

Lorentz Invariance and Origin of Symmetries

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We reconsider the role of Lorentz invariance in the dynamical generation of the observed internal symmetries. We argue that, generally, Lorentz invariance can be imposed only in the sense that all Lorentz noninvariant effects caused by the spontaneous breakdown of Lorentz symmetry are physically unobservable. The application of this principle to the most general relativistically invariant Lagrangian, with arbitrary couplings for all the fields involved, leads to the appearance of a symmetry and, what is more, to the massless vector fields gauging this symmetry in both Abelian and non-Abelian cases. In contrast, purely global symmetries are generated only as accidental consequences of the gauge symmetry.

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It is still a very attractive idea that a local symmetry for all the fundamental interactions of matter and the corresponding massless gauge fields could be dynamically generated (see [1] and extended references therein). In particular, there has been considerable interest [2] in the interpretation of gauge fields as composite Nambu-Jona-Lasinio (NJL) bosons [3], possibly associated with the spontaneous breakdown of Lorentz symmetry (SBLS). However, in contrast to the belief advocated in the pioneering works [2], there is a generic problem in turning the composite vector particles into genuine massless gauge bosons [4].

In this note, we would like to return to the role of Lorentz symmetry in a dynamical generation of gauge invariance. We argue that, generally, Lorentz invariance can be imposed only in the sense that all Lorentz noninvariant effects caused by its spontaneous breakdown are physically unobservable. We show here that the physical nonobservability of the SBLS, taken as a basic principle, leads to genuine gauge invariant theories, both Abelian and non-Abelian, even though one starts from an arbitrary relativistically in-

variant Lagrangian. In the original Lagrangian, the vector fields are taken as massive and all possible kinetic and interaction terms are included. However, when SBLS occurs and its nonobservability is imposed, the vector bosons become massless and the only surviving interaction terms are those allowed by the corresponding gauge symmetry. Thus, the Lorentz symmetry breaking does not manifest itself in any physical way, due to the generated gauge symmetry converting the SBLS into gauge degrees of freedom of the massless vector bosons. Remarkably, even global symmetries are not required in the original Lagrangian—the SBLS induces them automatically as accidental symmetries accompanying the generated gauge theory.

In order to consider general interactions between a vector field and fermionic matter, it is convenient to use 2-component left-handed Weyl fields ψ_{Li} to represent the fermions. For simplicity we shall consider the case of two Weyl fields ($i = 1, 2$), which will finally be combined to form a Dirac-like field $\psi = \begin{pmatrix} \psi_{L1} \\ \psi_{L2}^\dagger \end{pmatrix}$ in Weyl representation.

The most general Lagrangian density, only having terms of mass dimension four or less, for a theory containing a pure spin-1 vector field and two Weyl fermions is

$$L(A, \psi) = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} M^2 A_\mu^2 + i \sum_{j=1}^2 \psi_{Lj}^\dagger \sigma_\mu \partial_\mu \psi_{Lj} - \sum_{j,k=1}^2 \epsilon^{\alpha\beta} (m_{jk} \psi_{Lj\alpha} \psi_{Lk\beta} + m_{jk}^* \psi_{Lj\alpha}^\dagger \psi_{Lk\beta}^\dagger) + \sum_{j,k=1}^2 e_{jk} A_\mu \psi_{Lj}^\dagger \sigma_\mu \psi_{Lk} + \frac{f}{4} A_\mu^2 \cdot A_\mu^2, \quad (1)$$

with the Lorentz condition ($\partial_\mu A_\mu = 0$) imposed as an off-shell constraint, singling out a genuine spin-1 component in the four-vector A_μ . This constraint also ensures that, after an appropriate scaling, the kinetic term for A_μ can be written in the usual $-\frac{1}{4} F_{\mu\nu} F_{\mu\nu}$ form. Note that $m_{jk} = m_{kj}$, as a consequence of Fermi statistics. It is always possible to simplify this Lagrangian density, by defining two new left-handed Weyl spinor fields which transform the “charge term” $\sum_{j,k=1}^2 e_{jk} A_\mu \psi_{Lj}^\dagger \sigma_\mu \psi_{Lk}$ into the diagonal form $\sum_{k=1}^2 e_k A_\mu \psi_{Lk}^\dagger \sigma_\mu \psi_{Lk}$.

