Magic Numbers in Exotic Nuclei and Spin-Isospin Properties of the NN Interaction

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The magic numbers in exotic nuclei are discussed, and their novel origin is shown to be the spin-isospin dependent part of the nucleon-nucleon interaction in nuclei. The importance and robustness of this mechanism is shown in terms of meson exchange, G-matrix, and QCD theories. In neutron-rich exotic nuclei, magic numbers such as $N = 8$, 20, etc. can disappear, while $N = 6$, 16, etc. arise, affecting the structure of the lightest exotic nuclei to nucleosynthesis of heavy elements.

The magic number is the most fundamental quantity governing the nuclear structure. The nuclear shell model was started by Mayer and Jensen by identifying the magic numbers and their origin [1]. The study of nuclear structure has been advanced on the basis of the shell structure associated with the magic numbers. This study, on the other hand, has been made predominantly for stable nuclei, which are on or near the β-stability line in the nuclear chart. This is basically because only those nuclei have been accessible experimentally. In such stable nuclei, the magic numbers suggested by Mayer and Jensen remain valid, and the shell structure can be understood well in terms of the harmonic oscillator potential with a spin-orbit splitting.

Recently, studies on exotic nuclei far from the β-stability line have started owing to the development of radioactive nuclear beams [2]. The magic numbers in such exotic nuclei can be a quite intriguing issue. In this Letter, we show that new magic numbers appear and some others disappear when moving from stable to exotic nuclei in a rather novel manner due to a particular part of the nucleon-nucleon interaction.

In order to understand underlying single-particle properties of a nucleus, we can make use of *effective (spherical) single-particle energies (ESPE's)*, which represent mean effects from the other nucleons on a nucleon in a specified single-particle orbit. The two-body matrix element of the interaction depends on the angular momentum J, coupled by two interacting nucleons in orbits j_1 and j_2. Since we are investigating a mean effect, this J-dependence is averaged out with a weight factor $(2J + 1)$, and only diagonal matrix elements are taken. Keeping the isospin dependence, $T = 0$ or 1, the so-called monopole Hamiltonian is thus obtained with a matrix element [3,4]:

$$V_{j_1 j_2}^T = \frac{\sum_J (2J + 1)\langle j_1 j_2|V|j_1 j_2\rangle_{JT}}{\sum_J (2J + 1)}, \qquad (1)$$

where $\langle j_1 j_2|V|j_1' j_2'\rangle_{JT}$ stands for the matrix element of a two-body interaction, V.

The ESPE is evaluated from this monopole Hamiltonian as a measure of mean effects from the other nucleons. The normal filling configuration is used. Note that, because the J dependence is taken away, only the number of nucleons in each orbit matters. As a natural assumption, the lowest possible isospin coupling is assumed for protons and neutrons in the same orbit. The ESPE of an *occupied* orbit is defined to be the separation energy of this orbit with the opposite sign. Note that the separation energy implies the minimum energy needed to take a nucleon out of this orbit. The ESPE of an *unoccupied* orbit is defined to be the binding energy gain by putting a proton or neutron into this orbit with the opposite sign.

Figure 1 shows neutron ESPE's for ^{30}Si and ^{24}O, both of which have $N = 16$. The Hamiltonian and the single-particle model space are the same as those used in [4], where the structure of exotic nuclei with $N \sim 20$ has been successfully described within a single framework. The valence orbits are then $0d_{5/2,3/2}$, $1s_{1/2}$, $0f_{7/2}$, and $1p_{3/2}$.

