Array-Enhanced Coherence Resonance: Nontrivial Effects of Heterogeneity and Spatial Independence of Noise

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We demonstrate the effect of coherence resonance in a heterogeneous array of coupled Fitz Hugh–Nagumo neurons. It is shown that coupling of such elements leads to a significantly stronger coherence compared to that of a single element. We report nontrivial effects of parameter heterogeneity and spatial independence of noise on array-enhanced coherence resonance; especially, we find that (i) the coherence increases as spatial correlation of the noise decreases, and (ii) inhomogeneity in the parameters of the array enhances the coherence. Our results have the implication that generic heterogeneity and background noise can play a constructive role to enhance the time precision of firing in neural systems.

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Noise-induced effects in nonlinear systems, especially stochastic resonance (SR) [1] have drawn great research interests. The frontier of this interest has shifted to spatiotemporal systems recently [2,3]. One of the most interesting phenomena is *array-enhanced stochastic resonance* (AESR) [3], where the response of a bistable noisy oscillator to an external periodic signal is further optimized by coupling it locally to an array of identical oscillators. Potential symmetry is found to enhance SR in extended bistable systems [4]. Another generalization of SR to spatiotemporal systems is doubly stochastic resonance [5].

There is also great interest in the resonantlike phenomenon of the coherent motion induced *purely by noise without an external signal* [6–8], called coherence resonance (CR). Pure spatially independent noise can induce traveling waves [9], global oscillations, CR [10], and noise-enhanced phase synchronization [11] in excitable media. We have recently demonstrated that, very similar to AESR, the coherence can also be significantly enhanced when the noisy excitable elements are coupled and subjected to independent local noise. This behavior is called *array-enhanced coherence resonance* (AECR) [12].

In spatially extended systems, two further ingredients may have significant effects on the system's behavior: (i) noise correlation among the elements, and (ii) spatial heterogeneity in parameters of different elements. In particular, in neuroscience, heterogeneity is generic because neurons are never identical. Moreover, they also receive synaptic inputs from a large number of other neurons. These inputs exhibit randomlike behavior and might be highly correlated among a piece of neurons [13]. Most of the previous investigations considered the effect of spatially independent noise on parameter homogeneous media. The purpose of this contribution is, therefore, to study, in the context of AECR and noise-induced coherence of firing activity in neural systems, the following question: What is the most efficient for AECR, to couple the elements into a homogeneous or heterogeneous chain, and subject them to spatially correlated or independent noise? We will mainly show that the coherence of the spatiotemporal behavior of chain can be significantly enhanced by independent noise and heterogeneity.

We demonstrate these nontrivial effects of disorder in a chain of N locally coupled Fitz Hugh–Nagumo (FHN) neurons, which is a paradigmatic model of excitable systems, nerve pulses, CR [7], or the firing activity of electroreceptor cells of the electric fish [14]. The coupled system reads

$$\epsilon \dot{x}_i = x_i - x_i^3/3 - y_i + g(x_{i+1} + x_{i-1} - 2x_i), \quad (1)$$

$$\dot{y}_i = x_i + a_i + D\xi_i, \quad (2)$$

where $\epsilon = 0.01$, a_i is the parameter of the ith element, g is the coupling strength, and D is the intensity of the noise ξ_i. Periodic boundary condition $x_0 = x_N$ is employed in our simulations.

For a single FHN neuron, if $|a| > 1$, the system has only a stable fixed point corresponding to its quiescent state, while for $|a| < 1$ a limit cycle occurs. The quiescent neuron ($|a|$ slightly larger than 1.0) is excitable because it produces a firing pulse when perturbed away from the fixed point by external stimuli.

To study the effects of spatial heterogeneity, we compare a homogeneous chain (HC) where all a_i take the same value a_0, with an inhomogeneous chain (IHC) where a_i is a random variable uniformly distributed in $(a_0 - \delta a, a_0 + \delta a)$. We fix $a_0 = 1.05$, and set the heterogeneity degree $\delta a \leq a_0 - 1$ so that all the cells are in the subthreshold regime.

We introduce noise correlation among the cells in the following way: For simplicity, in a network consisting of a small number of neurons, highly correlated, randomlike synaptic inputs might be represented by a Gaussian noise common to all neurons [15]. The thermal fluctuations, or ion channel activity, however, give rise to spatially independent local noise. In our model, the noise consists of two components, $\xi_i(t) = \sqrt{R}\, e(t) + \sqrt{1 - R}\, \eta_i(t)$, where $e(t)$, a Gaussian noise with $\langle e(t)e(t - \tau)\rangle = \delta(\tau)$, is common to all elements, and η_i is the local Gaussian noise with $\langle \eta_i(t)\eta_j(t - \tau)\rangle = \delta_{i,j}\delta(\tau)$. Since $\langle \xi_i(t) \times \xi_j(t - \tau)\rangle = [R + (1 - R)\delta_{i,j}]\delta(\tau)$, the control parameter R measures the noise correlation between a pair of cells. Usually, spatial correlation of noise in extended systems is considered by a correlation length. The effects presented in the following are similar when the noise correlation is introduced by a correlation length [16].