Let us consider now the SBLS in some detail. We propose that the vector field A_μ takes the form

$$A_\mu = a_\mu(x) + n_\mu \quad (2)$$

when the SBLS occurs. Here the constant Lorentz four-vector n_μ is a classical background field appearing when the vector field A_μ develops a vacuum expectation value (VEV). Substitution of the form (2) into the Lagrangian (1) immediately shows that the kinetic term for the vector field A_μ translates into a kinetic term for a_μ ($F_{\mu\nu}^{(A)} = F_{\mu\nu}^{(a)}$), while its mass and interaction terms are correspondingly changed. As to the interaction term, one can always make a unitary transformation to two new Weyl fermion fields Ψ_{Lk}

$$\psi_{Lk} = \exp[ie_k \omega(x)] \Psi_{Lk}, \quad \omega(x) = n \cdot x, \quad (3)$$

so that the Lorentz symmetry-breaking term $n_\mu \cdot \sum_{k=1}^2 e_k \psi_{Lk}^\dagger \sigma_\mu \psi_{Lk}$ is exactly canceled in the Lagrangian density $L(a_\mu + n_\mu, \psi)$. This cancellation occurs due to the appearance of a compensating term from the fermion kinetic term, provided that the phase function $\omega(x)$ is chosen to be linear [5] in the coordinate four-vector x_μ [as indicated in Eq. (3)]. However, in general, the mass terms will also be changed under the transformation (3):

$$m_{jk} \psi_{Lj\alpha} \psi_{Lk\beta} \rightarrow m_{jk} \exp[i(e_j + e_k)n \cdot x] \Psi_{Lj\alpha} \Psi_{Lk\beta}. \quad (4)$$

If $e_j + e_k \neq 0$ for some nonzero mass matrix element m_{jk} , the transformed mass term will manifestly depend on n_μ through the translational noninvariant factor $\exp[i(e_j + e_k)n \cdot x]$, which in turn will visibly violate Lorentz symmetry. So our main assumption of the unobservability of SBLS implies that we can have only a nonzero value for m_{jk} when $e_j + e_k = 0$.

After imposing these conditions on the charges, the remaining traces of SBLS are contained in the vector field mass term and the $A_\mu^2 \cdot A_\mu^2$ term. Thus the remaining condition for the nonobservability of SBLS becomes

$$\{M^2 + f[a^2 + (n \cdot a) + n^2]\}(n \cdot a) = 0. \quad (5)$$

An extra gauge condition $n \cdot a \equiv n_\mu \cdot a_\mu = 0$ would be incompatible with the Lorentz gauge ($\partial_\mu A_\mu = 0$) already imposed on the vector field a_μ . Therefore, the only way to satisfy Eq. (5) is to take $M^2 = 0$ and $f = 0$. Otherwise, it would either represent an extra gauge condition on a_μ , or it would impose another dynamical equation in addition to the usual Euler equation for a_μ .

Thus imposing the nonobservability of SBLS, the Lorentz gauge restriction and the presence of terms of only dimension four or less has led us to the Lagrangian density for chiral electrodynamics, having the form

$$L = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + i \sum_{k=1}^2 \Psi_{Lk}^\dagger \sigma_\mu (\partial_\mu - ie_k a_\mu) \Psi_{Lk} - \sum_{j,k=1}^2 (m_{jk} \Psi_{Lj\alpha} \Psi_{Lk\beta} \epsilon^{\alpha\beta} + \text{H.c.}), \quad (6)$$

with the restriction that $m_{jk} = 0$ unless $e_j + e_k = 0$. In general, i.e., when $\sum_k e_k^3 \neq 0$, even this Lagrangian

density will lead to the observability of the SBLS, because of the presence of Adler-Bell-Jackiw anomalies [6] in the conservation equation for the current $j_\mu^A = \sum_k e_k \Psi_{Lk}^\dagger \sigma_\mu \Psi_{Lk}$ coupled to A_μ . We are now interpreting the Ψ_{Lk} as the physical fermion fields. However, in momentum representation, the transformation (3) corresponds to displacing the momentum of each fermion by an amount $e_k n_\mu$. This induces a breakdown of momentum conservation, which can be kept unobservable only as long as the charge associated with the current j_μ^A is conserved. This means that an anomaly in the current conservation will also violate momentum conservation by terms proportional to n_μ . Such a breaking of momentum conservation would also give observable Lorentz symmetry violation. So the only way to satisfy our nonobservability of SBLS principle is to require that the no gauge anomaly condition