The nucleus ^{30}Si has six valence protons in the sd shell and is a stable nucleus, while ^{24}O has no valence proton and is a neutron-rich exotic nucleus. For ^{30}Si, the neutron $0d_{3/2}$ and $1s_{1/2}$ are rather close to each other [see Fig. 1(a)]. For ^{24}O, as shown in Fig. 1(b), the $0d_{3/2}$ is lying much higher and is quite close to the pf shell, giving rise to a large gap ($\sim$6 MeV) between $0d_{3/2}$ and $1s_{1/2}$ [5]. On the other hand, for the stable nucleus ^{30}Si, a considerable gap ($\sim$4 MeV) is created between the $0d_{3/2}$ and the pf shell [See Fig. 1(a)]. Thus, the $N = 20$ magic structure is evident for ^{30}Si, whereas the $N = 16$ magic number arises in ^{24}O. In ^{24}O, the $0d_{3/2}$ is lying higher, reflecting the large spin-orbit splitting which is basically the same as that for ^{17}O. Although this high-lying $0d_{3/2}$ orbit is not so relevant to the ground state of lighter O isotopes, it should affect binding energies of nuclei around ^{24}O. Such an anomaly was pointed out by Ozawa *et al.* [6] in observed binding energy systematics.
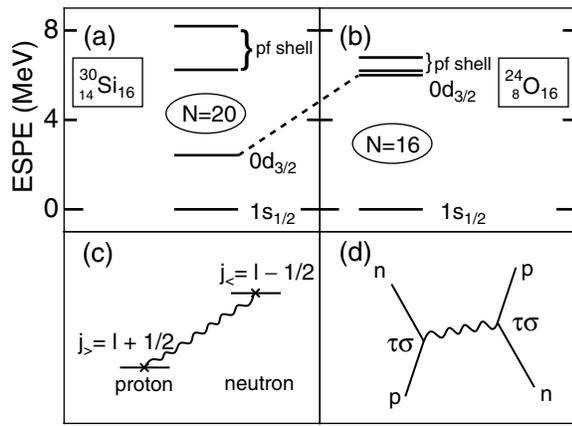
FIG. 1. Neutron ESPE's for (a) ^{30}Si and (b) ^{24}O, relative to $1s_{1/2}$. The dotted line connecting (a) and (b) is drawn to indicate the change of the $0d_{3/2}$ level. (c) The major interaction producing the basic change between (a) and (b). (d) The process relevant to the interaction in (c).

The dramatic change of ESPE's from ^{24}O to ^{30}Si is primarily due to the strongly attractive interaction between a proton in $0d_{5/2}$ and a neutron in $0d_{3/2}$. A schematic picture of this point is shown in Fig. 1(c) for the general cases comprised of a pair of orbits $j_> = l + 1/2$ and $j_< = l - 1/2$ with l being the orbital angular momentum. Note that $j_>$ and $j_<$ are nothing but spin-orbit coupling partners. The present case corresponds to $l = 2$. As one moves from ^{24}O to ^{30}Si, six valence protons are put into the $0d_{5/2}$ orbit. Consequently, due to the strong attraction shown in Fig. 1(c), a neutron in $0d_{3/2}$ is more bound in ^{30}Si, and its neutron $0d_{3/2}$ ESPE becomes very low, compared to that in ^{24}O where such attraction is absent.

The process illustrated in Fig. 1(d) produces the attractive interaction in Fig. 1(c). The nucleon-nucleon (NN) interaction in this process is written as

$$V_{\tau\sigma} = \tau \cdot \tau\sigma \cdot \sigma f_{\tau\sigma}(r). \qquad (2)$$

Here, the symbol ($\cdot$) denotes a scalar product, τ and σ stand for isospin and spin operators, respectively, r implies the distance between two interacting nucleons, and $f_{\tau\sigma}$ is a function of r. In the long range (or no r-dependence) limit of $f_{\tau\sigma}(r)$, the interaction in Eq. (2) can couple only a pair of orbits with the same orbital angular momentum l, i.e., the orbits $j_>$ and $j_<$.

The σ operator couples $j_>$ to $j_<$ (and vice versa) more strongly than $j_>$ to $j_>$ or $j_<$ to $j_<$. Therefore, the spin-flip process is more favored in the vertexes in Fig. 1(d), while spin nonflip contributions certainly exist. The same mathematical mechanism works for isospin: the τ operator favors charge exchange processes. By combining these two properties, $V_{\tau\sigma}$ produces large matrix elements for the spin-flip, isospin-flip processes: proton in $j_> \to$ neutron in $j_<$, and vice versa. This gives rise to the interaction in Fig. 1(c) with a strongly attractive monopole term for the appropriate sign of $V_{\tau\sigma}$. This feature is a general one

and is maintained with $f_{\tau\sigma}(r)$ in Eq. (2) with reasonable r dependences.