In a single excitable FHN neuron subjected to noise, the firing activity becomes most coherent for a certain noise intensity [7]. When coupled into an array, the firing of a neuron may propagate in the chain to excite its neighboring neurons. This mutual excitation may make some neurons fire in a synchronized fashion and reduce the fluctuations in pulse intervals.

To characterize the behavior in the chain, we introduce a phase [17] of the dynamics in each cell,

$$\phi_i(t) = 2\pi \frac{t - \tau_k^i}{\tau_{k+1}^i - \tau_k^i} + 2\pi k\,, \tag{3}$$

where τ_k^i is the time of the kth firing of the ith cell. We understand synchronization between two noisy cells as the appearance of peaks in the distribution of the cyclic relative phase $\Phi_{i,j} = (\phi_i - \phi_j)\mathrm{mod}2\pi$ [18]. Deviation of this distribution from a uniform one characterizes the strength of synchronization. We quantify this deviation by $\sigma_{i,j}^2$, which is the variance of the actual phase distribution divided by that of a uniform one. Choosing a reference cell n_0 in the chain, the spatial average,

$$\sigma_{\mathrm{syn}}^2 = \frac{1}{N - 1}\sum_{j \neq n_0} \sigma_{n_0,j}^2\,, \tag{4}$$

provides a measure of the synchronization strength in the chain. $\sigma_{\mathrm{syn}}^2 = 0$ means perfect global synchronization, and $\sigma_{\mathrm{syn}}^2 = 1$ indicates no synchronization in the chain.

To measure the temporal coherence of the noise-induced firing of a neuron, we examine the distribution of the pulse interval $T_k^i = \tau_{k+1}^i - \tau_k^i$. The sharpness S^i of this distribution,

$$S^i = \langle T_k^i\rangle_t \Big/ \sqrt{\mathrm{Var}(T_k^i)} \qquad (i = 1, 2, \dots, N)\,, \tag{5}$$

measures the coherence of the firing events. Here $\langle\cdot\rangle_t$ denotes average over time. This quantity is of biological significance because it is related to the timing precision of the information processing in neural systems [19]. We also compute the coherence factor S of the chain based on the distribution of pulse intervals of *all N* cells. S is defined similar to Eq. (5), but here the average is performed over both time and space. In the HC, there is $S^i = S$ for all the cells, while, in the IHC, S^i fluctuates around S due to the heterogeneity. Larger S is related to stronger coherence in the chain.

The results of our numerical experiments are common for lattices of different size N, e.g., $N = 50, 100, 200$. For $N = 100$, Fig. 1(a) exhibits clearly the feature of AECR where S reaches a maximal value at an optimal noise intensity $D_{\mathrm{opt}} \sim 10^{-1.5}$ and the coherence of the chain is much higher than that of the single element. D_{opt} shifts slightly to smaller values with decreasing R, and it is important to emphasize that these values are much smaller than that of the single element as a result of mutual excitation in the chain. The temporal coherence is also related to spatial coherence in the chain. Figure 1(b) displays the typical phenomenon of noise-enhanced phase synchronization [11] indicated by a minimal σ_{syn}^2 at a noise intensity close to the optimal one for S. Spatial phase synchronization is related to the coherence of the mean-field $X = (1/N)\sum x_i$ of the chain, which also displays the feature of CR [16]. The most interesting phenomenon here is that the correlation of noise has a stronger influence on temporal coherence than on spatial coherence: The temporal coherence increases rapidly with a decreasing noise correlation R over a broad range of noise intensity D, while, around D_{opt}, the synchronization degree is not degraded very much by decreasing R. As a result, spatiotemporal coherence is enhanced enormously by spatially independent noise. The common noise component $e(t)$ introduces a long-range interaction in the chain. However, its effect is to decrease the coherence, as is different to the effect of the coupling g which increases the coherence significantly both in AESR [3] and AECR [12] (and see Fig. 4 below). For global noise ($R = 1$), σ_{syn}^2 never reaches zero due to the spatial heterogeneity. There the maximal coherence $S \approx 7.0$ is rather close to $S \approx 5.3$ for the uncoupled single element with $a = a_0$, showing that spatial coupling of such elements subjected to global noise only weakly improves the coherence. Actually, this weak improvement is due to the spatial heterogeneity. The underlying mechanism will be discussed later.

