Quantum Pump for Spin and Charge Transport in a Luttinger Liquid

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We study two different parametric pumps—one for pumping spin currents, the other for charge currents—in interacting quantum wires. We find that, as a function of pumping frequency, the spin or charge pumped per cycle has a nonuniversal crossover—depending on pumping details—between two universal fixed point values of 0 and twice the electronic spin or charge quantum number. The direction of flow between these two fixed points depends on whether the interactions are repulsive or attractive, while the quantization itself is a signature of interactions.

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In recent years there has been a tremendous interest in electron transport via the mechanism of pumping, in which periodic perturbations of the system yield a dc current [1]. The idea of pumping charge by cyclic variation of external couplings was first introduced by Thouless [2], who showed that for a system of electrons in a periodic potential, an integer electronic charge is transported in an adiabatic pumping cycle as a consequence of the Fermi energy lying in a band gap. More recently, a generalization of this picture has led to the observation of charge pumping in open quantum dots where the Coulomb blockade is lifted [3]. The corresponding theory for noninteracting systems has been quite extensively developed [4,5]. For pumping through a quantum dot, quantization has been shown in the Coulomb blockade regime [6].

One of the questions that motivates our work is whether bulk electronic interactions in gapless systems, such as Luttinger liquids, could lead to quantization of charge transfer in a quantum pump. Further motivation to study pumping in interacting systems is provided by the recent developments in coherent spin transport in low-dimensional semiconductors [7,8]. The study of spin transport is important not only for constructing devices based on manipulation of spins [9], but also because it offers the possibility of addressing fundamental issues of spin-charge dynamics in low-dimensional strongly correlated systems. A mechanism to pump a spin current through a quantum wire would be an alternative approach to existing coherent spin transport methods relying on injection from ferromagnetic interfaces [10].

In this Letter we propose a quantum pump for charge transport, and another for spin transport through 1D systems such as quantum wires, metallic carbon nanotubes, or fractional quantum Hall edges. Our analysis focuses on the Luttinger liquid model, which describes the generic low energy behavior of 1D models of correlated quantum particles with gapless excitations [11], with additional time-dependent external interactions that drive the quantum pump. The specific geometry we have in mind, depicted schematically in Fig. 1, consists of two externally tunable perturbations that modify the local charge (spin) density of the wire. Charge (spin) transfer is achieved by periodically varying the perturbing potentials with a relative phase difference φ. While the setup of Fig. 1(a) (Q pump) allows for pumping charge, that of Fig. 1(b) (S pump) can pump a pure spin current under appropriate conditions described below.

The response to pumping is an average transfer of charge $Q_c = eN_c$ (or spin $Q_s = \hbar N_s$) in a cycle, so that we can define the corresponding *pumping conductances* as $\mathcal{G}_{c,s} = \frac{e^2}{h} N_{c,s}$. We find that for repulsive interactions, the pumping conductance $\mathcal{G}_c$, as well as $\mathcal{G}_s$ in the case of spin pumping, is quantized (at temperature $T = 0$) in the limit of slow pumping, so that the average charge pumped per cycle is $Q_c = 2e$ while the average spin pumped per cycle is $Q_s = \hbar$, irrespective of the strength of interactions. In the limit of fast pumping both these quantities go to zero. The picture for attractive interactions is reversed. Thus, in the slow pumping limit $\mathcal{G}_{c,s} = 0$, while in the limit of fast pumping both $\mathcal{G}_c$ and $\mathcal{G}_s$ are quantized ($Q_c = 2e$ and $Q_s = \hbar$) independent of the interaction strength. This behavior in the LL regime is to be contrasted with the noninteracting case where the two conductances are neither quantized nor dependent on the pumping frequency.

