Conditional Large Fock State Preparation and Field State Reconstruction in Cavity QED

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We propose a scheme for producing large Fock states in cavity QED via the implementation of a highly selective atom-field interaction. It is based on Raman excitation of a three-level atom by a classical field and a quantized field mode. Selectivity appears when one tunes to resonance a specific transition inside a chosen atom-field subspace, while other transitions remain dispersive, as a consequence of the field dependent electronic energy shifts. We show that this scheme can be also employed for reconstructing, in a new and efficient way, the Wigner function of the cavity field state.

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Harmonic oscillators have been, from the very beginning, at the core of the quantum theory. It was not until the invention of the laser [1], however, that their most interesting statistical properties could be tested in controlled experiments associated with electromagnetic fields. Since then, considerable theoretical and experimental efforts have been devoted to the production and characterization of nonclassical states of light, such as sub-Poissonian [2], squeezed [3], or Schrödinger cat states [4,5].

A great advance in the field came with the micromaser [6], where two-level Rydberg atoms interact with one mode of a high-Q cavity in an experimental realization of the Jaynes-Cummings (JC) model [7]. Many important experiments have followed, revealing the dynamical properties of this model, such as the observation of collapse and revivals [8] and the discrete character of the Rabi oscillations [9] of the atom-field doublets. The micromaser technique has also allowed the investigation of the most fundamental, nonclassical states of harmonic oscillators, especially the number (Fock) states. In particular, recent papers reported the quantum nondemolition measurement of the one photon Fock state [10] and the preparation of up to two photons [11] in the cavity mode.

The atom-field dynamics as well as the statistical properties of the field are observed, in cavity QED, through the detection of the atoms, which work either as a probe for the coupled system or as a measuring device for the light mode state, depending on the setup. Indeed, several theoretical results have shown that appropriate settings of this system enable a complete reconstruction of the quantum state of the cavity field [12,13]. However, despite the great advances in the latest years, the implementation of these proposals, as well as the generation and characterization of large Fock states, remain as challenging experimental problems in this field. In these cases, the characteristics of the JC model require the use of improved apparatus, such as cavities with higher quality factor and enhanced ways to control and manipulate the atoms.

Alternatively, different atom-field couplings could be used. In particular, selective interactions [14] present new ways to induce transitions in this system. Unlike the JC model, where the same resonant or dispersive regime applies for all initial atom-field states, selective interactions separate these states in subspaces with distinct coupling regimes. This property represents more versatility and the possibility to implement new classes of experiments even under current technological conditions.

In this paper, we show that it is possible to implement a selective interaction between three-level atoms, a classical field, and a quantized cavity mode. We study the feasibility of our scheme based on available experimental parameters and propose, as a first application, the preparation of large Fock states in the quantized mode. As a second relevant application, we show that selectivity in this system allows for the measurement of the probability distribution $P(n)$ of the quantized field state in the Fock state basis $|n\rangle$. Finally, we show that it is possible, as a natural consequence, to propose an efficient reconstruction method of the Wigner function [15] of the cavity field and, therefore, of its complete quantum state [16].

Our proposal relies on the Raman excitation of a three-level atom by a classical field of frequency ω_L and a quantized cavity mode of frequency ω_0, in a lambda configuration (see Fig. 1). The classical field drives dispersively the transition from level $|g\rangle$ to level $|h\rangle$, with coupling constant Ω_L and detuning $\delta = \omega_{hg} - \omega_L \gg |\Omega_L|$. The cavity mode couples level $|e\rangle$ to level $|h\rangle$, with coupling constant g and the same detuning

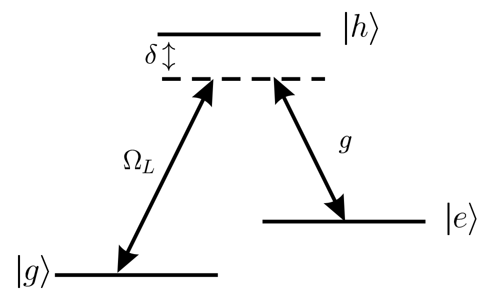

FIG. 1. Scheme for the Raman excitation of the three-level atom.

