Test of *CPT* and Lorentz Invariance from Muonium Spectroscopy

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Following a suggestion from Kostelecký *et al.*, we evaluated a test of *CPT* and Lorentz invariance from the microwave spectroscopy of muonium. Hamiltonian terms beyond the standard model violating *CPT* and Lorentz invariance would contribute frequency shifts $\delta\nu_{12}$ and $\delta\nu_{34}$ to ν_{12} and ν_{34}, the two transitions involving muon spin flip, which were precisely measured in ground state muonium in a strong magnetic field of 1.7 T. The shifts would be indicated by anticorrelated oscillations in ν_{12} and ν_{34} at the Earth's sidereal frequency. No time dependence was found in ν_{12} or ν_{34} at the level of 20 Hz, limiting the size of some *CPT* and Lorentz-violating parameters at the level of 2×10^{-23} GeV.

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Much current theoretical work is devoted to finding a more fundamental and general underlying theory from which the standard model of particle physics could be deduced as the low energy limit. String theory is a central candidate, with the feature that the assumptions underlying the *CPT* theorem are invalid for strings, which are extended objects. *CPT* violation then becomes a possible signature of more fundamental underlying theories, which can be probed by sensitive experimental tests of *CPT* invariance.

Some years ago Kostelecký and co-workers [1–3] developed a plausible extension of the standard model based on spontaneous breaking of Lorentz and *CPT* symmetry in an underlying theory without gravity. Their low energy effective theory provides a theoretical basis for establishing quantitative bounds on *CPT* invariance. The analysis was done in the context of conventional relativistic quantum mechanics and quantum field theory in four dimensions, retaining the usual gauge structure and renormalizability. The Lorentz- and *CPT*-violating additions to the standard model Lagrangian are highly suppressed to remain compatible with established experimental bounds.

As applied to muonium ($\mu^+ e^-$ bound state), the most relevant terms in the extension to the standard model are those in the QED limit, involving only muons, electrons, and photons. The additional terms in the Lorentz-violating Lagrangian lead to a modified Dirac equation and are given by [2]

$$\begin{aligned}\mathcal{L} = & -a^l_\alpha \bar{\psi}_l \gamma^\alpha \psi_l - b^l_\alpha \bar{\psi}_l \gamma_5 \gamma^\alpha \psi_l \\ & - \tfrac{1}{2} H^l_{\alpha\beta} \bar{\psi}_l \sigma^{\alpha\beta} \psi_l + \tfrac{1}{2} i c^l_{\alpha\beta} \bar{\psi}_l \gamma^\alpha \overleftrightarrow{D}^\beta \psi_l \\ & + \tfrac{1}{2} i d^l_{\alpha\beta} \bar{\psi}_l \gamma_5 \gamma^\alpha \overleftrightarrow{D}^\beta \psi_l .\end{aligned} \tag{1}$$

The lepton fields are denoted by $l = e^-, \mu^-$, and $iD_\alpha = i\partial_\alpha - qA_\alpha$, where $q = -|e|$. All terms are Lorentz violating, while a and b are *CPT* odd, and H, c, and d are *CPT* even. The lepton number violating terms allowed in the extension are not relevant here and have been omitted.

To predict the perturbations to muonium, we consider the energy level diagram of the ground state of muonium in a magnetic field (Fig. 1). The transition frequencies ν_{12} and ν_{34} have been measured [4] with high precision and are used to determine the hyperfine structure interval $\Delta\nu$ and the ratio μ_μ/μ_p of the muon magnetic moment to the proton magnetic moment.

Leading-order Lorentz-violating energy shifts $\delta\nu_{12}$ and $\delta\nu_{34}$ due to the new terms in the Lagrangian can be calculated using perturbation theory and relativistic two-fermion techniques. For our observed transitions at the strong magnetic field of 1.7 T, dominantly only muon spin flip occurs so these energy shifts are characterized by the muon parameters alone of the extended theory. This approach results in

$$\delta\nu_{12} \approx -\delta\nu_{34} \approx \tilde{b}^\mu_3/\pi , \tag{2}$$

where $\tilde{b}^\mu_3 \equiv b^\mu_3 + d^\mu_{30} m_\mu + H^\mu_{12}$ [5] are laboratory frame parameters. High precision experiments on muonium (M) can measure or set limits on the parameters of these symmetry-violating terms, which are sensitive at the Planck scale level [5].