In the presence of the externally tunable interactions indicated in Fig. 1, the Hamiltonian gets an explicitly time-dependent term:

$$\delta\mathcal{H}(t) = \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int dx\, V_{\sigma\sigma'}(x,t)\psi^\dagger_\sigma(x)\psi_{\sigma'}(x)\,. \quad (1)$$

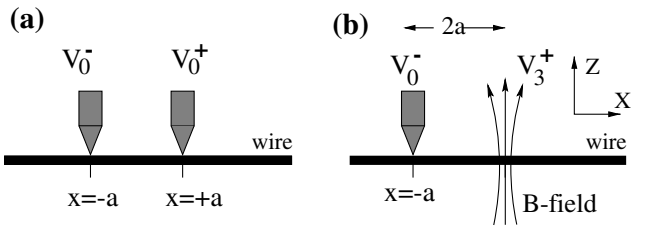

FIG. 1. (a) Geometry for a charge pump. Two gates placed a distance $2a$ apart are biased with ac voltages of the same frequency, ω_0, and relative phase φ. (b) Geometry for a spin pump. In addition to a gate as in (a), a inhomogeneous magnetic field points in the z direction near $x = a$, and oscillates with frequency ω_0 and a shifted phase φ.

For the Q pump, $V_{\sigma\sigma'}(x,t) = V_0^+(x,t)\delta_{\sigma\sigma'} + V_0^-(x,t)\delta_{\sigma\sigma'}$ is the sum of the two potentials arising from the gate voltages, with $V_0^{\pm}(x,t)$ being essentially zero outside the gate's point of contact ($x = \pm a$). The S pump has $V_{\sigma\sigma'} = V_0^-(x,t)\delta_{\sigma\sigma'} + V_i^+(x,t)\tau^i_{\sigma\sigma'}$, where τ^i is the ith Pauli spin matrix, and V_i^+ is the coupling of the local magnetic field (in the i direction) to the electron spin. A general periodic time dependence of the potentials $V_\nu^{\pm}$, for example the harmonic form $V_\nu^{\pm}(x,t) = V_\nu^{\pm}(x)\cos(\omega_0 t \pm \varphi/2)$ with a nonzero phase difference φ, operates the quantum pump yielding a dc current I_p. The current originates solely from the nonequilibrium backscattering of carriers, due to the explicit time dependence in the external Hamiltonian in Eq. (1). This is in contrast to the dc current $I = I_d - I_b$ due to a dc source-drain voltage, where there are two distinct contributions: (i) a direct part I_d arising from the applied dc voltage; the resulting conductance is ge^2/h [12,13], and (ii) a backscattered part I_b. In the case of a quantum pump, no source-drain voltage is applied, so there is no direct contribution ($I_d = 0$). Therefore, all the current arises from the backscattering term: $I_p = -I_b$.

To proceed we look at the Hamiltonian in the canonical bosonization scheme [14], wherein the fermion fields, linearized about the two Fermi points ($\pm k_F$), are written as $\psi_\sigma(x) = e^{ik_F x}\psi_{R,\sigma}(x) + e^{-ik_F x}\psi_{L,\sigma}(x)$. Here $\sigma = \uparrow,\downarrow$, and $\psi_{R,L;\sigma}$ are the right and left moving *chiral* fields, which are represented as normal ordered exponentials of bosonic fields, $\psi_{R,\sigma} =: e^{i\sqrt{4\pi}\phi_{R,\sigma}(x,\tau)} :, \psi_{L,\sigma} =: e^{-i\sqrt{4\pi}\phi_{L,\sigma}(x,\tau)} :$. The combinations $\phi_{R,\uparrow} + \phi_{L,\uparrow} = (\Phi_c + \Phi_s)/2$ and $\phi_{R,\downarrow} + \phi_{L,\downarrow} = (\Phi_c - \Phi_s)/2$ separate the bulk Hamiltonian $\mathcal{H}_0$ into independent spin ($\mathcal{H}_s$) and charge ($\mathcal{H}_c$) sectors.

$$\mathcal{H}_0 = \mathcal{H}_c + \mathcal{H}_s = \int dx \left\{ \frac{v_c}{2g_c}\left[(\partial_x\Phi_c)^2 + \frac{1}{v_c^2}(\partial_t\Phi_c)^2\right] + \frac{v_s}{2g_s}\left[(\partial_x\Phi_s)^2 + \frac{1}{v_s^2}(\partial_t\Phi_s)^2\right]\right\}, \quad (2)$$