$$\sum_k e_k^3 = 0 \quad (7)$$

be fulfilled. For the simple case of just two Weyl fields, this means that the two charges must be of equal magnitude and opposite sign, $e_1 + e_2 = 0$. This is also precisely the condition that must be satisfied for a nonzero mass matrix element $m_{12} = m_{21} \neq 0$. If the charges are nonzero, the diagonal (Majorana) mass matrix elements vanish, $m_{11} = m_{22} = 0$, and the two Weyl fields correspond to a massive particle described by the Dirac field $\Psi = \begin{pmatrix} \Psi_{L1} \\ \Psi_{L2}^\dagger \end{pmatrix}$. Thus we finally arrive at gauge invariant QED as the only version of the theory which is compatible with physical Lorentz invariance when SBLS occurs.

Let us now consider the many-vector field case which can result in a non-Abelian gauge symmetry. We suppose there are a set of pure spin-1 vector fields, $A_\mu^i(x)$ with $i = 1, \dots, N$, satisfying the Lorentz gauge condition, but not even proposing a global symmetry at the start. The matter fields are collected in another set of Dirac fields $\psi = (\psi^{(1)}, \dots, \psi^{(r)})$. Here, for simplicity, we shall neglect terms violating fermion number and parity conservation. The general Lagrangian density $L(A_\mu^i, \psi)$ describing all their interactions is given by

$$L = -\frac{1}{4} F_{\mu\nu}^i F_{\mu\nu}^i + \frac{1}{2} (M^2)_{ij} A_\mu^i A_\mu^j + \alpha^{ijk} \partial_\nu A_\mu^i \cdot A_\mu^j A_\nu^k + \beta^{ijkl} A_\mu^i A_\nu^j A_\mu^k A_\nu^l + i \bar{\psi} \gamma \partial \psi - \bar{\psi} m \psi + A_\mu^i \bar{\psi} \gamma_\mu T^i \psi. \quad (8)$$

Here $F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i$, while $(M^2)_{ij}$ is a general $N \times N$ mass matrix for the vector fields and α^{ijk} and β^{ijkl} are dimensionless coupling constants. The $r \times r$ matrices m and T^i contain the still arbitrary fermion masses and coupling constants describing the interaction between the fermions and the vector fields (all the numbers mentioned are real and the matrices Hermitian, as follows in this case from the Hermiticity of the Lagrangian density).

We assume that the vector fields A_μ^i each take the form

$$A_\mu^i(x) = a_\mu^i(x) + n_\mu^i \quad (9)$$

when SBLs occurs; here the constant Lorentz four-vectors $n_\mu^i (i = 1, \dots, N)$ are the VEVs of the vector fields. Substitution of the form (9) into the Lagrangian density (8) shows that the kinetic term for the vector fields A_μ^i translates into a kinetic term for the vector fields $a_\mu^i (F_{\mu\nu}^A = F_{\mu\nu}^{(a)})$, while their mass and interaction terms are correspondingly changed. Now we consider at first just infinitesimally small n_μ^i four vectors. Furthermore, we introduce a stronger form of the nonobservability of SBLs principle, requiring exact cancellations between non-Lorentz invariant terms of the same structure in the Lagrangian density $L(a_\mu^i + n_\mu^i, \psi)$ for *any* set of infinitesimal vectors n_μ^i . Then we define a new set of vector fields \mathbf{a}_μ^i by the infinitesimal transformation

$$a_\mu^i = \mathbf{a}_\mu^i - \alpha^{ijk} \omega^j(x) \mathbf{a}_\mu^k, \quad \omega^i(x) = n_\mu^i \cdot x_\mu, \quad (10)$$

which includes the above coupling constants α^{ijk} and the linear ‘‘gauge’’ functions $\omega^i(x)$. We require that the Lorentz symmetry-breaking terms in the cubic and quartic self-interactions of the vector fields \mathbf{a}_μ^i , including those arising from their kinetic terms, should cancel for *any* infinitesimal vector n_μ^i . This condition is satisfied if and only if the coupling constants α^{ijk} and β^{ijkl} satisfy the following conditions (a) and (b):

(a) α^{ijk} is totally antisymmetric (in the indices i, j , and k) and obeys the structure relations

$$\alpha^{ijk} \equiv \alpha^{[ijk]} \equiv \alpha^i_{[jk]}, \quad [\alpha^i, \alpha^j] = -\alpha^{ijk} \alpha^k, \quad (11)$$

where the α^i are defined as matrices with elements $(\alpha^i)^{jk} = \alpha^{ijk}$.

