Particle Segregation in Vibrofluidized Beds Due to Buoyant Forces

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We utilize two-dimensional discrete element computer simulations to investigate the equilibrium position of an impurity in a vibrofluidized bed of otherwise homogeneous particles. The steady state equilibrium height of the impurity increases with increasing vibration velocity amplitude and decreases with the impurity to the surrounding particle density ratio. A simple model whereby the impurity weight is balanced by a "buoyant" force due to the surrounding particle impacts makes a good prediction of the impurity position.

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A vibrofluidized bed (VFB) of a granular material is one in which vibration, as opposed to a flowing gas, is used to fluidize the particulates. In a VFB, particles are sufficiently agitated such that sustained contacts rarely occur. VFBs are commonly used in industries for various thermal processing operations such as drying, heating, cooling, and agglomerating [1,2]. The vibrofluidized bed is one of the most successful modifications of the conventional fluidized bed and is especially suited for processing sticky granular materials which cannot be fluidized conventionally [3].

Despite their common use, few have investigated how particulate materials segregate in a VFB. Previous work by Ohtsuki *et al.* [4,5] found that, when an impurity is introduced into a VFB, the impurity moves to a particular equilibrium position depending on the impurity diameter, density, and the bed vibration characteristics. They reported that the equilibrium position could not be predicted using a force balance consisting of the granular pressure force acting on the impurity and the impurity weight [4].

This Letter seeks to further clarify the influence of size, density, and excitation parameters on an impurity's equilibrium position within a VFB. We have used a two-dimensional, discrete element (DE) computer simulation to measure the equilibrium position for a variety of impurity to surrounding particle diameter and density ratios as well as for a range of oscillation amplitudes and frequencies. In addition, we propose and verify a model whereby the impurity position is, in fact, predicted by a balance between the net granular pressure, or "buoyant," force within the VFB and the impurity weight.

A two-dimensional, soft-particle discrete element simulation is used to model the vibrofluidized bed. The simulated granular material consists of 300 homogeneous, frictionless, inelastic circular disks of diameter, d, and density, ρ, and a single impurity of diameter, d_I, and density, ρ_I. The particle-particle coefficient of restitution and the particle-wall coefficient of restitution are 0.95. The particles are placed within a rectangular container of width, $20d$, that has a rigid base that oscillates sinusoidally with amplitude, a, and radian frequency, ω, giving a dimensionless acceleration of $\Gamma = a\omega^2/g$, where g is the acceleration due to gravity. Periodic side boundaries are used to eliminate side-wall effects such as convection [6]. To verify that the VFBs are completely fluidized, the duration of all particle collisions are measured in the simulations. Only those simulations in which all of the collisions have a duration equal to that of a two-particle impact are considered completely fluidized. For computing the stresses at various vertical positions in the simulated bed, the container is divided into several horizontal "bins," each of which is three-particle diameters in height. The fluctuating horizontal and vertical velocities, u' and v', respectively, are calculated for each bin.

The simulations were first used to investigate the vertical position of the impurity as a function of the impurity diameter, density, and vibration parameters. Figure 1 shows the instantaneous impurity vertical position above the base as a function of an oscillation cycle for a dimensionless frequency of $\omega\sqrt{(d/g)} = 5.81$ and dimensionless acceleration of $\Gamma = 23.2$. After a short transient resulting from

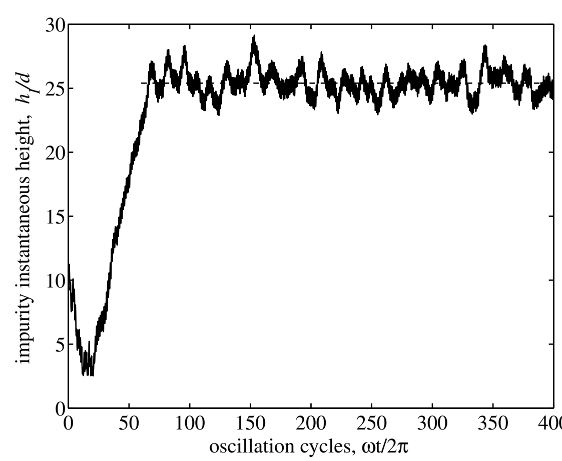

FIG. 1. A plot of the instantaneous height of the impurity, h_I/d, as a function of the number of oscillation cycles, $\omega t/(2\pi)$. The diameter ratio for the simulation is $d_I/d = 5$, the density ratio is $\rho_I/\rho = 0.4$, and the oscillation amplitude and frequency are, respectively, $a/d = 0.69$ and $\omega\sqrt{d/g} = 5.81$. The dashed line indicates the impurity average position.

