized through a feedback loop under the hypothesis that the drift of their clocks is sufficiently slow.

As shown in Fig. 2, Alice operates the parametric down converter and generates the twin beams she sends through the interferometer. The beams travel up to Bob's position and are reflected back to Alice. She makes them interfere at a 50-50 beam splitter and measures the coincidence rate of the photodetector clicks at the outputs of the beam splitter, as a function of the optical path length difference δl. She obtains a flat coincidence rate with a very narrow dip for $\delta l = 0$ (as in Fig. 3).

As will be shown in Sect. III, in order to check whether their clocks are synchronized, Alice and Bob must introduce time varying delays $\delta l_a^S(t)$, $\delta l_a^I(t)$, $\delta l_b^S(t)$, and $\delta l_b^I(t)$ both on the idler I and signal S beam. (As examples of varying delays consider moving mirrors or time varying

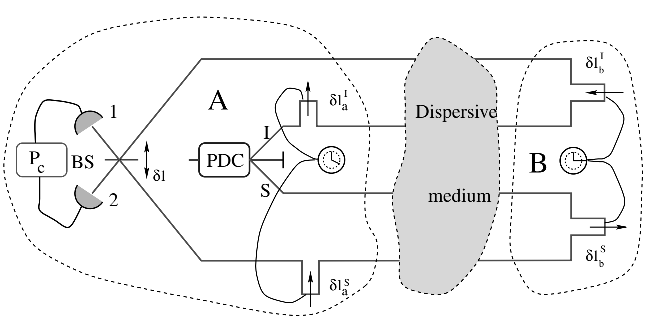

FIG. 2. Sketch of the experimental setup for the all-orders dispersion cancellation synchronization. Alice on the left and Bob on the right are separated by a dispersive medium. The frequency entangled state is produced by a parametric down converter crystal (PDC) and is made to interfere at the 50-50 beam splitter (BS). The coincidence rate P_c is measured as a function of the position of the beam splitter δl. Alice and Bob introduce timing information in the setup through the time varying delays δl_a^I, δl_b^I, δl_a^S, and δl_b^S. Notice that two delays (δl_a^I and δl_b^S) increase in time, while the other two (δl_a^S and δl_b^I) decrease.

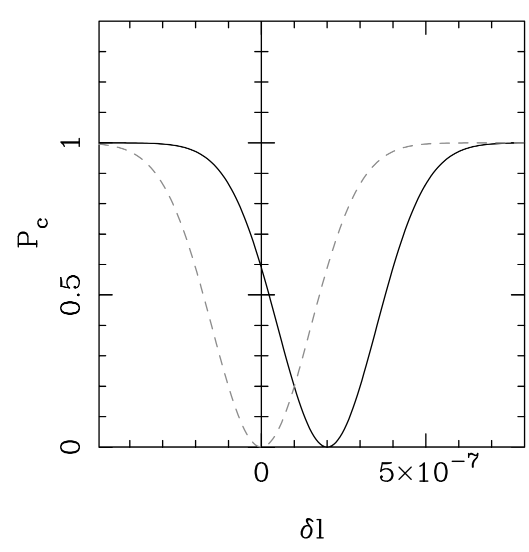

FIG. 3. Plot of P_c vs δl. The dashed line refers to the case $v = 0$, and the continuous line to the case $v = 50\ \frac{\mathrm{m}}{\mathrm{s}}$. In this graph, the Gaussian spectrum of the down-converted beams has a bandwidth $\Delta\omega = 10^{15}\ \mathrm{s}^{-1}$ and $\tau = 1$ ns.

phase shifters.) These delays play the role of adding or subtracting the sand from the conveyor belt. In order to have dispersion cancellation, four delays are needed instead of only the two that are sufficient for the synchronization. The protocol requires these delays to be linked to the time measured by Alice's clock and Bob's clock, respectively. Consider, for example, the case of linear time dependence, i.e.,

$$\begin{aligned} \delta l_a^I(t) &= v(t - t_0^a), \qquad \delta l_b^I(t) = -v(t - t_0^b), \\ \delta l_a^S(t) &= -v(t - t_0^a), \qquad \delta l_b^S(t) = v(t - t_0^b), \end{aligned} \tag{1}$$

where t_0^a and t_0^b are the times at which Alice and Bob start their clocks and v is the delay rate (e.g., the speed of the moving mirrors). As shown in Fig. 2, Alice places one of her delays at the beginning and the other one at the end of the transmission line. Bob applies both his delays in the middle of the two paths. We might assume that Alice and Bob have identical clocks and only want to find out the time difference $\tau = t_0^b - t_0^a$ between them. Because of the introduction of the delays (1), the final path length difference as measured by the HOM interferometer will be affected by a factor dependent on τ and Alice can measure it by observing the shift of the dip position in the photon coincidence rate P_c as shown in Fig. 3.