(b) β^{ijkl} takes the factorized form

$$\beta^{ijkl} = -\frac{1}{4} \alpha^{ijm} \cdot \alpha^{klm}. \quad (12)$$

It follows from (a) that the matrices α^k form the adjoint representation of a Lie algebra, under which the vector fields transform infinitesimally as given in Eq. (10). In the case when the matrices α^i can be decomposed into a block diagonal form, there appears a product of symmetry groups rather than a single simple group.

Let us turn now to the mass term for the vector fields in the Lagrangian $L(a_\mu^i + n_\mu^i, \psi)$. When expressed in terms of the transformed vector fields \mathbf{a}_μ^i (10) it contains SBLs remnants, which should vanish, of the type

$$(M^2)_{ij} (\alpha^{ikl} \omega^k \mathbf{a}_\mu^l \mathbf{a}_\mu^j + \mathbf{a}_\mu^i n_\mu^j) = 0. \quad (13)$$

Here we have used the symmetry feature $(M^2)_{ij} = (M^2)_{ji}$ for a real Hermitian matrix M^2 and have retained only the first-order terms in n_μ^i . These two types of remnant have different structures and hence must vanish independently. One can readily see that, in view of the antisymmetry of the structure constants, the first term in Eq. (13) may be written in the following form containing the commutator of the matrices M^2 and α^k :

$$[M^2, \alpha^k]_{jl} \omega^k \mathbf{a}_\mu^l \mathbf{a}_\mu^j = 0. \quad (14)$$

It follows that the mass matrix M^2 should commute with all the matrices α^k , in order to satisfy Eq. (14) for all sets of ‘‘gauge’’ functions $\omega^i = n_\mu^i \cdot x_\mu$. Since the matrices α^k have been shown to form an irreducible representation of a (simple) Lie algebra, Schur’s lemma implies that the matrix M^2 is a multiple of the identity matrix, $(M^2)_{ij} = \mathbf{M}^2 \delta_{ij}$, thus giving the same mass for all the vector fields. It then follows that the vanishing of the second term in Eq. (13) leads to the simple condition

$$\mathbf{M}^2 (n^i \cdot \mathbf{a}^i) = 0 \quad (15)$$

for any infinitesimal n_μ^i . Since the Lorentz gauge condition $(\partial_\mu \mathbf{a}_\mu^i = 0)$ has already been imposed, we cannot impose extra gauge conditions of the type $n^i \cdot \mathbf{a}^i = n_\mu^i \cdot \mathbf{a}_\mu^i = 0$. Thus, we are necessarily led to

(c) massless vector fields, $(M^2)_{ij} = \mathbf{M}^2 \delta_{ij} = 0$.

Finally, we consider the interaction term between the vector and fermion fields in the ‘‘shifted’’ Lagrangian density $L(a_\mu^i + n_\mu^i, \psi)$. In terms of the transformed vector fields \mathbf{a}_μ^i (10), it takes the form

$$(\mathbf{a}_\mu^i - \alpha^{ijk} \omega^j \mathbf{a}_\mu^k + n_\mu^i) \cdot \bar{\psi} \gamma_\mu T^i \psi. \quad (16)$$

It is readily confirmed that the Lorentz symmetry-breaking terms (the second and third ones) can be eliminated, when one introduces a new set of fermion fields Ψ using a unitary transformation of the type

$$\psi = \exp[iT^i \omega^i(x)] \Psi, \quad \omega^i(x) = n^i \cdot x. \quad (17)$$

One of the compensating terms appears from the fermion kinetic term and the compensation occurs for any set of ‘‘gauge’’ functions $\omega^i(x)$ if and only if

(d) the matrices T^i form a representation of the Lie algebra with structure constants α^{ijk} :

$$[T^i, T^j] = i \alpha^{ijk} T^k. \quad (18)$$

In general, this will be a reducible representation but, for simplicity, we shall take it to be irreducible here. This means that the matter fermions Ψ are all assigned to an irreducible multiplet determined by the matrices T^i . At the same time, the unitary transformation (17) changes the mass term for the fermions to

$$\bar{\Psi} (m + i \omega^k [m, T^k]) \Psi. \quad (19)$$

The vanishing of the Lorentz noninvariant term (the second one) in Eq. (19) for any set of ‘‘gauge’’ functions $\omega^i(x)$ requires that the matrix m should commute with all the matrices T^k . According to Schur’s lemma, this means that the matrix m is proportional to the identity, thus giving

(e) the same mass for all the fermion fields within the irreducible multiplet determined by the matrices T^i :

$$m_{rs} = \mathbf{m} \delta_{rs}. \quad (20)$$

In the case when the fermions are decomposed into several irreducible multiplets, their masses are equal within each multiplet.

