A New Determination of G Using Two Methods

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We present the results of a measurement of G made with a torsion-strip balance used in two substantially independent ways. The two results agree to within their respective uncertainties; the correlation coefficient of the two methods is -0.18. The result is $G = 6.675\,59(27) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with a standard uncertainty of 4.1 parts in 10^5. Our result is 2 parts in 10^4 higher than the recent result of Gundlach and Merkowitz.

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There has in recent years been considerable uncertainty as to the correct value of the Newtonian gravitational constant, G, despite precision measurements extending back two centuries [1,2]. We note in particular the value from Michaelis *et al.* [3] of the PTB (Braunschweig) that differed from the 1986 CODATA value by $(0.6 \pm 0.008)\%$. There is still no explanation for this large discrepancy, although we present here one possible effect that could have led to an error of this magnitude. A number of recent papers [2,4] give values rather closer to the CODATA value, particularly the paper by Gundlach and Merkowitz that gives a result with the very low uncertainty of 14 parts per million (ppm). We report here a new determination of G, which has a standard uncertainty of 41 ppm. Our value is unique in that it is based on two results obtained using the same apparatus but with different methods of measurement. Our result does, however, differ from that of Gundlach and Merkowitz by some 200 ppm.

The BIPM torsion balance [5] (see Fig. 1) has the following principal features: (1) a four-mass configuration to give a much reduced sensitivity to external gravitational fields; (2) a torsion strip to give much improved stability with practically no dependence on the material properties of the strip; (3) a gravitational signal torque of 1.7×10^{-8} N m, some 4 orders of magnitude larger than in most previous comparable experiments; this improves the ratio of gravitational signal to nongravitational noise and allows a very precise measurement to be made in a short time; (4) three possible methods of operation, (a) electrostatic servo control, (b) free-deflection (Cavendish method), and (c) change in period of free oscillation; the result presented here is based on (a) and (b), the timing precision of the third method not at present being sufficient to give a useful result; (5) dimensional metrology that is quick and accurate by having the whole apparatus mounted on the base of a coordinate measuring machine (CMM).

The source and test masses are made from Cu-0.7%Te free-machining alloy. They are right-circular cylinders with heights equal to their diameters and with masses of 12 and 1.2 kg, respectively. The test masses are mounted on a radius of about 120 mm around the periphery of an aluminum-alloy disk suspended from the torsion strip inside a vacuum chamber.

The torsion strip is made from Cu-1.8%Be dispersion-hardened alloy of thickness $t = 30\ \mu$m, width $b = 2.5$ mm, and length $L = 160$ mm. It is loaded to 800 MPa, about 80% of its yield strength. The torsion constant, c, is given by $c = bt^3F/3L + Mgb^2/12L$, where F is the shear modulus of elasticity and g is the local gravitational acceleration. For our strip, the second (gravitational) term is 27 times larger than the first (elastic) term, which thus makes up less than 4% of the total. Anelasticity in the suspension is thus much reduced, leading to a high mechanical Q of the system, 3×10^5, and very small zero drift of less than 1 μrad per week. The high Q removes as significant sources of concern systematic biases caused by the anelastic aftereffect [6], or a frequency-dependent torsion constant [7]. The natural period of torsional oscillations is 125 sec; thus the balance has a ring-down time of some five months.

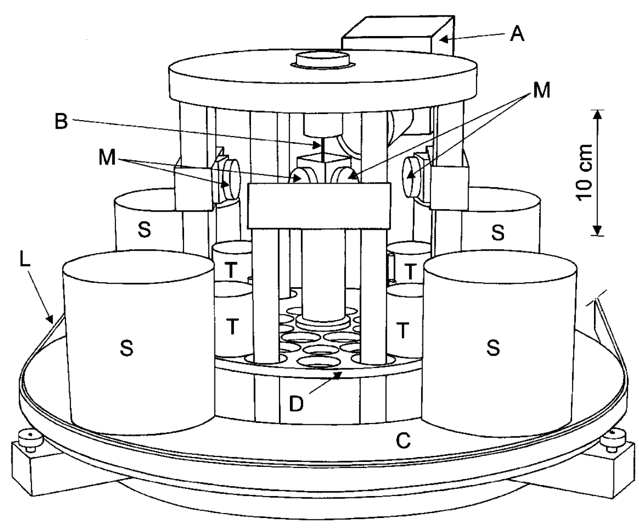

FIG. 1. Outline of apparatus: T, test masses; S, source masses; D, torsion balance disk; B, torsion strip; C, carousel; L, drive belt; M, mirrors for sixfold multiplying optics; A, autocollimator.