III. Analysis of the experiment.—In this section the main result is derived. It will be shown that the dip in the coincidence rate graph is located at the position $\delta l_0 = 4v\tau$, which shows that by measuring the Mandel dip position [9], one can recover the time difference between Alice's clock and Bob's clock. There is no dependence on the distance between Alice and Bob or on any property of the intervening medium. The only two hypotheses that are required are (a) the travel time of the twin beam state from Alice to Bob is the same as from Bob to Alice (which gives a lower time limit for the fluctuations of the intervening medium) and (b) the medium acts in the same way on the beams traveling in both directions (which gives a lower limit for the spatial inhomogeneities of the medium: the distance between the beams traveling in the two directions must be smaller than the inhomogeneity characteristic length).

The twin beam state at time $t = 0$ at the output of the crystal (cw pumped at frequency $2\omega_0$) is given by the frequency maximally entangled state

$$|\psi\rangle = \int d\omega\, \phi(\omega)|\omega_0 + \omega\rangle_I |\omega_0 - \omega\rangle_S\,, \tag{2}$$

where $|\omega\rangle_S$ and $|\omega\rangle_I$ denote the signal and idler "frequency states" (i.e., state in which there is only one photon at frequency ω and the vacuum for all the other frequencies), and $\phi(\omega)$ is the spectral function of the down-converted light, centered in $\omega = 0$ and characterized by the bandwidth $\Delta\omega$. The coincidence rate at the photodetectors is given by the Mandel formula for photodetection [10]

$$P_c \propto \int_T dt_1\, dt_2\, \langle\psi|E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)}|\psi\rangle\,, \tag{3}$$

where T is the integration time window of the detectors. In Eq. (3) the electromagnetic fields at time t_j at the output of the beam splitter, assuming for the sake of simplicity a linear polarization, are given by

$$\begin{cases} E_j^{(+)} = i \int d\omega \sqrt{\frac{\hbar\omega}{4\pi c A}}\, a_j(\omega) e^{-i\omega(t_j - x_j/c)} \\ E_j^{(-)} = (E_j^{(+)})^\dagger \qquad \text{for } j = 1, 2, \end{cases} \tag{4}$$

where A is the beam cross section and x_j is the position of the detector. It is possible to match the output fields of the beam splitter with the fields before the moving mirrors by applying the beam splitter transformation on the mirrors' reflected fields. This latter may be obtained by performing some Lorentz transformations on the input fields: first from the source to the moving-mirror frame and then back to the laboratory frame. This procedure yields the transformations for the field annihilation operators when the fields are bounced off the moving mirrors. Analyze the idler beam first. One finds that the annihilation operator $a_I(\omega)$ at the crystal position is evolved into $a_I'(\omega)$ at Bob's position (at a distance L from the crystal) and into $a_I''(\omega)$ at the beam splitter position (at a distance L' from Bob), with

$$\begin{aligned} a_I'(\omega) &= \sqrt{\chi}\, a_I(\chi\omega) e^{-i\omega[2\beta(t_0^a - x_0/c)/(1-\beta) - L/c] + i\kappa_t^I(\omega)}, \\ a_I''(\omega) &= a_I(\omega) e^{-i\omega[2\beta(t_0^a - t_0^b - x_0/c)/(1+\beta) - (L/\chi + L')/c] + i\kappa_t^I(\omega/\chi) + i\kappa_f^I(\omega)}, \end{aligned} \tag{5}$$

where $\beta = \frac{v}{c}$, $\chi = \frac{1+\beta}{1-\beta}$ is associated with the Doppler shift introduced by the moving mirrors, and x_0 is the distance of Alice's delay from the crystal. In Eq. (5) the terms κ_t^I and κ_f^I take into account the effect of the dispersive medium on the idler beam on their way *to* and *from* Bob, respectively. Notice that because of the Doppler shift introduced by the first mirror (which, for $\beta > 0$, reduces the frequency of the beam after the mirror), κ_t^I is evaluated at ω/χ. On the other hand, because of the compensation due to the second mirror, κ_f^I is evaluated at frequency ω. Analogous procedure applies to the signal beam, resulting in the operator transformations

$$\begin{aligned} a_S'(\omega) &= \sqrt{\chi}\, a_S(\chi\omega) e^{-i\omega[2\beta(t_0^b - L/c)/(1-\beta) - L/c] + i\kappa_t^S(\chi\omega)}, \\ a_S''(\omega) &= a_S(\omega) e^{i\omega[2\beta(t_0^a - t_0^b + x_0/c)/(1+\beta) + (L/\chi + L')/c] + i\kappa_t^S(\omega) + i\kappa_f^S(\omega/\chi)}, \end{aligned} \tag{6}$$

where here a_S^l has been evaluated after the delay δl_b^S.