the initial configuration of the system, the impurity vertical position approaches an equilibrium height, h_I. Fluctuations about this equilibrium position occur due to the oscillations of the base and impacts with the surrounding particles. Only those simulations in which these fluctuations are less than two-particle diameters are considered in this Letter. If the collision frequency is large the impurity will not fluctuate much about its equilibrium position since the restoring pressure force working against the impurity weight will be nearly continuous. If the collision frequency is small, however; the impurity position will fluctuate considerably since the restoring force for the particle weight will occur in discrete impacts rather than as a continuous pressure. If the impurity mass is sufficiently large, the impurity will not remain suspended in the VFB but will instead oscillate on the vibrating base with the surrounding particles acting to dampen the impurity motion. The equilibrium height of the impurity is determined by time averaging the vertical position over 200 oscillation cycles. The time average does not include the initial 50 oscillation cycles in order to avoid the initial transient.

First, the effect of impurity density ratio, ρ_I/ρ, is presented. As shown in Fig. 2, the equilibrium height decreases as the density ratio increases. The scatter bars in the figures are one standard deviation above and below the equilibrium position and reflect the fluctuations in the impurity position. The data plotted in Fig. 2 correspond to a range of mass ratios, M_I/M, between 5 and 15 and diameter ratios, d_I/d, between 5 and 8.

Second, the effect of the oscillation parameters on the impurity equilibrium position were also investigated. The impurity height increases with increasing dimensionless velocity amplitude, $a\omega/\sqrt{gd}$, as shown in Fig. 3. The data correspond to a range of dimensionless accelerations from 10 to 46.5 and dimensionless frequencies from 3.87 to 7.75 (corresponding, for example, to a dimensional frequency of 62 to 124 Hz for 1 mm diam. particles). The height of the impurity varies nearly linearly with the oscillation velocity for the parameters investigated here. Previous experiments [7] and simulations [8] have shown that the overall bed expansion increases with the square of the oscillation velocity. Although in our simulations the bed expansion does indeed increase with the square of the velocity, the impurity height only increases linearly with the velocity. As the coefficient of restitution decreases, the bed height and the impurity height decrease. At sufficiently low coefficients of restitution, the bed behaves as a dense bed characterized by a large number of nonbinary collisions.

Momentum transfer in a granular bed occurs in two modes. The "streaming" or "kinetic" mode is the transport of momentum as a particle moves through the material carrying its momentum with it. The "collisional" mode is the transport of momentum by interparticle collisions. Both mechanisms can make significant contributions to the stress tensor in a vibrofluidized bed. The streaming mode dominates at low densities where collisions are infrequent and the collisional mode is dominant at high densities as the particles cannot move far between collisions. In the vibrofluidized beds investigated here, both of these components have been taken into account.

The streaming normal stresses in a two-dimensional bed are given by $\tau_{S,xx} = \rho\nu\langle u'^2\rangle$ and $\tau_{S,yy} = \rho\nu\langle v'^2\rangle$ [9], where ρ is the particle mass density, ν is the mean solid fraction in the bin, and u' and v' are the fluctuating velocities in the horizontal and vertical directions. The angle brackets, $\langle\cdot\rangle$, represent the average of the quantities sampled at regular time intervals over a long period of system time.

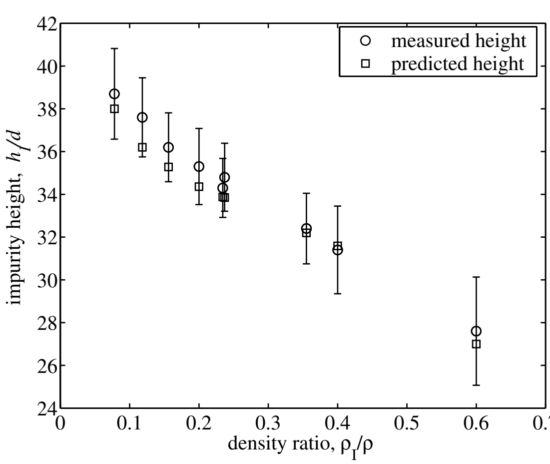

FIG. 2. Plots of the dimensionless steady state impurity height, h_I/d, as a function of density ratio, ρ_I/ρ, for $a/d = 1.03$ and $\omega\sqrt{d/g} = 5.81$. The data are for a range of mass ratios, M_I/M, from 5 to 15 and diameter ratios, d_I/d, between 5 and 8. The actual position and the position predicted by the pressure profiles are shown.