In stable nuclei with $N \sim Z$ with ample occupancy of the $j_>$ orbit in the valence shell, the proton (neutron) $j_<$ orbit is lowered by neutrons (protons) in the $j_>$ orbit. In exotic nuclei, this lowering can be absent, and then the $j_<$ orbit is located rather high, not far from the upper shell. In this sense, the proton-neutron $j_>$-$j_<$ interaction enlarges a gap between major shells for stable nuclei with the proper occupancy of relevant orbits. This interaction has been known, for instance [7,8], to play important roles also in other issues, e.g., the onset of the deformation.

The origin of the strong $V_{\tau\sigma}$ is quite clear. The one-boson-exchange-potentials (OBEP) for π and ρ mesons have this type of terms as major contributions. While the OBEP is one of the major parts of the effective NN interaction, the effective NN interaction in nuclei can be provided by the G-matrix calculation with core polarization corrections and other various effects. Such effective NN interaction will be called simply G-matrix interaction for brevity. The G-matrix interaction should maintain the basic features of meson exchange processes, and, in fact, existing G-matrix interactions generally have quite large matrix elements for the cases shown in Fig. 1(c), including strongly attractive monopole terms [9].

We would like to point out that the $1/N_c$ expansion of QCD by Kaplan and Manohar indicates that $V_{\tau\sigma}$ is one of three leading terms of the long range part of the NN interaction [10]. Since the next order of this expansion is smaller by a factor $(1/N_c)^2$, the leading terms should have a rather distinct significance. One of the other two leading terms in the $1/N_c$ expansion [10] is a central force,

$$V_0 = f_0(r), \qquad (3)$$

where f_0 is a function of r. The last leading term is a tensor force. We shall come back to these forces later.

Figure 2 shows the effective $0d_{3/2}$-$1s_{1/2}$ gap, i.e., the difference between ESPE's of these orbits, in $N = 16$ isotones with $Z = 8 \sim 20$ for three interactions: "Kuo" means a G-matrix interaction for the sd shell calculated by Kuo [11], and USD was obtained by adding empirical modifications to "Kuo" [12]. The present shell-model interaction is denoted SDPF hereafter, and its sd-shell part is nothing but USD with small changes [4]. A steep decrease of this gap is found in all cases, as Z departs from 8 to 14. In other words, a magic structure can be formed at about $Z = 8$, but it should disappear quickly as Z deviates from 8 because the gap decreases very fast. The slope of this sharp drop is determined by $V_{0d_{5/2}0d_{3/2}}^{T=0,1}$ in Eq. (1), where the dominant contribution is from $T = 0$. Note that $V_{0d_{5/2}0d_{3/2}}^{T=0}$ is most attractive among all $V_{j_1 j_2}^T$'s in the sd shell for each of the three interactions. In SDPF, for instance, its magnitude exceeds by ~ 1 MeV that of the second most attractive one.

We shall now discuss briefly the present gap in other approaches. The gap can be calculated from the
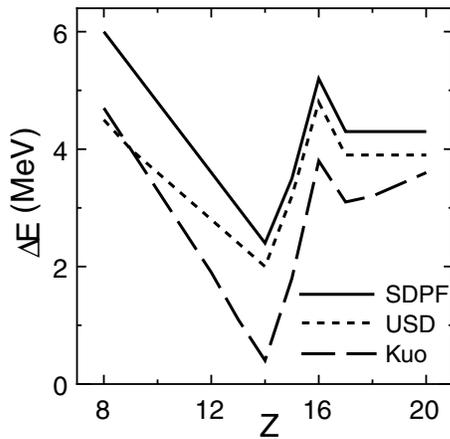
FIG. 2. The effective $1s_{1/2}$-$0d_{3/2}$ gap in $N = 16$ isotones as a function of Z. The shell-model Hamiltonians, SDPF, USD, and "Kuo," are used. See the text.

Woods-Saxon potential which is a standard modeling of a single-particle structure for stable nuclei. The resultant gap is rather flat, and is about half of the SDPF value for $Z = 8$. With Skyrme Hartree-Fock (HF) interactions, the gap changes more smoothly and gradually, too.