Outside the vacuum chamber, the four source masses are mounted symmetrically on a radius of about 214 mm on an aluminum-alloy carousel belt-driven by a stepping motor. When aligned radially with the test masses, the source masses produce no torque on the balance. When rotated in either direction by 18.7° the gravitational torque is at its maximum. Angular deflection of the balance is measured by an Elcomat 2000 autocollimator through sixfold multiplying mirror optics.

In the servo-controlled method, the gravitational torque of the source masses is balanced by an electrostatic torque acting directly on the test masses. Control of the balance is by applying ac voltages, V, at a frequency of 1 kHz between the test masses themselves and a pair of thin vertical cylindrical copper electrodes placed about 1 mm from the masses. This geometry produces a very linear variation of capacitance, C, between the electrodes and the test masses. The electrostatic torque constant is calibrated directly in SI units by determining $dU/d\theta$, the change in total electrostatic energy as a function of angle where $U = (1/2)\sum_{ij} C_{ij}(V_i - V_j)^2$, and C_{ij} is the capacitance between one copper electrode and the other electrode, the masses and the vacuum can. Calibration is accomplished by measurement of all $dC_{ij}/d\theta$. The relation between G and the measured and calculated quantities is $G = \tau/\Gamma$, where τ is the measured torque and Γ is the gravitational coupling constant between the torsion balance and source masses. By holding the balance at a fixed angular position during the torque measurement, anelastic effects in the suspension are eliminated, although as noted below in the case of the Cavendish method, anelasticity in our suspension leads to insignificant error. We at first used a dc servo system but were unable to eliminate the possibility of frequency-dependent losses that could render the calibration of $dC_{ij}/d\theta$ at 1 kHz inconsistent with the $dC_{ij}/d\theta$ at 1 mHz during the servo control. Biases in the calibration were such as to increase the apparent value of G in contradiction to the result of our recent theoretical study [8]. In this study we considered only loss mechanisms due to surface films on the electrodes. However, it can be shown that a grounded lossy dielectric located in the electric field will add, in parallel, a frequency-dependent capacitance such that measurement of $dC_{ij}/d\theta$ at high frequency overestimates the calibration constant. We encountered the possibility of such a bias in our early work by noting that the Q of the torsion balance was halved on application of 2 kV dc to the electrodes. We later identified a coaxial cable affected by the fringing field of a pair of electrodes. We suggest that a similar effect may have been present in the PTB measurement since in their experiment the calibration and measurement frequencies differed by orders of magnitude, as was the case in our preliminary work. Such problems are eliminated by using an ac servo.

In the Cavendish method the torsion balance is allowed to move in response to the movements of the source masses. At equilibrium, the applied gravitational torque is balanced by the suspension stiffness. The angular deflection, θ (about 80 μrad), is related to the applied torque τ by Hooke's law: $\tau = c\theta$, where c is the stiffness of the suspension. Also $\tau = G\Gamma$, where Γ is the same function of the mass distribution as for the servo-controlled method. We obtain c from the period of free oscillation T and the moment of inertia I of the suspended system (calculated from dimensional and mass metrology) using the relation $c = I\omega^2$.

Common to both methods is the calculation of Γ, the gravitational interaction between the source and test masses; this was done by two different methods. The first uses the analytical expression for the radial acceleration field of a right cylinder [9], integrated numerically over the volume of a test cylinder to find the force between a source and test mass. The second uses a multipole expansion to determine the force between a source and test mass [10,11]. Corrections were applied for the gravitational attraction of the air displaced by the source masses.