$\delta = \omega_{he} - \omega_0 \gg |g|$. In the interaction picture, the interaction Hamiltonian in the rotating wave approximation is given by

$$\hat{H}_{\mathrm{int}} = \hbar\Omega_L \hat{\sigma}_{hg} e^{-i\delta t} + \hbar g \hat{\sigma}_{he} \hat{a} e^{-i\delta t} + \mathrm{H.c.}, \qquad (1)$$

where $\hat{\sigma}_{jm} \equiv |j\rangle\langle m|$ is an electronic flip operator, and $\hat{a}$ is the annihilation operator of the quantized cavity mode. Since level $|h\rangle$ is coupled dispersively with both levels $|g\rangle$ and $|e\rangle$, it can be adiabatically eliminated giving rise to an effective second order anti–Jaynes-Cummings Hamiltonian

$$\hat{H}_{\mathrm{eff}} = \hbar \frac{|\Omega_L|^2}{\delta} \hat{\sigma}_{gg} + \hbar \frac{|g|^2 \hat{a}^\dagger \hat{a}}{\delta} \hat{\sigma}_{ee} + \hbar \frac{|g\Omega_L^*|}{\delta} (\hat{\sigma}_{eg} \hat{a}^\dagger + \hat{\sigma}_{ge} \hat{a}), \qquad (2)$$

where we chose Ω_L in phase with g. The first two terms correspond to dynamical energy shifts of levels $|g\rangle$ and $|e\rangle$, and the last two terms describe transitions between these levels, accompanied by creation or annihilation of a photon in the cavity mode. Notice that the difference of the energy shifts of level $|g\rangle$ and $|e\rangle$, which depends explicitly on the number n of photons in the cavity mode, will determine the effective resonance frequency of the $|g\rangle \leftrightarrow |e\rangle$ transition.

The Hamiltonian (2) is block separable in the subspaces spanned by the states $\{|g, n\rangle, |e, n + 1\rangle\}$ of the atom-field system. There is a specific difference of energy shifts Δ_{eg}^n, associated to each one of these subspaces, which may be compensated by external action on either the atom (dc Stark shift) or the cavity mode (by shifting its frequency). In this way, transitions inside a chosen subspace may be tuned to resonance, while other transitions remain dispersive, producing a selective interaction in the atom-field Hilbert space. Once this frequency adjustment is made for one specific subspace $\{|g, N_o\rangle, |e, N_o + 1\rangle\}$, the detunings associated with the remaining subspaces ($n \neq N_o$) change to

$$\Delta_n \equiv \Delta_{eg}^n - \Delta_{eg}^{N_o} = \frac{|g|^2}{\delta} (n - N_o). \qquad (3)$$

By controlling the ratio between g and Ω_L, the detunings Δ_n can be made large enough for considering the effective interaction in the remaining subspaces as dispersive, i.e., $\Delta_n \gg (|g\Omega_L^*|)/\delta$. In this case, if the atom enters the cavity in state $|g\rangle$, it can only experiment a Rabi flip to level $|e\rangle$ if the cavity has N_o photons.

Hamiltonian (2) can be easily diagonalized. In particular, after the frequency adjustment made in Eq. (3), its stationary eigenstates are given by the ground state $|e, 0\rangle$ and the doublets

$$|\pm, n\rangle = \frac{G_n |g, n\rangle + \lambda_{\pm,n} |e, n + 1\rangle}{\sqrt{\lambda_{\pm,n}^2 + G_n^2}}, \qquad (4)$$

with respective eigenvalues 0 and $\lambda_{\pm,n} = \frac{\Delta_n}{2} \pm \Omega_n$. In Eq. (4), $G_n = [(|g\Omega_L^*|)/\delta]\sqrt{n + 1}$ and $\Omega_n = \sqrt{\Delta_n^2/4 + G_n^2}$.