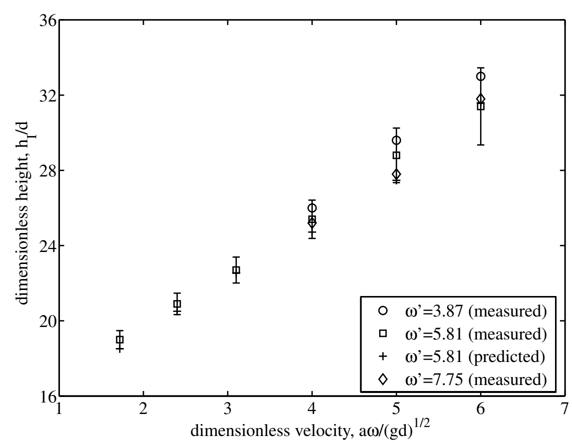

FIG. 3. Plots of the dimensionless height, h_I/d, as a function of dimensionless velocity amplitude, $a\omega/\sqrt{gd}$, for an impurity with $M_I/M = 10$ and $d_I/d = 5$. The data correspond to a range of dimensionless frequencies from $\omega' = \omega\sqrt{d/g} = 3.87$ to 7.75. The scatter bars and predicted height using the pressure profile are shown for $\omega' = 5.81$.

The collisional component of the stress tensor is found from the impulse of particle collisions. In our system the particles are frictionless disks; hence, the impulse acts in the direction along a line connecting the particle centers. Between time t and $t + \Delta t$, the impulse, $\mathbf{J}_{ji}$, that a particle j exerts on particle i during a collision is $\mathbf{J}_{ji} = \int_t^{t+\Delta t} \mathbf{F}_{ji}\, dt$, where $\mathbf{F}_{ji}$ is the force particle j exerts on particle i. The collisional normal stresses, $\tau_{C,xx}$ and $\tau_{C,yy}$, for a particular bin are [6]

$$\tau_{C,xx} = \frac{\sum 2R\mathbf{J} \cdot \hat{e}_x}{(W\Delta y_{\mathrm{bin}})\Delta t}, \tag{1a}$$

$$\tau_{C,yy} = \frac{\sum 2R\mathbf{J} \cdot \hat{e}_y}{(W\Delta y_{\mathrm{bin}})\Delta t}, \tag{1b}$$

where the summation is over all collisions occurring in time Δt in the bin of interest, R is the particle radius, and $\hat{e}_x$ and $\hat{e}_y$ are unit vectors pointing in the x and y directions. Here $\mathbf{J}$ is the magnitude of the momentum exchange between the two particles. The total stress in the granular bed is the sum of the collisional and streaming stress components while the granular pressure is defined as $p = \frac{1}{2}(\tau_{S,xx} + \tau_{S,yy} + \tau_{C,xx} + \tau_{C,yy})$. At equilibrium, the net vertical force acting on the impurity due to collisions with the surrounding particles will balance with the impurity's weight. The net pressure force acting in the vertical direction, F_p, is given approximately as $F_p \approx \frac{dp}{dy} d_I^2$, where d_I is the impurity diameter and the impurity is assumed to be sufficiently small. The weight of the impurity is $M_I g$, where M_I is the impurity mass and g is the acceleration due to gravity. Equating the net pressure force with the weight and noting that the weight acts in the direction opposite to the net pressure force gives

$$\frac{dp}{dy} = \frac{-M_I g}{d_I^2} = -\frac{\pi}{4}\rho_I g\,. \tag{2}$$

Thus the equilibrium position of the impurity, of known density ρ_I, can be found with the aid of Eq. (2) if the pressure variation in the vertical direction is known. Note that the present analysis assumes that the impurity does not affect the pressure distribution in the VFB. Hence, the pressure distribution for a VFB without the impurity can be used to determine the impurity's equilibrium position. This assumption was verified by using the simulations.

Figure 4(a) shows the pressure distribution, made dimensionless by using the surrounding particle diameter, d, density, ρ, and gravitational acceleration, g, in a VFB (without impurity) for an oscillation amplitude of $a/d = 0.69$ and frequency of $\omega\sqrt{d/g} = 5.81$. The pressure increases with increasing depth from the free surface of the bed and reaches a maximum at the floor. The pressure profile in a VFB is similar to the pressure profile of a compressible fluid in a gravity field.