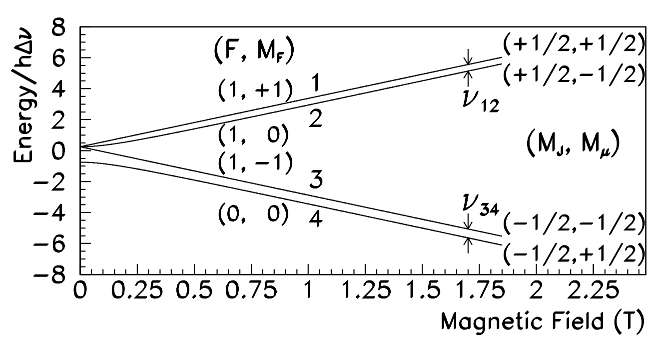

FIG. 1. Breit-Rabi energy level diagram of ground state muonium. At high fields, the indicated transitions, ν_{12} and ν_{34}, are essentially muon spin flip transitions.

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Predictions of the values of ν_{12} and ν_{34} from standard theory—dominantly the QED terms—require values for many atomic constants including m_μ, μ_μ, and $\Delta\nu$ as well as the calculation of higher order QED radiative corrections. The relevant constants and calculations are not known to as high accuracy as the experimental determinations of ν_{12} and ν_{34}. Indeed several of these constants—$\Delta\nu$ and μ_μ/μ_p—are obtained from the muonium experiment. Comparing predictions for ν_{12} and ν_{34} (based on independent determinations of the required atomic constants) with the experimental results results in poor sensitivity to the nonstandard model energy shifts $\delta\nu_{12}$ and $\delta\nu_{34}$.

However, the theory with *CPT* and Lorentz violation involves spatial components in a celestial frame of reference, and, since the laboratory rotates with the Earth, these spatial components vary with time, and consequently the experimentally observed ν_{12} and ν_{34} may oscillate about a mean value at the Earth's sidereal frequency $\Omega = 2\pi/23$ h 56 min with amplitudes $\delta\nu_{12}$ and $\delta\nu_{34}$. No such signal would be obtained from the standard model. In the nonrotating celestial frame of reference with equatorial axes $\{\hat{X}, \hat{Y}, \hat{Z}\}$ where $\hat{Z}$ is oriented along the Earth's rotational North Pole, an experimental constraint on $\delta\nu_{12}$ implies [5]

$$\frac{1}{\pi} |\sin\chi| \sqrt{(\tilde{b}_X^\mu)^2 + (\tilde{b}_Y^\mu)^2} \le \delta\nu_{12}, \qquad (3)$$

in which $\chi \sim 90°$ is the angle between $\hat{Z}$ and the quantization axis defined by the laboratory magnetic field at Los Alamos where the muonium experiment was performed. The transformation from the lab frame quantity $\tilde{b}_3^\mu$ to the nonrotating celestial frame quantities $\tilde{b}_J^\mu$ (where $J = X, Y, Z$) is given by

$$\tilde{b}_3^\mu = \tilde{b}_Z^\mu \cos\chi + (\tilde{b}_X^\mu \cos\Omega t + \tilde{b}_Y^\mu \sin\Omega t)\sin\chi. \qquad (4)$$

The nonrotating frame quantities, $\tilde{b}_J^\mu$, are defined by $\tilde{b}_J^\mu \equiv b_J^\mu + m_\mu d_{J0}^\mu + \frac{1}{2}\epsilon_{JKL}H_{KL}^\mu$ [5]. The experiment has no sensitivity to the celestial frame parameter $\tilde{b}_Z^\mu$, but ideal sensitivity to the pair $\tilde{b}_X^\mu$ and $\tilde{b}_Y^\mu$.

The sum of the transition frequencies, $\nu_{12} + \nu_{34}$, is equal to the ground state hyperfine splitting, $\Delta\nu$, and, since we expect [see Eq. (2)] $\delta\nu_{12} + \delta\nu_{34} \approx 0$, no sidereal variation is expected in the hyperfine interval. At the high field strengths of this experiment, the difference in transition frequencies $\nu_{12} - \nu_{34}$ is almost proportional to the magnetic moment of the muon. By looking for a sidereal variation in the difference $\delta\nu_{12} - \delta\nu_{34} \approx 2\tilde{b}_3^\mu/\pi$, we are essentially probing a possible sidereal variation of the magnetic moment of the muon (while the g factor remains constant).