Next we demonstrate the important role of spatial heterogeneity on AECR. We compare a HC ($\delta a = 0$)

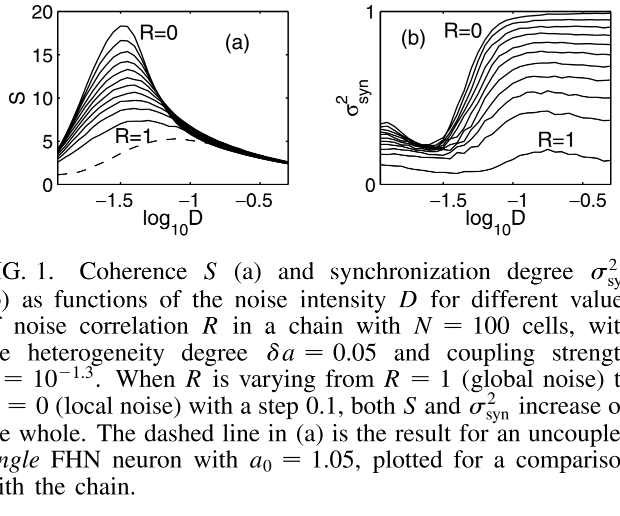

FIG. 1. Coherence S (a) and synchronization degree σ_{syn}^2 (b) as functions of the noise intensity D for different values of noise correlation R in a chain with $N = 100$ cells, with the heterogeneity degree $\delta a = 0.05$ and coupling strength $g = 10^{-1.3}$. When R is varying from $R = 1$ (global noise) to $R = 0$ (local noise) with a step 0.1, both S and σ_{syn}^2 increase on the whole. The dashed line in (a) is the result for an uncoupled *single* FHN neuron with $a_0 = 1.05$, plotted for a comparison with the chain.

with an IHC ($\delta a = 0.05$) subjected to local noise ($R = 0$). The coupling strength $g = 10^{-1.3}$ is the same as in Fig. 1, and the noise intensity $D = 10^{-1.5}$ is chosen close to D_{opt} of the IHC in Fig. 1. As shown by the average pulse interval $\langle T_k^i \rangle_t$ in Fig. 2(a), the IHC fires more frequently than the HC. In spite of the heterogeneity, the IHC has achieved a regime of frequency locking with the same value of $\langle T_k^i \rangle_t$ for all its cells. The variances of the pulse intervals $\text{Var}(T_k^i)$, however, fluctuate strongly, but it is important to emphasize that their values are significantly smaller than those of the HC. As a result, the coherence factors S^i [Eq. (5)] of *all* cells in the IHC are larger than those in the HC [Fig. 2(b)]. Furthermore, the coherence S increases monotonically with increasing heterogeneity degree δa (Fig. 3).

Now we compare more systematically the coherence of the HC and IHC in the parameter space of coupling strength g and noise intensity D for local noise $R = 0$. As seen in Figs. 4(a) and 4(b), for rather weak coupling g, both the HC and IHC have a coherence very close to that of the uncoupled single element with parameter a_0. AECR sets in for strong enough coupling, i.e., when the noise-induced pulses of a cell are able to excite neighboring cells in the chain. The coherence decreases again for very strong coupling and large enough noise. Now we analyze in more detail the appearance of the valley, the peak, and the plateau in Fig 4(c) which shows $\Delta S = S_{\text{IHC}} - S_{\text{HC}}$, the difference between the coherence factors of the IHC and HC. (i) Valley: In the IHC, those cells with a_i rather above the critical point $a = 1.0$ are more difficult to excite by coupling or noise than those closer to $a = 1.0$. For quite weak coupling, the former cells may not be excited by their neighboring cells; and they can give rise only to rather sparse and irregular pulses in the presence of relatively weak noise. While in the HC, mutual excitation has enhanced its coherence with the same coupling strength and noise intensity, resulting in a valley of ΔS. (ii) Peak: When moving to strong enough coupling, the former cells in the IHC become excitable by the latter cells in their neighborhood which generate frequent and rather regular pulses even at a weaker noise level. The whole IHC now achieves frequency locking to this quicker and more coherent firing activity, and a peak of ΔS in the region of stronger coupling and weaker noise is seen. With an increasing heterogeneity degree, more cells are getting closer to $a = 1.0$, resulting an enhanced coherence shown in Fig. 3. (iii) Plateau: With further increasing of the coupling, quick diffusion dominates the firing process in the chain and the heterogeneity becomes unimportant. The whole lattice becomes more rigid, and both the HC and IHC tend to act as a single element with decreasing temporal coherence. When too strong noise dominates the firing activity, one again sees no significant differences between the HC, the IHC, and the single element. The above-described behavior is similar for spatially correlated noise, i.e., $R \neq 0$.