The occupation number of the neutron $1s_{1/2}$ is calculated by a Monte Carlo shell model [13] with full configuration mixing for the nuclei shown in Fig. 2. It is nearly 2 for ^{24}O, as expected for a magic nucleus, but decreases sharply as Z increases. It remains smaller (<1.5) in the middle region around $Z = 14$, and finally goes up again for $Z \sim 20$. This means that the $N = 16$ magic structure is broken in the middle region of the proton sd shell, where deformation effects also contribute to the breaking. The $N = 16$ magic number is thus quite valid at both ends. It is of interest that the gap becomes large again for larger Z, due to other monopole components.

We now turn to exotic nuclei with $N \sim 20$. The ESPE was evaluated for them in [4]. The small effective gap between $0d_{3/2}$ and the pf shell for neutrons is obtained, and is found to play essential roles for various anomalous features. This small gap is nothing but what we have seen for ^{24}O in Fig. 1(b). Thus, the disappearance of the $N = 20$ magic structure in $Z = 9 \sim 14$ exotic nuclei and the appearance of the new magic structure in ^{24}O have the same origin.

A very similar mechanism works for p-shell nuclei. We consider the structure of a stable nucleus ^{13}C, and exotic nuclei ^{11}Be and ^{9}He, all of which have $N = 7$. Shell-model calculations are performed with the PSDMK2 Hamiltonian [14] in the $p + sd$ shell-model space on top of the ^{4}He core. Figure 3 indicates that the experimental levels are reproduced well for ^{13}C [15], whereas notable deviations are found in ^{11}Be and ^{9}He [15,16]. Here, for experimental levels in the continuum, their resonance(like) energies are compared to the shell-model results.

The p-shell part of PSDMK2 is the Cohen-Kurath (CK) Hamiltonian [17]. The single-particle energies of $0p_{3/2}$
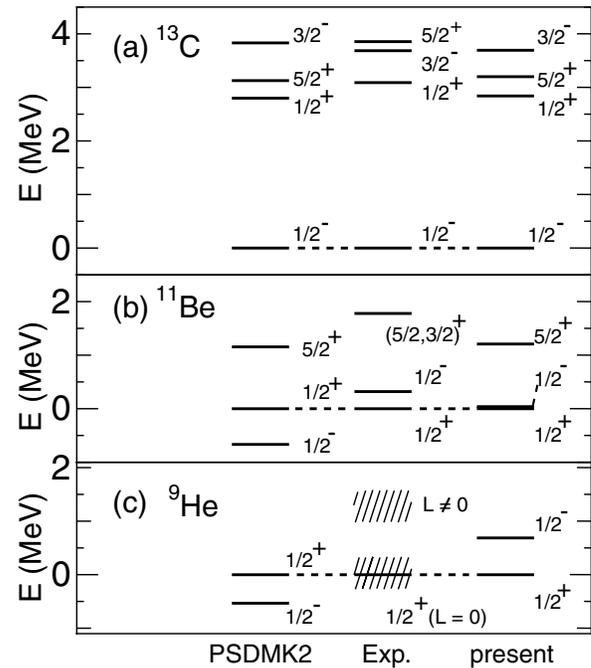
FIG. 3. Energy levels for (a) ^{13}C, (b) ^{11}Be, and (c) ^{9}He, relative to the state corresponding to the experimental ground state. The results by the PSDMK2 and present Hamiltonians are compared to experiments. Hatched areas indicate continuum states with resonance(like) structures.

and $0p_{1/2}$ are 1.38 and 1.68 MeV, respectively, in PSDMK2. These energies correspond to the observed spectra of ^{5}He: 1.15 for $0p_{3/2}$ and ~ 5 MeV for $0p_{1/2}$. We use these observed values as single-particle energies, while $0p_{1/2}$ is quite low in PSDMK2. As compared to the G-matrix interaction [9], the interaction in Fig. 1(c) for the p shell is too weak in CK, and is enlarged by shifting all $\langle 0p_{3/2}0p_{1/2}|V|0p_{3/2}0p_{1/2}\rangle_{J,T=0}$'s by -2 MeV independently of J. Thus, the Hamiltonian is modified only for three parameters. Since we are focusing on basic trends and would like to eliminate spurious center-of-mass components, all shell-model calculations for $N = 7$ isotones are made in the so-called $0\hbar\omega$ or $1\hbar\omega$ space. Figure 3(a) indicates that the levels of ^{13}C calculated from the present Hamiltonian are similar to the ones obtained from PSDMK2, because the higher-lying $0p_{1/2}$ is pulled down by V as discussed above.