The accurate measurements of ν_{12} and ν_{34} were done in a microwave magnetic resonance experiment [4]. Muonium was formed by electron capture by muons stopping in a krypton gas target. Resonance lines were observed by varying the magnetic field with fixed microwave frequency and by varying the microwave frequency with fixed magnetic field. A line narrowing technique was used involving observation of a transition signal only from M atoms which have lived considerably longer than $\tau_\mu \sim 2.2\ \mu$s (Fig. 2). The values reported for ν_{12} and ν_{34} at a magnetic field strength corresponding to a free proton precession frequency of 72.320 000 MHz were

$$\nu_{12}(\text{exp}) = 1\,897\,539\,800\,(35)\ \text{Hz}\ (18\ \text{ppb}), \qquad (5)$$

$$\nu_{34}(\text{exp}) = 2\,565\,762\,965\,(43)\ \text{Hz}\ (17\ \text{ppb}), \qquad (6)$$

in which one standard deviation error includes both statistical and systematic errors. To search for a time dependence of ν_{12} and ν_{34}, we employed the following algorithm. Data from each resonance line run (each lasting about one-half an hour) are fit at the measured magnetic field strength and Kr pressure to determine provisional line centers for ν_{12} and ν_{34}. These line centers were then transformed to their values in a magnetic field strength corresponding to a free proton precession frequency of 72.320 000 MHz. The data were taken at Kr pressures of 0.8 and 1.5 atm, so the line centers were corrected for a small quadratic pressure shift, and then were extrapolated linearly to their values at zero pressure, using a pressure shift coefficient determined from the data. Line centers for ν_{12} and ν_{34} obtained in this way were then grouped as a function of sidereal time, where time zero was set as the time in 1995 when we obtained our first data.

Nonzero values for $\delta\nu_{12}$ and $\delta\nu_{34}$ could arise from systematic effects which lead to variations in the parameters affecting the line centers—particularly variations having a period of $\approx$24 h. Principal concerns are possible day-night variations of the magnetic field strength, and of the density and temperature of the Kr gas.

Day-night variations of several degrees centigrade in the temperature of the experimental hall lead to oscillations in

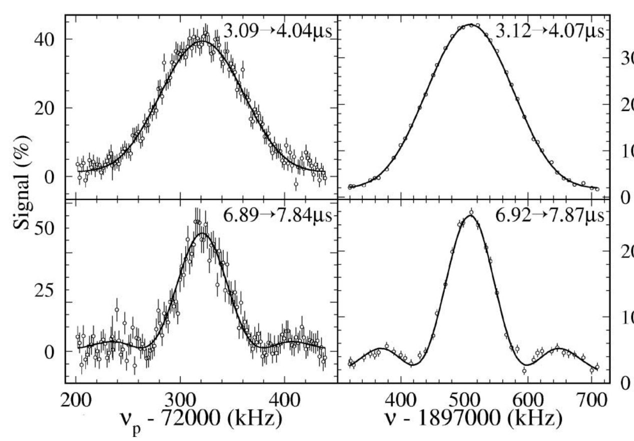

FIG. 2. Muonium resonance lines (data and fit) for ν_{12} taken by using magnetic field sweep on the left and microwave frequency sweep on the right, for muonium atoms which have decayed in selected time intervals after formation.

the magnetic field strength of the persistent-mode superconducting solenoid of ≤ 1 ppm. Changes of 0.05 ppm in the field strength were easily resolved, and the oscillation's effects on the line centers were accounted for by extracting the line centers. Temperature changes also affected the diamagnetic shielding constant of the water in the NMR probes used to monitor the field (the probes were not temperature stabilized, but were in good thermal contact with the microwave cavity which was temperature stabilized to 0.1 °C). The maximum conceivable 2 °C day-night changes in water temperature would change the NMR frequencies by 0.02 ppm, leading to errors in the line centers of about 2.5 Hz; of opposite sign for ν_{12} and ν_{34}. This potential effect is well below the statistical sensitivity for sidereal variations of 12 to 15 Hz.