A critical parameter in all measurements of G is the uniformity of density of the attracting masses. For both source and test masses we measured the density inhomogeneities by hydrostatic weighing of samples cut from the original ingots and, for the source masses, by determination of center of gravity from measurements of period of free oscillation when supported on an air bearing. Samples were afterwards cut from the source masses themselves. We found that the density varied linearly across the diameter of the source masses by an amount, $\Delta\rho_0$ from axis to perimeter, that increased from about 1 to 2 parts in 10^4 along the length of the ingot. Such a transverse linear density gradient results in a $\cos\phi$ dependence of density in each source mass. Inner mass multipole moments, e.g., q_{11}, were therefore present in the source masses that were not accounted for in the analysis described above. A more detailed analysis (following the method given by Urso and Adelberger [10]) shows that these inner moments interact with outer moments, e.g., Q_{22}, of the test masses. These moments can, in turn, be calculated from the inner moments of the test masses using results from Trenkel and Speake [11]. This calculation shows that the torque between the two masses, whose centers are a distance a apart, has an extra term,

$$\frac{\Delta\Gamma}{\Gamma} = \frac{5}{6}\frac{R_s}{a}\frac{\Delta\rho_0}{\rho_0}\cos\phi_0\cos\phi\,, \qquad (1)$$

where R_s is the radius of the source mass, ϕ is the orientation of the q_{11} multipole, and ϕ_0 is the angle between the line joining the centers of the source and test masses and that between the centers of source mass and torsion balance; 21.3° for a source mass offset of 18.7°. Using the measured density homogeneities, the maximum fractional error is about 90 ppm for one source mass. When the torques due to all source and mass pairs are summed, this is reduced to -32, -0.4, and 36 ppm, depending on the orientation of the source masses. Thus the errors of the torques at three angular positions of the test masses 120° apart average almost to zero. A linear axial variation of density amounting to 100 ppm over the length of each source mass was also found by hydrostatic weighing. This introduces an axial shift in the centers of mass of about 1 μm, which in turn produces a negligible change in torque.

In the (far smaller) billet from which the test masses were cut, a significant azimuthal dependency was not found to within the uncertainty of our measurements. The accuracy of the density measurements were limited to ± 5 ppm but, owing to the shape of the samples used, linear variations of 9 ppm could not be ruled out. Using Eq. (1) to calculate the analogous torque error for the test mass inhomogeneity, we find a maximum fractional change in torque for a single mass pair of about 1.7 ppm. We would expect that, when summed over all pairs, this would be reduced to less than 1 ppm. This was considered negligible and was not included in the uncertainty budget.

The variation of density across a radius would shift the center of gravity of the test masses, and this would introduce an error in the calculation of the moments of inertia. If we have a density gradient across the diameter of a cylinder with $\Delta\rho = \Delta\rho_0(r/R_i)\cos\phi$, the shift in the center of mass of the cylinder can be calculated as $\delta r = (\Delta\rho_0/\rho)(R_t/4)$. Assuming a random orientation of the masses, the first-order fractional change in the moment of inertia of the torsion balance is given as $\Delta I/I \approx \delta r/d\sqrt{2}$ where d is the average radial distance of the center of the masses from the axis of rotation. This amounts to a change of much less than 1 ppm in the moment of inertia which is, again, negligible. This systematic effect, if present, would show up differently in the Cavendish and servo methods.

The servo value of G is based on the 25 measurements of torque shown in Fig. 2. All data points were used. Each was corrected for the small effect of density inhomogeneity appropriate to the orientation of the source masses given in Fig. 2.