For arbitrary values of the detunings Δ_n, if the atom enters the cavity in state $|g\rangle$, and the field is initially in state $|\Phi_0\rangle = \sum_n c_n |n\rangle$, the state of the system evolves, after an interaction time t, to

$$|\Psi(t)\rangle = \sum_n c_n e^{-i[(\Delta_n t)/2]} \Big[\Big(\cos\Omega_n t + \frac{i\Delta_n}{2\Omega_n} \sin\Omega_n t \Big) \times |g, n\rangle - \frac{iG_n}{\Omega_n} \times \sin\Omega_n t |e, n + 1\rangle \Big]. \qquad (5)$$

In particular, if the atom is found in state $|e\rangle$, after it has interacted with the light fields during a time interval $\tau = \pi\delta/2(|g\Omega_L^*|\sqrt{N_o + 1})$, the correlated state of the cavity mode is

$$|\Phi_e(\tau)\rangle = \frac{c_{N_o} |N_o + 1\rangle + \sum_{n \neq N_o} b_n |n + 1\rangle}{\sqrt{|c_{N_o}|^2 + \sum_{n \neq N_o} |b_n|^2}}. \qquad (6)$$

The coefficients b_n are given by

$$b_n = \frac{c_n (-i) e^{-i[(\Delta_n \tau)/2]} \sin\frac{\pi}{2}\sqrt{q/(N_o + 1)}}{\sqrt{q}}, \qquad (7)$$

where $q = [r^2(n - N_o)^2]/[4(n + 1)] + 1$, and $r = \frac{|g|}{|\Omega_L|}$. Note that, as q increases, the coefficients b_n become negligible compared to the coefficient c_{N_o} and $|\Phi_e(\tau)\rangle$ tends to the Fock state $|N_o + 1\rangle$. For a chosen N_o, this condition is satisfied if $r \gg 2\sqrt{N_o + 2}$. In this limit, the Fock state $|N_o + 1\rangle$ could be produced in the cavity field, as long as $c_{N_o} \neq 0$. In principle, one could use this scheme to produce any Fock state in the quantized mode. In practice, however, one is limited by the decay time τ_c of the cavity divided by N_o, which must be much longer than the interaction time, τ. Typically, for Rydberg atoms interacting with high-Q microwave cavities, $g/2\pi \sim 50$ kHz [10]. For $N_o \sim 10$, $\delta/2\pi \sim 1$ MHz, and $\Omega_L \sim \frac{g}{30}$ ($r \sim 30$), τ will be of the order of 1 ms. Interaction times of this order are of the same order of the decaying time of the open cavities used on these experiments, suggesting that cavities with slightly higher Q factor are needed to implement experimentally the proposed scheme. Closed cavities, with longer decay time, may also be used ($\tau_c \sim 0.3$ s) [17]. However, those cavities do not allow the use of highly excited circular atoms and, therefore, the atomic decay time becomes a concern. Furthermore, it is also more difficult to apply the external classical field to them.

We may define the fidelity of generating the selected Fock state $|N_o + 1\rangle$ as $F \equiv |\langle N_o + 1|\Phi_e(\tau)\rangle|^2$. F is approximately given by $1 - \sum_n (|b_n|^2/|c_{N_o}|^2)$ and it approaches unity when the coefficients b_n's go to zero. Initial states for which $c_{N_o} > c_n$ enhance the protocol efficiency. In this sense, good candidates for initial cavity state are coherent states with mean number of photons around N_o. Not only do they satisfy $c_{N_o} > c_n$, but they are also easily produced in microwave cavities, by just coupling them to a microwave generator [5]. In Fig. 2, we show an example for the preparation of large Fock states in the cavity mode, after only one atom has interacted with the fields. From an initial coherent state $|\alpha\rangle$ with $|\alpha|^2 = 5$, the Fock state $|6\rangle$ is prepared in the cavity with a fidelity higher than 0.99.