The effect on these data of the variation of Kr pressure with time was evaluated. The front end window to the Kr stopping target was of 76 μm mylar, and flexed with day-night variations of the external atmospheric pressure. This induced fractional day-night oscillations in the Kr gas target pressure which were measured to be about 2.5×10^{-4}. Through pressure shift coefficients of about -16.5 kHz/atm for ν_{12} and -19.5 kHz/atm for ν_{34}, the resulting shifts in the line centers (typically 7.5 Hz in ν_{34} and 6 Hz in ν_{12}) were automatically accounted for in performing the linear extrapolation to zero pressure, and should not contribute any significant time variation to ν_{ij}. The pressure shift coefficients depend on the average velocities of the atoms, and so are functions of temperature. The fractional changes in the transition frequencies with temperature (measured in hydrogen and its isotopes [6]) are roughly 1×10^{-11} °C^{-1} Torr^{-1}. Given the temperature stability of the Kr gas of about 0.1 °C, temperature dependent errors in the extrapolation of the line centers to their vacuum values would be limited to a few Hz, well below the statistical sensitivity of our test.

Other potential concerns involve the two frequency references used in the experiment—the proton precession frequency forming the basis of the magnetic field determination, and the Loran-C 10 MHz frequency reference used for the NMR and microwave frequency synthesizers. The Loran-C standard is based on hyperfine transitions in Cs with $m_F = 0$, and thus is insensitive to any preferred spatial orientation, and would not introduce a signature for Lorentz violation into the spectroscopic measurements. Time dependent gravitational redshifts or blueshifts of the frequency standard, caused by tidal distortions of the Earth, are below the 10^{-16} level and too small to be seen. Finally, bounds on clock comparisons of ^{199}Hg and ^{133}Cs [7,8] place limits on the Lorentz-violating energy shifts in the precession frequency of a proton of 10^{-27} GeV, which imply that the NMR measurements are free of shifts well below the Hz level.

All the data obtained in 1995 and 1996 are plotted as a function of time measured as a function of a sidereal day in Fig. 3, where twelve points at $\approx$2 h. intervals are plotted, and the vertical scale is in Hz. The data for ν_{12} and ν_{34} were fit separately by the functions

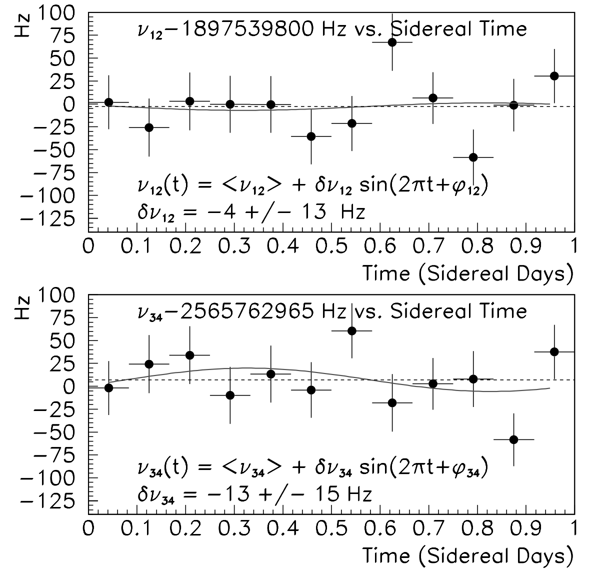

FIG. 3. Two years of data on ν_{12} and ν_{34} are shown binned versus sidereal time and fit for a possible sinusoidal variation. The amplitudes are consistent with zero.

$$\nu_{ij}(t) = \langle \nu_{ij} \rangle + \delta\nu_{ij} \sin(2\pi t + \phi_{ij}), \tag{7}$$

where t is the time in sidereal days, and the fit parameters are $\langle \nu_{ij} \rangle$, the amplitude of the possible time variation $\delta\nu_{ij}$, and the phase ϕ_{ij} (where no phase relation was assumed between ν_{12} and ν_{34}). The amplitudes for $\delta\nu_{12}$ and $\delta\nu_{34}$ are consistent with zero, -4 ± 13 Hz and -13 ± 15 Hz, respectively.