The dimensionless pressure gradient corresponding to Fig. 4(a) is shown in Fig. 4(b). The expected equilibrium location of an impurity can be determined using Fig. 4(b) and Eq. (2). As can be seen in the figure, there are two heights at which the net pressure force will balance the impurity weight. By considering small perturbations to these equilibrium heights, the stable position of the impurity is seen to occur where $\frac{d^2p}{dy^2} > 0$. For example, if the impurity position is slightly greater than the larger equilibrium height, the net pressure force will be smaller than the impurity weight and the impurity will fall. However, if the impurity position is slightly less than the larger equilibrium height, the net pressure force will be greater than the impurity weight and the impurity will rise. Hence, the larger equilibrium height is a stable position. Similar considerations for the smaller equilibrium height show that it is an unstable position. Note that if the impurity moves away from the unstable equilibrium position towards the base of the container it will eventually impact the base and, if the impurity velocity after impact is large enough, the impurity will be thrown to a height larger than the unstable equilibrium position. The impurity will then move towards the stable equilibrium position.

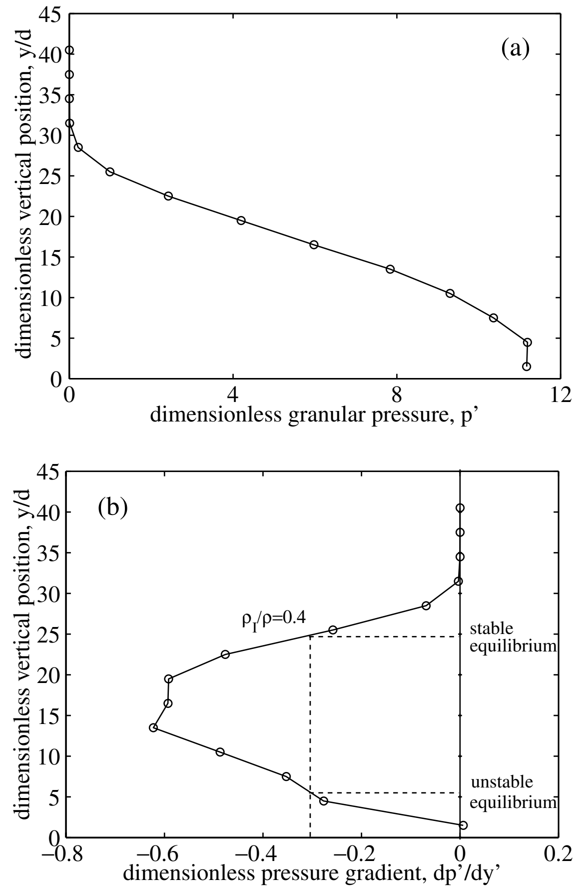

FIG. 4. Plots of the steady state (a) dimensionless granular pressure, $p' = p/(\rho g d)$, and (b) dimensionless pressure gradient, $dp'/dy' = (dp/dy)/(\rho g)$, for $a/d = 0.69$ and $\omega\sqrt{d/g} = 5.81$. The stable and unstable equilibrium positions of an impurity with a density ratio of $\rho_I/\rho = 0.4$ are shown in the figure.

If the pressure gradient required to balance the impurity weight is greater than the maximum gradient existing in the bed, then there is no preferred location for the impurity as it cannot be supported by particle collisions anywhere in the bed. For these conditions the impurity oscillates on the vibrating base, and the surrounding particles act to dampen the impurity motion. The maximum density that can be supported decreases with increasing velocity amplitude and increasing coefficient of restitution.

The expected equilibrium positions of the impurity determined from the pressure gradient profiles for the varying density ratio are also included in Fig. 2. As can be observed in this figure, the model accurately predicts the impurity equilibrium position. The near linear dependence of the equilibrium position on the density ratio occurs due to the approximately linear pressure gradient profile in the stable region of the bed (where $d^2p/dy^2 > 0$). The expected equilibrium positions for varying oscillation parameters are shown in Fig. 3. Again, the proposed model accurately predicts the measured equilibrium positions. These results indicate that the buoyant force analysis presented here accurately predicts the equilibrium height of the impurity.

The simulation results presented here indicate that an impurity in an otherwise homogeneous vibrofluidized bed will move to an equilibrium position within the bed such that the net pressure, or buoyant, force balances the particle weight. This conclusion stands in contrast to the claim by Ohtsuki *et al.* [4] that they were not able to predict an impurity's equilibrium position from the granular pressure profile. Our model also predicts the qualitative behavior observed by Hong *et al.* [10] in their hard-particle simulation studies of "reverse buoyancy." In the cases where the density ratio, ρ_I/ρ, increases, the larger (impurity) particles migrate to the bottom of the container. However when the density ratio decreases, the larger particles move toward the free surface. Direct comparisons with the reverse buoyancy work of Shinbrot and Muzzio [11] are not appropriate here since they investigated deep granular beds which are characterized by a large percentage of nonbinary contacts, where convective and interstitial air effects may play an important role. Although in this paper the pressure gradient in the bed is determined from computer simulations, granular kinetic theory [12–14] could also be used to predict the pressure gradient profile.

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