As shown in Fig. 2, the modes which are detected at the output of the beam splitter are obtained as

$$\begin{cases} a_1(\omega_1) = \frac{1}{\sqrt{2}}[i a_I''(\omega_1) e^{-i\omega_1 \delta l/c} + a_S''(\omega_1)], \\ a_2(\omega_2) = \frac{1}{\sqrt{2}}[i a_S''(\omega_2) + a_I''(\omega_2) e^{-i\omega_2 \delta l/c}], \end{cases} \quad (7)$$

where δl is the delay that Alice introduces in order to scan the path length dependence of P_c. Replacing Eqs. (2) and (7) into (3), and taking the limit $T \to \infty$, which corresponds to wide time window at the detection [5,6], one obtains (recalling that the state $|\psi\rangle$ contains only two photons)

$$P_c \propto \int d\omega_1 \, d\omega_2 \, |\langle 0|a_1(\omega_1) a_2(\omega_2)|\psi\rangle|^2, \quad (8)$$

where the usual limit $\Delta\omega \ll 2\omega_0$ was employed to simplify the frequency dependence of the field (4). Assuming the symmetry $\phi(\omega) = \phi(-\omega)$ of the spectral function, the matrix element in (8) is given by

$$\langle 0|a_1(\omega_1) a_2(\omega_2)|\psi\rangle = \tfrac{1}{2}\delta(\omega_1 + \omega_2 - 2\omega_0) e^{i\varphi} \phi(\omega_1 - \omega_0)[1 - e^{2i(\omega_1 - \omega_0)[4\beta(t_0^b - t_0^a)/(1+\beta) - \delta l/c] - i\Delta\kappa(\omega_1)}], \quad (9)$$

where φ is an overall phase term that will disappear taking the modulus, and

$$\begin{aligned} \Delta\kappa(\omega) = {} & \kappa_t^S(\omega) - \kappa_f^I(\omega) + \kappa_f^I(2\omega_0 - \omega) \\ & - \kappa_t^S(2\omega_0 - \omega) + \kappa_t^I\left(\frac{2\omega_0 - \omega}{\chi}\right) \\ & - \kappa_f^S\left(\frac{2\omega_0 - \omega}{\chi}\right) + \kappa_f^S\left(\frac{\omega}{\chi}\right) - \kappa_t^I\left(\frac{\omega}{\chi}\right) \end{aligned} \quad (10)$$

is the contribution of the dispersion terms. It is immediate to see that $\Delta\kappa$ vanishes altogether at all orders by requiring that $\kappa_t^S = \kappa_f^I$ and $\kappa_f^S = \kappa_t^I$, which can be satisfied by allowing the *from* idler beam to propagate at a distance less than the spatial inhomogeneities of the medium from the *to* signal beam and, equivalently, by allowing the *to* idler beam to propagate near the *from* signal beam. In this case any effect of the medium will be erased. Replacing Eq. (9) into (8), one obtains

$$P_c = \int d\omega \, |\phi(\omega)|^2 \left[1 - \cos\left(2\frac{\omega}{c}(\delta l_0 - \delta l)\right)\right], \quad (11)$$

where $\delta l_0 = 4v\tau/(1 + \beta)$ or $\delta l_0 \simeq 4v\tau$ in the nonrelativistic regime. Finally, in the simple case of a Gaussian spectrum $|\phi(\omega)|^2$ with variance $\Delta\omega^2$, Eq. (11) becomes

$$P_c = 1 - e^{-2\Delta\omega^2(\delta l - \delta l_0)^2/c^2}, \quad (12)$$

which, as shown in Fig. 3, features a dip of width $1/(4\Delta\omega)$ centered in $\delta l = \delta l_0$. By measuring the dip position, one can recover the time difference τ between Alice's clock and Bob's clock.

Notice that, if the parameter v is known with an accuracy Δv, then, in the nonrelativistic regime, the error in the determination of τ is given by

$$\Delta\tau = \sqrt{\frac{1}{(4\Delta\omega\beta)^2} + \left(\frac{\Delta v}{v}\right)^2 \tau^2}, \quad (13)$$

where the first term refers to the intrinsic accuracy in the dip position measurement. Suppose, for example, that the clocks are initially synchronized up to $\tau = 1$ ns and the twin beam bandwidth is $\Delta\omega = 10^{15}$ Hz (as in [4]), then to achieve an accuracy $\Delta\tau \sim 10^{-1}$ ns one has to use $v \sim 500$ m/s with $\Delta v \lesssim 50$ m/s.

In conclusion, the proposed protocol allows Bob to measure the time difference τ between his clock and Alice's clock, without being affected by the dispersion of the intermediate medium. The accuracy of the scheme is dependent only on the bandwidth $\Delta\omega$ of the twin beam and on the delay rate v; namely he can recover τ with an error $\sim 1/(\Delta\omega v/c)$.

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