where the velocities of the bosonic fields $\Phi_{c(s)}$ are denoted by $v_{c(s)}$. The spin isotropic point with a global U(1) × SU(2) symmetry corresponds to $g_s = 2$, and the noninteracting fermion limit is recovered for $g_s = g_c = 2$. In the absence of the backscatterers, the dc two-terminal conductance is $G_c = g_c e^2/h$, while the spin conductance is $G_s = g_s e^2/h$ [12]. The time-dependent Hamiltonian $\delta\mathcal{H}(t)$ in Eq. (1) describes both the backscattering and forward scattering processes by the two point contacts. The two contacts in the Q and S pumps can be reduced to an effective single contact as long as the pumping frequency $\omega_0 \ll \Omega_c = v_F/a$ (where v_F is the Fermi velocity), as shown in the context of fractional quantum Hall edges in Ref. [15]. We can thus form a low energy theory, for which Ω_C is an upper cutoff, having a single scatterer at $x = 0$ with an effective backscattering amplitude $\int dx\, V_{\sigma\sigma'}(x,t)e^{-i2k_F x}$. Also, since the pumping current is determined entirely by the periodic variation in backscattering processes, we can drop the forward scattering part of the interactions from the Hamiltonian. As a result, the time-dependent term in the Hamiltonian can be written in a matrix form with a unified notation for the Q and S pumps

$$\delta\mathcal{H}(t) = \Psi^\dagger(0)\mathbf{\Upsilon}(t)\Psi(0), \quad (3)$$

where $\Psi^\dagger = (\psi^\dagger_{R,\uparrow}\psi^\dagger_{R,\downarrow}\psi^\dagger_{L,\uparrow}\psi^\dagger_{L,\downarrow})$, and

$$\mathbf{\Upsilon}(t) = \sum_\nu \begin{bmatrix} 0 & \Gamma^*_\nu(t)\tau^\nu \\ \Gamma_\nu(t)\tau^\nu & 0 \end{bmatrix}.$$

For the Q pump, the only nonvanishing term is $\Gamma_0(t) = e^{-i2k_F a}\tilde{V}_0^-(2k_F,t) + e^{+i2k_F a}\tilde{V}_0^+(2k_F,t)$, while the S pump has $\Gamma_0(t) = e^{-i2k_F a}\tilde{V}_0^-(2k_F,t)$ and also $\Gamma_i(t) = e^{+i2k_F a}\tilde{V}_i^+(2k_F,t)$. The $\tilde{V}_\nu^{\pm}(k,t)$ are the Fourier modes of the $V_\nu^{\pm}(x,t)$ potentials. Let us denote the two parameters whose periodic variations operate these pumps as $X_1(t)$ and $X_2(t)$. These parameters are identified as $X_1(t) = e^{-i2k_F a}\tilde{V}_0^-(2k_F,t)$ for both pumps; $X_2(t) = e^{+i2k_F a}\tilde{V}_0^+(2k_F,t)$ for the Q pump, while $X_2(t) = e^{+i2k_F a}\tilde{V}_i^+(2k_F,t)$ for the S pump.

The response to this parametric variation in the charge sector is given by the charge backscattering current: $\hat{I}_b^0 = i[\hat{N}_L, \delta\mathcal{H}] = -i[\hat{N}_R, \delta\mathcal{H}]$, where $\hat{N}_{R,L}$ is the charge density of right (left) movers [16]. This expression can be generalized to include spin currents and written in the following form

$$\hat{I}_b^\lambda = -\frac{1}{2}\, i\Psi^\dagger(0)[\mathbf{M}^\lambda, \mathbf{\Upsilon}]\Psi(0), \quad (4)$$

where $\mathbf{M}^\lambda = \tau^\lambda \otimes \mu^3$, the μ^3 matrix being a Pauli matrix in the *chiral* space.

Consider first the effect of harmonic variations of the parameters $X_1(t)$ and $X_2(t)$ perturbatively, for weak barriers. The leading order contribution to the dc pumping current is $I_p^\lambda \simeq i\int_{-\infty}^t dt' \langle[\hat{I}_b^\lambda(t), \delta\mathcal{H}(t')]\rangle_{\mathcal{H}_0}$. Evaluating this at the spin isotropic point, we get

$$I_p^\lambda \simeq \sum_{\mu,\nu} \mathrm{Tr}\{\{\tau^\lambda,\tau^\mu\},\tau^\nu\} \times \int dt'\, \mathrm{Im}[\Gamma_\mu(t)\Gamma^*_\nu(t')]\,\mathrm{Im}G^R(t-t'), \quad (5)$$