The Fock state preparation is conditioned to finding the atom in state $|e\rangle$. From Eqs. (5), (6), and (7), it is easy to show that, as r becomes larger, the probability P_e of measuring the atom in the excited state approximates the probability P_{N_o} of finding N_o photons in the initial cavity field state. If r cannot be made large enough due to experimental limitations, the state one wants to prepare is polluted by marginal Fock state populations. In this case, one only needs to send a second atom in state $|g\rangle$ and set the experimental parameters to the transition $|g, N_o + 1\rangle \rightarrow |e, N_o + 2\rangle$. The probability of finding both atoms in the excited state becomes closer to P_{N_o}, and the field state produced will be, with very high fidelity, state $|N_o + 2\rangle$.

The equivalence between P_e and P_{N_o} for large r suggests a very practical and easy way to obtain the photon statistics P_n of an arbitrary state in the cavity mode. In fact, for each selected transition $|g, N\rangle \leftrightarrow |e, N + 1\rangle$, the proportion of atoms measured in state $|e\rangle$, P_e, gives directly P_N, for all possible values of N. Combined with the possibility of coherently displacing microwave cavity fields, this allows one to fully reconstruct the Wigner function of the state of the quantized mode. Since it does not rely on additional devices, such as Ramsey interferometers, this scheme simplifies the task of field state reconstruction, as we will discuss below.

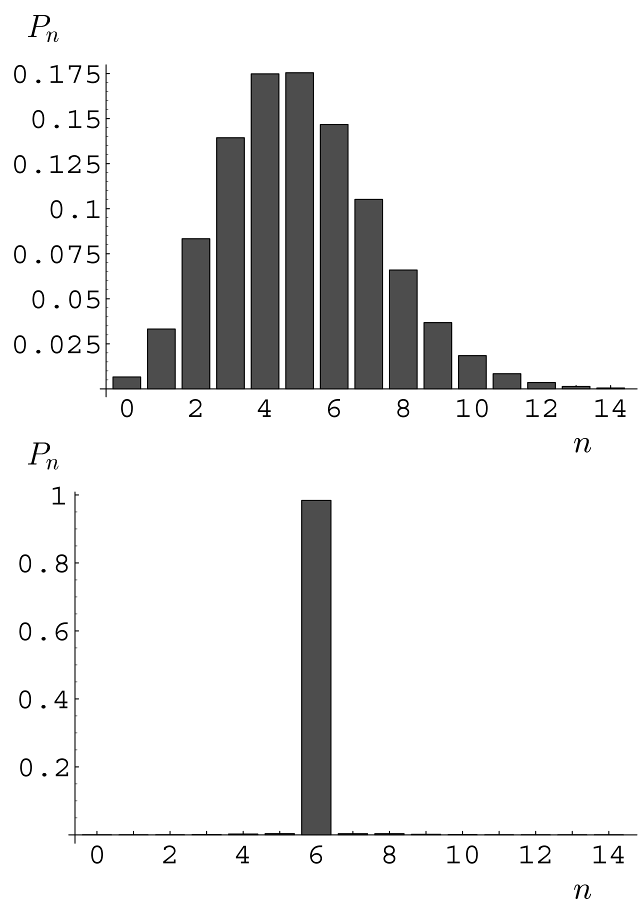

FIG. 2. Preparation of the Fock state $|6\rangle$ in the cavity mode by measuring the atom in its excited state after it passed through the cavity. The cavity field was initially in a coherent state with $|\alpha|^2 = 5$. The value of the parameter r was set to $r = 30$.

The Wigner function of the state $\hat{\rho}$ of a harmonic oscillator can be written as