The Cavendish value of G is based on 38 measurements (each of 4 h duration) of angular deflection. All data points

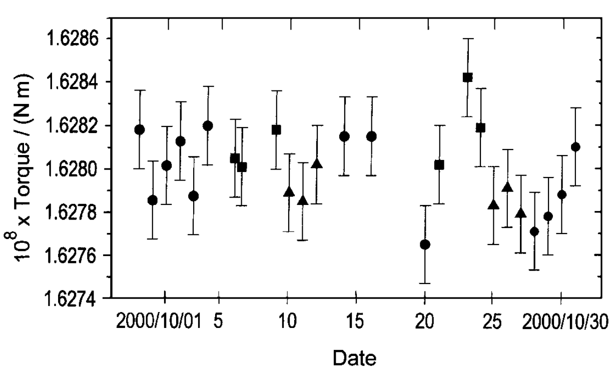

FIG. 2. Results of the servo measurements: the bars represent the combination of the uncertainties of the fit of the 10 h of data and the calibrations of the servo before and after each run; the different orientations of the source masses are indicated.

were used and each was corrected for residual anelastic effects in the strip (-13 ± 4) ppm and for the density gradient in the source masses $(+32 \pm 6)$ ppm at the single orientation used. Both the servo and Cavendish data were corrected (1.000 268 5) for the refractive index of air [12], whose effect was to decrease the measured value of G in the servo method and increase it in the Cavendish method.

Among the many additional checks and auxiliary measurements were the following: (1) the source mass positions were permuted with respect to the test mass positions in 90° increments; (2) the source masses were removed from the carousel and the torque of their kinematic mounts on the balance was measured; (3) the test masses were removed from the disk so that the torque of the source masses on all the other components of the torsion balance could be measured; this test also demonstrated the absence of a tilt perturbation correlated with the movement of the source masses; (4) a second CMM was used to verify the reproducibility of about 1 μm for the dimensional measurements; (5) parasitic magnetic forces were estimated to be negligible based on the measured properties of all materials used in the apparatus. In the most critical cases, these conclusions were tested by experiment.

Our final result for the servo method is $G = 6.675\,53 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with a standard uncertainty of 6.0 parts in 10^5 and for the Cavendish method is $G = 6.675\,65 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with a standard uncertainty of 6.7 parts in 10^5. The uncertainty budget for the whole experiment is given in Table I.

A detailed analysis shows that the uncertainties of the two measurements have a correlation coefficient of -0.18. This coefficient, defined as the covariance of the two measurements divided by the geometric mean of their variances, is a measure of the independence of the two methods. Note that the servo-control method relies essentially on electrical measurement while the Cavendish method relies on timing and that the mass of the test cylinders is eliminated in the Cavendish method since it appears in both τ and Γ. The same relative angle error would produce an equal but opposite error in the servo-control and Cavendish methods and thus would be eliminated in the mean of the two results. However, there is not complete cancellation of the angle uncertainties here because in the Cavendish method there is an additional uncertainty due to nonlinearities in the autocollimator at small angles.

The combined final result, the mean of (a) and (b) is thus $G = 6.675\,59(27) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with a standard uncertainty, taking account of correlations, of 4.1 parts in 10^5.

In conclusion, the close agreement of the results of our two substantially independent methods is evidence for the absence of many of the systematic errors to which a G measurement is subject. Nevertheless, the two most accurate measurements of G, this one and that of Gundlach and Merkowitz, differ by more than 4 times their combined standard uncertainty, Fig. 3. We examined the possibility

TABLE I. Uncertainty budget.

Source	Contribution to uncertainty in G (parts in 10^5)
(a) Servo method:	
Test mass value	0.25
Source mass value	0.25
Test and source mass coordinates	3.2
Angle calibration	2
Calibration of electrical instruments	4
Mean of torque measurements ($n = 25$)	2.3
Subtotal	6.0
(b) Cavendish method	
Test mass value	0
Source mass value	0.25
Test and source mass coordinates	2.4
Timing	3.5
Angle calibration	4
Anelasticity	0.4
Source-mass density inhomogeneity	0.6
Mean of deflection measurements ($n = 38$)	3.2
Subtotal	6.7
Average of (a) and (b), taking account of correlations	
Mass values	0.3
Test and source mass coordinates	2.2
Source-mass density inhomogeneity	0.3
Anelasticity	0.2
Capacitance calibration	2
Angle calibration	1
Timing	1.75
Means of torque and deflection	2.0
Combined uncertainty	4.1

that this discrepancy could be put down to a failure of the inverse square law, since the effective distances apart of the source and test masses are different. Considering a plot of the strength versus range of a new interaction modeled