where $G^R(t-t')$ is the retarded Green's function of the bosonized operator $\psi^\dagger_{R;\sigma}(t)\psi_{L;\sigma}(t)$. For the Q pump $\mu = \nu = 0$, so that the only nonzero component of the generalized current is the charge current $I_p^0 \simeq \frac{2}{\pi}\frac{\mathcal{A}}{\Gamma([g_c+2]/2)}(\omega_0/\omega_\Gamma)^{2([g_c+2]/2-2)}\omega_0$, where ω_Γ is a crossover energy scale set by the details of the path described by the amplitudes $\Gamma_\nu(t)$ [17]. With $X_1(t) = X_1\cos(\omega_0 t - \varphi/2)$ and $X_2(t) = X_2\cos(\omega_0 t + \varphi/2)$, we have $\mathcal{A} = \mathrm{Im}[X_1 X_2^*]\sin\varphi$—the area enclosed in a pumping cycle by the parameters $X_1(t)$-$X_2(t)$. For the S pump, with the magnetic field in the $\hat{z}$ direction, we get only a spin-current I_p^3 having the same expression as I_p^0 above. The reason is that terms giving a nonvanishing contribution to a dc current require $\mu \neq \nu$, in which case the trace term is nonzero only for $\lambda = 3$.

The perturbative expansion is meaningful for $g_c > 2$ only in the IR limit ($\omega_0 \ll \omega_\Gamma$), and for $g_c < 2$ only in the UV limit ($\omega_0 \gg \omega_\Gamma$). In both these limits $\mathcal{G}_{c,s} = 0$. For noninteracting electrons ($g_c = g_s = 2$), we get charge pumping in the Q pump with a frequency independent pumping conductance: $\mathcal{G}_c \equiv \frac{e^2}{h}\frac{2\pi}{\omega_0} I_p^0 = \frac{e^2}{h}\sin\varphi\,\mathrm{Im}[4X_1X_2^*]$, similar to Ref. [4]. Also, for noninteracting electrons, the S pump operates as a pure spin pump, with a spin pumping conductance $\mathcal{G}_s$ identical in form to $\mathcal{G}_c$ above. Both these expressions display nonuniversal behavior, being dependent on the form of the external perturbations.

The nonperturbative regime of repulsive (attractive) interactions in the IR (UV) limit of pumping can be addressed by exact solutions. For the Q pump we need only consider the case of *spinless* electrons, where $g < 1$ for repulsive interactions. For the special case of $g = 1/2$ the problem can be mapped into that of a time dependent scattering problem involving free chiral fermions and an impurity state. We have solved this problem exactly for a periodic function $\Gamma_0(t + T_0) = \Gamma_0(t)$ [18]. The pumping current I_p^0 is given by

$$I_p^0(t) = \frac{e}{2}|\Gamma_0(t)|^2\Bigg[1 - \sum_\omega 4n_\omega \mathrm{Re}\Bigg\{\int_{-\infty}^{t} dt_0 \frac{\Gamma_0(t_0)}{\Gamma_0(t)} \times e^{i\omega(t-t_0)}\exp\Bigg(2\int_t^{t_0} dt'|\Gamma_0(t')|^2\Bigg)\Bigg\}\Bigg], \tag{6}$$

where n_ω is the equilibrium fermion occupation number. In the UV limit ($T_0 \to 0$) the charge-pumping conductance $\mathcal{G}$ vanishes, as anticipated by the perturbative calculation. In the IR limit ($T_0 \to \infty$) the charge pumped in a cycle is

$$Q_c = eN_c = \int_0^{T_0} dt\, I_p^0 = e\frac{1}{2\pi i}\oint \frac{d\Gamma_0}{\Gamma_0} = ne\,, \tag{7}$$

where the integer n is the winding number of the Γ_0 contour. Thus, in the IR limit for $g = 1/2$, a quantum of charge is pumped in a cycle of the Q pump for a contour that winds once around the origin, regardless of all other details of $\Gamma_0(t)$. This universality allows us to define the charge-pumping conductance for *spinless* electrons:

$$\mathcal{G} \equiv \frac{e^2}{h}N_c = e^2/h\,. \tag{8}$$

We can extend these results to other values of g along the lines of the renormalization group (RG) arguments by Kane and Fisher [12] for spinless electrons. In this picture, for single impurity interactions, there are two fixed points: (i) the perfectly transmitting limit, and (ii) the perfectly backscattering limit of the Luttinger liquid. For repulsive interactions the barrier is a relevant perturbation for fixed point (i) and is irrelevant for fixed point (ii). As a result, for $g < 1$, the dc conductance $G = 0$ in the IR limit. Thus, for a small applied dc voltage we get $I = I_d - I_b = 0$ and all the current is backscattered. For the Q pump, where only the backscattering current matters, this picture implies the maximal pumping response in the adiabatic (IR) limit. To calculate this we note (as suggested by the $g = 1/2$ case) that in the adiabatic limit the pumping current should be independent of the form of the pumping path. Therefore choosing $\Gamma_0(t) = X_+ e^{i\omega_0 t}$ should pump the same charge per cycle as would any other form of $\Gamma_0(t)$. For the purpose of calculating the pumping or backscattering current, this particular form of pumping corresponds to applying an effective source-drain voltage $V_{\mathrm{eff}} = -\omega_0/g$ [16]. Consequently, the backscattering current, which is also the (negative) pumping current, should be $I_b = I_d = gV_{\mathrm{eff}}$. We then recover a quantized charge pumped in a cycle, $N_c = -\frac{2\pi}{\omega_0} I_b = 1$. Expression (8) defines $\mathcal{G}$, in the repulsive regime, independent of interaction strength.

For attractive interactions ($g > 1$) the weak barrier perturbation is irrelevant for the fixed point (i) while it is relevant for the fixed point (ii). Consequently $I_b \to 0$ in the IR limit, upholding our earlier conclusion, based on perturbation theory, that $\mathcal{G} = 0$ for attractive interactions in this limit. To access the behavior in the nonperturbative UV limit of pumping, we note that at $T = 0$, and for an effective dc source-drain voltage, there exists an exact $g \to 1/g$ duality such that the backscattering current satisfies the relation: $I_b(V_{\mathrm{eff}}, g) = \frac{e^2 g V_{\mathrm{eff}}}{h} - g^2 I_b(V_{\mathrm{eff}}, 1/g)$ [19]. At least in the particular case of $\Gamma_0(t) = X_+ e^{i\omega_0 t}$, this duality implies that the UV limit of pumping conductance for $g > 1$, should be equal to the IR limit for $g < 1$, which is given by (8). We thus get a complete picture of the universal behavior of the charge pumping conductance $\mathcal{G}$. A comparison with dc source-drain conductance G, shown in Table I, reveals a complementary behavior with frequency.

In the case of pumping in a fractional quantum Hall bar, the charge pumped *per* cycle (in the IR limit) is always the electron charge e, *irrespective* of the filling fraction ν ($= g$). This follows for the particular geometry we studied; other pumping geometries, operating through antidots [20], can be designed so as to pump fractional charge *per* cycle.

Including spins in our description of the Q pump, the physics at the bulk spin-isotropic point ($g_s = 2$) is governed by the same fixed points as in the spinless case [12]. However, the duality relation is changed; also, the special form of pumping $\Gamma_0(t) = X_+ e^{i\omega_0 t}$, corresponds to $V_{\mathrm{eff}} = -2\omega_0/g_c$ [21]. Consequently, the behavior of $\mathcal{G}_c$ is the same as that of $2\mathcal{G}$.

TABLE I. The IR and UV fixed point values, for repulsive ($g < 1$) and attractive ($g > 1$) interaction regimes for *spinless* electrons, of the charge pumping conductance $\mathcal{G}$. The corresponding values for the dc charge conductance G from Ref. [12] are also shown.