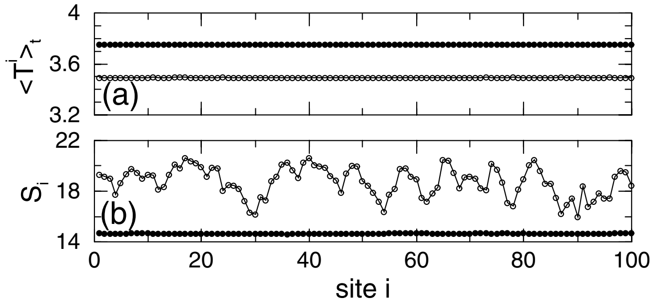

FIG. 2. Comparison of IHC (empty circles, $\delta a = 0.05$) and HC (filled circles, $\delta a = 0.0$). (a) Average of pulse interval T_k^i; (b) Coherence factor S^i of the cells.

Actually, the competition between pulse propagation induced by coupling and pulse excitation induced by noise also underlies the mechanism of enhanced coherence by spatially independent noise. In the HC subjected to global noise, the whole chain acts as a single uncoupled element without an enhancement of the coherence. If the noise has an independent local component, which generates firing events independently at some different cells and destroys perfect global synchronization, a cell which is not excited by noise may be excited by its neighboring cells. As a whole, the coherence is enhanced by mutual excitation. For smaller noise correlation, independent firing can be invoked by a weaker noise, so that D_{opt} in the chain shifts to slightly smaller values with decreasing R [Fig. 1a]. Parameter heterogeneity destroys perfect global synchronization and introduces mutual excitation even in the presence of global noise, thus the coherence can be improved as shown in Fig. 1(a). The most efficient optimization of the coherence by coupling occurs in the IHC with local noise.

In conclusion, we have demonstrated various nontrivial noise-induced behavior in a chain of coupled FHN neurons, especially array-enhanced coherence resonance and noise-enhanced synchronization. The parameter heterogeneity and spatially independent noise in the chain are found to play a decisive role in creating an efficient AECR. A detailed description of the mechanism and an extension from 1D chains to 2D lattices will be presented in a forthcoming paper [16]. We expect that similar properties can be observed in AESR where the chain is subjected to external periodic driving. We have studied here a system with

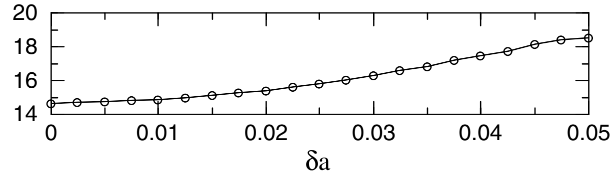

FIG. 3. Coherence factor S of the IHC as a function of the heterogeneity degree δa.

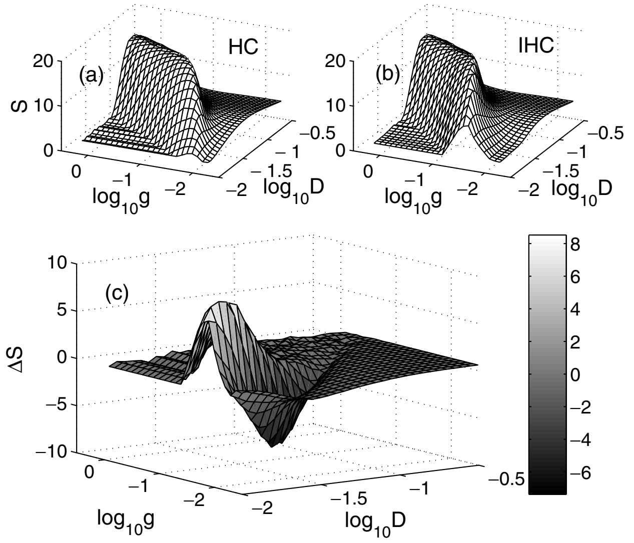

FIG. 4. Comparison of the coherence factors of the HC and IHC with respect to the coupling strength g and the noise intensity D. (a) S of the HC; (b) S of the IHC with heterogeneity $\delta a = 0.05$; and (c) the difference $\Delta S = S_{\text{IHC}} - S_{\text{HC}}$.

homogeneous coupling strength. It has been shown that a selective coupling can enhance SR in extended bistable systems [20]. Since selective coupling also affects the competition between pulse propagation and noise-induced excitation, it may also play a nontrivial role in AECR, which is under study.

Our finding may be of importance in neuroscience, where generic heterogeneity and spatially uncorrelated noise due to thermal fluctuation or ion channel activities may play an active role to increase the order and timing precision of an ensemble of interacting neurons in biological information processing. To make a stronger connection to neurobiology, future models should incorporate more biologically realistic ingredients. In particular, we will study the case where noise is colored both in time and space, e.g., the noise generated by a neural network [21] and the FHN neurons are coupled as a small-world network [22] whose connectivity topology is placed somewhere between a regular and a complete random connectivity.

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