$$W(-\alpha) = \frac{2}{\pi} \sum_n (-1)^n P_n(\alpha), \tag{8}$$

where $P_n(\alpha) = \langle n|\hat{D}(\alpha)\hat{\rho}\hat{D}^{-1}(\alpha)|n\rangle$ is the number distribution of state $\hat{\rho}$ displaced coherently in the phase space by α [16,18]. This tells us that, to obtain the Wigner function of the cavity field on each point of the phase space, all one needs to know is the number distribution of the field, after it has been displaced in the phase space. In fact, the first experimental reconstruction of the quantum state of a harmonic oscillator made use of Eq. (8) [19]. The number distribution P_n was obtained by analyzing the time evolution of the population of the internal states of a trapped ion coupled via the Jaynes-Cummings interaction with its vibrational degree of freedom. A similar scheme could also, in principle, be applied in cavity QED. In our case, the coherent displacement of the cavity field can be easily implemented by coupling the cavity to a microwave generator. After this step, one can use the selective scheme discussed above to measure $P_n(\alpha)$ and, then, Eq. (8) to calculate $W(-\alpha)$. This method is exact for large values of r, and it represents an experimentally simple way to measure the photonic statistics of the cavity field and to reconstruct its Wigner function. It requires neither the preparation of atoms in a coherent superposition of upper and lower states nor the exact control of interaction times of the atoms with Ramsey zones, as in usual schemes that rely on Ramsey interferometry [20]. As an example, we show in Fig. 3 the efficient reconstruction of the Wigner function of the Fock state $|6\rangle$ for the realistic parameter $r = 30$. By subtracting the exact Wigner function from the reconstructed one, we also show that the errors introduced by supposing that transitions occur only inside each selected subspace $|g, N\rangle \leftrightarrow |e, N + 1\rangle$ (perfect selectivity) are negligibly small.

In conclusion, we have proposed a scheme to implement a selective interaction in cavity QED, between three-level atoms, a classical field, and a quantized cavity mode. It relies on the possibility of tuning to resonance a specific transition inside a chosen atom-field subspace, while other transitions remain dispersive, as a consequence of the field dependent electronic energy shifts. As a first relevant application of this scheme, we have proposed a method for generating large Fock states in the cavity mode. Additionally, we have shown that this scheme allows for the reconstruction of the Wigner function of an arbitrary cavity

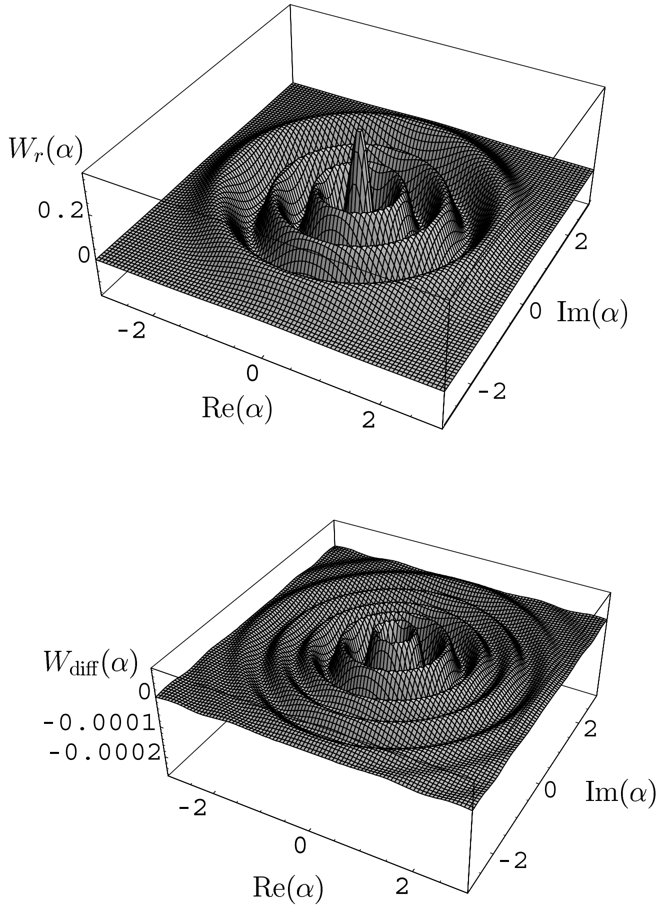

FIG. 3. Plot of the reconstructed Wigner function of the Fock state $|6\rangle$ with the use of the selective scheme (above) and its difference to the exact one (below). The value of the parameter r was set to $r = 30$.

field state, with simplified experimental setup. Many other application examples could be imagined, especially those exploring the entanglement created between the atoms and different field states, as is the case in quantum logic and quantum communication schemes.

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