Now, collecting together the conditions (a)–(e) derived from the nonobservability of the SBLs for *any* set of infinitesimal vectors n_μ^i applied to the general Lagrangian density (8), we arrive at a truly gauge invariant Yang-Mills theory for the new fields \mathbf{a}_μ^i and Ψ :

$$L_{YM} = -\frac{1}{4} \mathbf{F}_{\mu\nu}^i \mathbf{F}_{\mu\nu}^i + i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \mathbf{m} \bar{\Psi} \Psi + \mathbf{g} \mathbf{a}_\mu^i \bar{\Psi} \gamma_\mu \mathbf{T}^i \Psi. \quad (21)$$

Here $\mathbf{F}_{\mu\nu}^i = \partial_\mu \mathbf{a}_\nu^i - \partial_\nu \mathbf{a}_\mu^i + \mathbf{g} \mathbf{a}^{ijk} \mathbf{a}_\mu^j \mathbf{a}_\nu^k$ and \mathbf{g} is a universal gauge coupling constant extracted from the corresponding matrices $\alpha^{ijk} = \mathbf{g} \mathbf{a}^{ijk}$ and $\mathbf{T}^i = \mathbf{g} \mathbf{T}^i$.

Let us now consider the generalization of the vector field VEVs from infinitesimal to finite background classical fields n_μ^i . Unfortunately, one cannot directly generalize the SBLs form (9) to all finite n_μ^i vectors. Otherwise, the n_μ^i for the different vector fields might not commute under the Yang-Mills symmetry and might point in different directions in Lorentz space, giving rise to a nonvanishing field strength $F_{\mu\nu}^k$ in the corresponding vacuum. Such a vacuum would not be Lorentz invariant, implying a real physical breakdown of Lorentz symmetry. This problem can be automatically avoided if the finite SBLs shift vector n_μ^i in the basic equation (9) takes the factorized form $n_\mu^i = n_\mu \cdot f^i$ where n_μ is a constant Lorentz vector as in the Abelian case, while f^i ($i = 1, 2, \dots, N$) is a vector in the internal charge space. Using the Lagrangian density (21) derived for infinitesimal VEVs, it is now straightforward to show that there will be no observable effects of SBLs for *any* set of finite factorized VEVs $n_\mu^i = n_\mu \cdot f^i$. For this purpose, we generalize Eq. (10) to the finite transformation:

$$a_\mu \cdot \alpha = \exp[(\omega \cdot \alpha)] \mathbf{a}_\mu \cdot \alpha \exp[-(\omega \cdot \alpha)]. \quad (22)$$

In conclusion, we have shown that gauge invariant Abelian and non-Abelian theories can be obtained from the requirement of the physical nonobservability of the SBLs rather than by using the Yang-Mills gauge principle. Thus the vector fields become a source of the symmetries, rather than local symmetries being a source of the vector fields as in the usual formulation. Imposing the condition that the Lorentz symmetry breaking be unobservable of course restricts the values of the coupling constants

and mass parameters in the Lagrangian density. These restrictions may naturally also depend on the direction and strength of the Lorentz symmetry breaking vector field VEVs, whose effects are to be hidden. This allows us a choice as to how strong an assumption we make about the nonobservability requirement. Actually, in the Abelian case, we just assumed this nonobservability for the physical vacuum (2) that really appears. However, we needed a stronger assumption in the non-Abelian case: the SBLs is unobservable in any vacuum for which the vector fields have VEVs of the factorized form $n_\mu^i = n_\mu \cdot f^i$. This factorized form is a special case in which the n_μ^i commute with each other.

We did not specify here mechanisms which could induce the SBLs—rather we studied general consequences for the possible dynamics of the matter and vector fields, requiring it to be physically unobservable. We address this and other related questions elsewhere [7].

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