	($g < 1$)		($g > 1$)	
Conductance	IR	UV	IR	UV
G	0	g	g	0
$\mathcal{G}$	1	0	0	1

We now turn to the behavior in the nonperturbative regimes for the S pump. The external potential $\delta\mathcal{H}$, can be written as

$$\delta\mathcal{H}(t) = |X_1|\cos\omega_0 t\cos\sqrt{\pi}\,\Phi_c(0)\cos\sqrt{\pi}\,\Phi_s(0) + |X_2|\cos(\omega_0 t + \varphi) \times \sin[\sqrt{\pi}\,\Phi_c(0) + \chi]\sin\sqrt{\pi}\,\Phi_s(0), \quad (9)$$

where χ is the constant phase difference between X_1 and X_2. From the RG analysis of Ref. [12] we know that, for $g_s = 2$, $g_c < 2$, the most relevant perturbation due to a single barrier at $x = 0$ is $v_e\cos\sqrt{\pi}\,\Phi_c(0)\cos\sqrt{\pi}\,\Phi_s(0)$, and the system is a spin and charge insulator. Consequently, in the IR limit $I_b^{0,3} = I_d^{0,3}$, which means that for the S pump, the pumping current $I_p^3 = g_s V_{\text{eff}}/2\pi$, where V_{eff} is the "voltage" that couples to the spin in the action. Such a "voltage" gives the barrier term a time dependence: $v_e\cos(\sqrt{\pi}\,\Phi_c)\cos(\sqrt{\pi}\,\Phi_s)\cos(g_s V_{\text{eff}} t/2) + v_e\cos(\sqrt{\pi}\,\Phi_c)\sin(\sqrt{\pi}\,\Phi_s)\sin(g_s V_{\text{eff}} t/2)$, and can only yield a spin current. This time-dependent barrier is the same as Eq. (9) when $\chi = \pi/2 = \varphi$, and $|X_1| = |X_2|$, so that we can identify $V_{\text{eff}} = -2\omega_0/g_s$. Thus, for this particular form of pumping, $I_p^0 = 0$ and $I_p^3 = 2\omega_0/2\pi$. If in the IR limit of pumping the spin transferred per cycle is independent of the form of the perturbing parameters, as was argued earlier for the spinless charge pump, then the S pump has an IR fixed point spin conductance $\mathcal{G}_s = 2e^2/h$, the same as the Q pump's $\mathcal{G}_c$. Furthermore, the approximate duality of Ref. [12] seems to imply that the other nonperturbative regime for attractive interactions ($g_c > 2$, $g_s = 2$) in the UV limit of pumping also has a fixed point value of $\mathcal{G}_s = 2e^2/h$.

Finally, we consider the pumped current flow for the quantum wire connected to reservoirs. We analyzed the dc spin/charge current driven by the periodic driving of the two pumping parameters, assuming that the only contribution to the pumping current I_p comes from the backscattering—$-I_b$, without a direct contribution I_d. This is valid for adiabatically contacted reservoirs so that the charge (spin) pumped can flow without accumulation and backscattering. Any accumulation of charge (spin) in the contacts will generate a voltage across the terminals and lead to a nonzero backflow I_d. In practice, the degree of backflow will depend on the nature of the contacts. In particular, we would like to point out that for open wires, running the pump would lead to a nonzero voltage across the terminals of the wire so that the backflow completely compensates the pumping current. While this may not be of particular use for producing a voltage or imbalance of chemical potential for charges, the imbalance between chemical potentials for up and down spins so generated in the S pump is an interesting realization of a "spin battery." Let us also note here that one can operate the $Q(S)$ pump without contact to reservoirs, as in a ring geometry, and measure the current flow (for the charge case) as in persistent current experiments, with a SQUID.

In conclusion, we have proposed a charge and a spin pump in a Luttinger liquid wire which shows universal behavior in the IR and UV limits of pumping. As can be deduced from our result (6) for $g = 1/2$, the frequency dependent crossover regime is nonuniversal, depending on details of the path that the pumping amplitude $\Gamma_0(t)$ traces on the complex plane. Finding the exact response to pumping, for all g and over the full range of frequencies, is an open nonequilibrium problem. It is not obvious that techniques, such as the Bethe ansatz—which give the exact dc conductance-voltage relationships—will be applicable here. The quantization of pumped charge (spin), arises in the asymptotic limits of pumping when the "impurity" effectively separates the system into two LL wires, since the tunneling density of states (TDOS) vanishes; it is absent for a noninteracting system where the TDOS is a constant. Thus, the reason for this quantization is distinct from that in gapped Fermi systems shown by Thouless [2]. We note that including long range interactions in our LL model does not qualitatively change this picture of the vanishing TDOS, so that we expect the quantization to be robust.

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