The ground state of ^{11}Be and ^{9}He is known for the *inversion* between $1/2^+$ and $1/2^-$ [18]. The present calculation reproduces this inversion for both nuclei, whereas the PSDMK2 fails in either case. With the present Hamiltonian, the neutron effective $1s_{1/2}$-$0p_{1/2}$ gap decreases from 8.6 MeV for ^{13}C down to 0.8 MeV for ^{9}He. Dynamical correlations finally invert $1/2^+$ and $1/2^-$ eigenstates [19]. Thus, we achieve a reasonable description of stable and exotic nuclei with $N = 7$. Although the result appears promising, we have investigated only possible improvements, and further studies are needed for an overall description of p-sd nuclei. For instance, in comparison to

experiments, the present interaction gives a $3/2^-$ state in ^{15}O ~2 MeV too high [15], and ^9He is too unbound relative to ^8He by ~4.5 MeV [16]. The latter may be related to coupling to continuum and/or to nuclear-size dependence of two-body interaction.

The neutron $0p_{1/2}$ orbit becomes higher as the nucleus loses protons in its spin-flip partner $0p_{3/2}$. In nuclei such as He, Li and Be, the $N = 8$ magic structure then disappears. In some cases, $N = 6$ becomes magic: for instance, bound ^8He and unbound ^9He are obtained, similarly to the case of bound ^{24}O and unbound ^{25}O.

As the neutron $0p_{1/2}$ is shifted quite high in He, Li, and Be, this orbit becomes close to the $1s_{1/2}$ which may come down due to halo structure. This is a situation like the Efimov state [20]. The pairing between these two orbits, including contribution from the tensor force, may stabilize the two-neutron halo and provide us with bound ^{11}Li, etc. Thus, the present mechanism plays a crucial role in the structure of drip line nuclei. Without this mechanism, it is usually difficult to make $0p_{1/2}$ and $1s_{1/2}$ so close to each other, except for an empirical shift of the single-particle energy.

Moving back to heavier nuclei, from the strong interaction in Fig. 1(c), we can predict other magic numbers, for instance, $N = 34$ associated with the $0f_{7/2}$-$0f_{5/2}$ interaction. In heavier nuclei, $0g_{7/2}$, $0h_{9/2}$, etc. are shifted upward in neutron-rich exotic nuclei, disturbing the magic numbers $N = 82, 126$, etc. It is of interest how the r process of nucleosynthesis is affected by it.

In conclusion, we showed how magic numbers are changed in nuclei far from the β-stability line: $N = 6$, 16, 34, etc. can become magic numbers in neutron-rich exotic nuclei, while usual magic numbers, $N = 8, 20, 40$, etc., may disappear. Since such changes occur as a result of the nuclear force, similar changes should occur for the same Z values in mirror nuclei due to isospin symmetry. The mechanism of this change can be explained by the strong $V_{\tau\sigma}$ interaction which has robust origins in OBEP, G matrix, and QCD. In fact, a simple structure such as magic numbers should have a simple and sound basis. Since it is unlikely that a mean central potential can simulate most effects of $V_{\tau\sigma}$, we should treat $V_{\tau\sigma}$ rather explicitly. It is nice to build a bridge between very basic features of exotic nuclei and the basic theory of hadrons, QCD. In existing Skyrme HF calculations, except for those with Gogny force, effects of $V_{\tau\sigma}$ may not be well enough included, because the interaction is truncated to be of the δ-function−type. The relativistic mean field calculations must include pion degrees of freedom to be consistent with $V_{\tau\sigma}$. Thus, the importance of $V_{\tau\sigma}$ opens new directions for mean field theories of nuclei. In addition, $f_0(r)$ in Eq. (3) and $f_{\tau\sigma}(r)$ in Eq. (2) should have different ranges. Since the latter has smaller contri-

butions in exotic nuclei, this difference should produce very interesting effects on nuclear size. Loose-binding or continuum effects are important in some exotic nuclei. By combining such effects with those discussed in this Letter, one may draw a more complete picture of the structure of exotic nuclei. Finally, we would like to mention once more that the $V_{\tau\sigma}$ interaction should produce large, simple, and robust effects on various properties, and may change the landscape of nuclei far from the β-stability line in the nuclear chart.

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