As stated above, the theory being tested requires $\delta\nu_{12} \approx -\delta\nu_{34}$. A plot of $\nu_{12} - \nu_{34}$ versus sidereal time is shown in Fig. 4, and fit for a sinusoidal variation [as in Eq. (7)], where a common phase is assumed between ν_{12} and ν_{34}. The data exhibit no variation with time within ± 20 Hz, which corresponds to a 68% confidence level (1σ) limit on the nonrotating frame components [see Eq. (2)]

$$\sqrt{(\tilde{b}_X^\mu)^2 + (\tilde{b}_Y^\mu)^2} \leq 2 \times 10^{-23}\ \text{GeV}. \tag{8}$$

The figure of merit of these results as a test of *CPT* violation is taken as

$$2\sqrt{(\tilde{b}_X^\mu)^2 + (\tilde{b}_Y^\mu)^2}/m_\mu \lesssim \frac{2\pi|\delta\nu_{12}|}{m_\mu} \approx 5 \times 10^{-22}. \tag{9}$$

The limits on $\delta\nu_{34}$ and $\delta(\nu_{34} - \nu_{12})$ yield similar values. Other choices for the denominator in the figure of merit such as ν_{ij} or $\Delta\nu$ are inappropriate since the former are B-field dependent and the latter is not a fundamental property of the muon. It is natural to take the muon mass as the basic energy scale since it is an input parameter of the standard model from which the atomic properties are

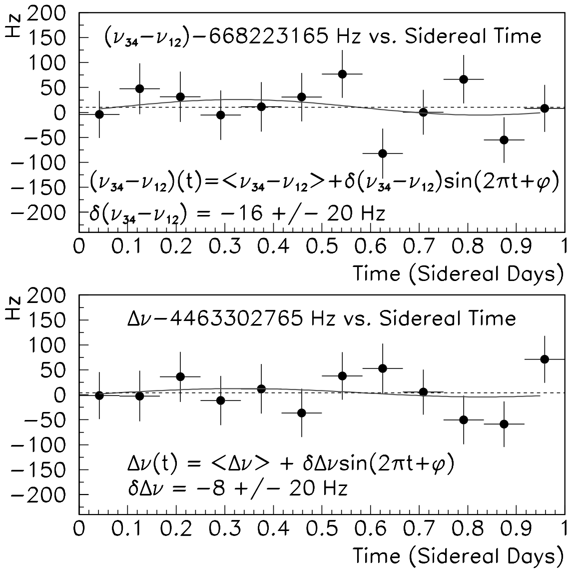

FIG. 4. Two years of data on $\nu_{12} - \nu_{34}$ and $\nu_{12} + \nu_{34} = \Delta\nu$ are shown binned versus sidereal time and fit for a possible sinusoidal variation. The amplitudes are consistent with zero.

derivable. The ratio of Lorentz- and *CPT*-violating deviations of muonium energy levels to the muon mass might be expected to be suppressed by the ratio of the low energy (standard model) scale to the Planck scale of the underlying theory [9]. In this context the result achieved here, 5×10^{-22}, is comparable to the dimensionless scaling factor $m_\mu/M_P \sim 10^{-20}$.

Bounds on a different linear combination of the muon parameters b_J^μ, d_{J0}^μ, and H_{KL}^μ from muonium can also be obtained from precision measurements of the anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$ [5], where one can search for sidereal variations of the anomalous precession frequency ω_a^μ (the difference between spin and cyclotron frequencies). Also, the parameter $\tilde{b}_Z^\mu$, which is not tested in the latest muonium experiment, is bound at the level of 10^{-22} GeV by the measurements of the anomalous precession frequencies ω_a^μ of μ^+ and μ^- measured at CERN and BNL [5,10]. Further improvements in the Lorentz and *CPT*-violating parameters for the muon, coming from muonium, will require higher intensity muon sources, as the uncertainties are predominantly statistical.

In conclusion, no unambiguous violation of *CPT* or Lorentz invariance has been observed although tests have been done on many systems since 1960 [11]. The results presented here represent Planck scale sensitivity, and almost an order of magnitude improvement in sensitivity over previous limits on the muon parameters of the theory. They also represent the first test for sidereal variations in the second generation of leptons, where Lorentz- and *CPT*-violating effects may be enhanced [5]. Higher sensitivity tests as well as tests on different physical systems will surely be made [3,12].

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