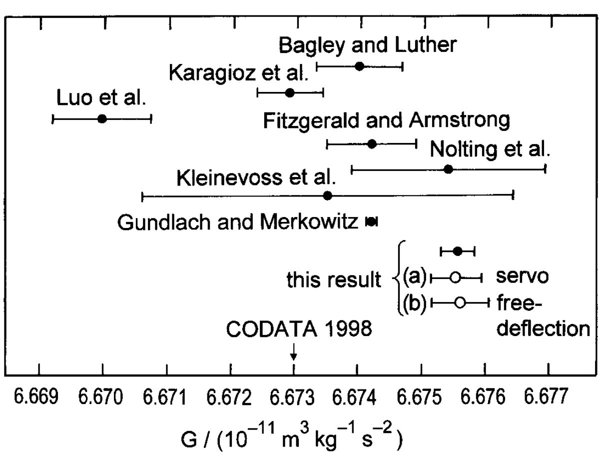

FIG. 3. The present result compared with measurements of G published since 1997 [13].

as a Yukawa potential; at 7 cm the violation of the inverse square law required to eliminate the discrepancy between the results at the 1σ level is a factor of about 3 above the 1σ limit set by Spero *et al.* [14]. The difference is thus likely still to be due to systematic errors, at the level of one or two parts in 10^4, hidden in one or both of the measurements. Nevertheless, it now seems certain that the 1996 result of Michaelis *et al.,* discrepant by 0.6%, was indeed subject to some systematic error, perhaps one related to losses in the dc servo system as we suggest above.

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[1] T. J. Quinn, Nature (London) **408**, 919 (2001).
[2] P. J. Mohr and B. N. Taylor, J. Phys. Chem. Ref. Data **28**, No. 6 (1999); Rev. Mod. Phys. **178**, No. 2 (2000).
[3] W. Michaelis, H. Haars, and R. Augustin, Metrologia **32**, 267 (1995/96).
[4] J. H. Gundlach and S. M. Merkowitz, Phys. Rev. Lett. **85**, 2869 (2000).
[5] S. J. Richman, T. J. Quinn, C. C. Speake, and R. S. Davis, Meas. Sci. Technol. **10**, 460 (1999).
[6] T. J. Quinn, C. C. Speake, and L. M. Brown, Philos. Mag. A **65**, 261 (1992).
[7] K. Kuroda, Phys. Rev. Lett. **75**, 2796 (1995).
[8] C. C. Speake, R. S. Davis, T. J. Quinn, and S. J. Richman, Phys. Lett. A **263**, 219 (1999).
[9] Y. T. Chen and A. H. Cook, *Gravitational Experiments in the Laboratory* (Cambridge University Press, Cambridge, 1993).
[10] C. D'Urso and E. G. Adelberger, Phys. Rev. D **55**, 7970 (1997).
[11] C. Trenkel and C. C. Speake, Phys. Rev. D **60**, 107501 (1999).
[12] G. W. Kaye and T. H. Laby, *Tables of Physical and Chemical Constants* (Longman, London, 1995), 16th ed., p. 130.
[13] C. H. Bagley and G. G. Luther, Phys. Rev. Lett **78**, 3047 (1997); O. V. Karagioz *et al.,* Gravitation Cosmol. **4**, 239 (1998); J. Luo *et al.,* Phys. Rev. D **59**, 042001 (1999); M. P. Fitzgerald and T. R. Armstrong, Meas. Sci. Technol. **10**, 439 (1999); F. Nolting *et al., ibid.* **10**, 487 (1999); U. Kleinvoss *et al., ibid.* **10**, 492 (1999).
[14] R. E. Spero, J. K. Hoskins, R. Newman, J. Pellam, and J. Schultz, Phys. Rev. Lett. **44**, 1654 (1980).