

## Remote State Preparation

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Quantum teleportation uses prior entanglement and forward classical communication to transmit one instance of an unknown quantum state. Remote state preparation (RSP) has the same goal, but the sender knows classically what state is to be transmitted. We show that the asymptotic classical communication cost of RSP is one bit per qubit—half that of teleportation—and even less when transmitting part of a known entangled state. We explore the tradeoff between entanglement and classical communication required for RSP, and discuss RSP capacities of general quantum channels.

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A principal goal of quantum information theory is understanding the resources necessary and sufficient for intact transmission of quantum states. In quantum teleportation [1] an unknown state is transmitted from a sender (“Alice”) to a receiver (“Bob”) using classical communication and prior entanglement. Two bits of forward classical communication and one ebit of entanglement (a maximally entangled pair of qubits) per teleported qubit are both necessary and sufficient, and neither resource can be traded off against the other. In remote state preparation (RSP) the goal is the same—for Bob to end up with a single specimen of a state—but here Alice starts with complete classical knowledge of the state.

Pati [2] and Lo [3] showed that for special ensembles of states (e.g., qubit states on the equator of the Bloch sphere) RSP requires less classical communication than teleportation, but Lo conjectured that for general states the classical communication costs of the two tasks would be equal. Here we show that, in the presence of a large amount of prior entanglement, the asymptotic classical communication cost of RSP for general states is one bit per qubit, half that of teleportation. Most of this entanglement is not destroyed, but, as we will show, can be recovered afterward using *backward* classical communication from Bob to Alice, a resource that is entirely unhelpful for teleportation.

We show that RSP is unlike teleportation in that it exhibits a nontrivial tradeoff between classical communication and entanglement, the classical cost of preparing a generic qubit state ranging from one bit in the high-entanglement limit to infinitely many without prior entanglement (if any finite classical message, say of  $k$  bits, sufficed, Bob could use that message to make infinitely many copies, determine the state’s amplitudes to more than  $k$  bits precision, and thereby violate causality).

We introduce two new kinds of channel capacity, reflecting a general quantum channel’s asymptotic ability to be used for remote state preparation, with or without prior entanglement, and relate these capacities to the regular quantum and classical capacities with or without prior

entanglement. Finally, we discuss remote preparation of states entangled between Alice and Bob.

*RSP in the high-entanglement limit.*—To see how a large amount of shared entanglement enables general states to be remotely prepared at an asymptotic cost of one bit per qubit, it is helpful first to consider an exact (nonasymptotic) RSP protocol for the special ensemble mentioned earlier: equatorial states. Assume Alice and Bob share a number of singlets, i.e., pairs of qubits in the state  $|\Psi^-\rangle = |01\rangle - |10\rangle$  (we will often omit the normalization  $1/\sqrt{2}$ ). To remotely prepare an equatorial state  $|\psi\rangle = |0\rangle + e^{i\phi}|1\rangle$ , Alice takes one of her singlets and measures it [2] in the basis  $(\psi, \psi^\perp)$  where  $\psi^\perp$  denotes the antipodal (orthogonal) state to  $\psi$ . If the outcome is  $\psi^\perp$  she knows (by the properties of the singlet state) that Bob’s remaining half of the singlet is in the desired state  $\psi$ . But equally often Alice’s outcome is  $\psi$ , leaving Bob with  $\psi^\perp$ , the antipode of the state Alice wished to prepare. For equatorial states, Bob can correct  $\psi^\perp$  to  $\psi$  by applying the Pauli operator  $\sigma_z$ , a  $180^\circ$  rotation about the  $z$  axis. Thus Alice can remotely prepare an arbitrary equatorial state known to her by measuring a shared singlet in the basis determined by that state, and sending Bob the one-bit measurement result, which tells him whether to apply  $\sigma_z$ . But for general, nonequatorial states, the corrective transformation  $\psi^\perp \rightarrow \psi$  is antiunitary, and Bob cannot perform it by any physical means.

Now suppose Alice wishes to remotely prepare a large number of general qubit states  $\psi_1, \psi_2, \dots, \psi_n$ , and that she and Bob share an unlimited supply of singlets. For each  $j = 1 \dots n$ , Alice measures  $m = 2^{n+\log n}$  of her singlets in the basis  $\{\psi_j, \psi_j^\perp\}$ , and stores the results as one row of an  $n \times m$  table  $T$ , writing  $T(j, k) = 1$  for a success (meaning Bob’s half of that singlet is in the desired state  $\psi_j$ ) and  $T(j, k) = 0$  for a failure (meaning Bob’s half is in the antipodal state  $\psi_j^\perp$ ). Alice does all this without telling Bob anything, obtaining a large table of  $mn$  independent random zeros and ones. When she is done making all the measurements, she looks for a column of all ones, and uses

$n + \log n$  bits to tell Bob its index. Bob keeps the states in the successful column and discards all the others. If [with probability  $o(1)$ ] no successful column exists, Alice tells Bob so, then uses  $n$  more singlets and  $2n$  classical bits to simply teleport the states to Bob. Thus one bit per qubit is asymptotically sufficient for RSP; it is also necessary [3] by causality.

This protocol can be generalized from qubits to states in a  $d$ -dimensional Hilbert space, allowing them to be remotely prepared at an asymptotic classical communication cost of  $\log_2 d$  bits per state. Instead of singlets, Alice and Bob use maximally entangled pairs of the form  $|\Phi_d^+\rangle = |00\rangle + |11\rangle + \dots + |(d-1)(d-1)\rangle$ . Alice and Bob prearrange  $mn$  such states with  $m \gg d^n$  in an array of  $n$  rows and  $m$  columns. For each row  $j$ , Alice measures her halves of the pairs in a basis including  $\psi_j^*$ , the complex conjugate of the state she wishes to remotely prepare. If (with probability  $1/d$ ) her measurement outcome is  $\psi_j^*$ , Bob's half of the entangled pair will be left in the desired state  $\psi_j$ , and Alice enters a 1 in her success/failure table; otherwise she enters a 0.

The high-entanglement RSP protocol described above uses a large number of ebits, approximately  $2^n$  per state sent if  $n$  qubits are transmitted. But, using back communication, this protocol can be modified so that only a constant number of ebits are needed per state transmitted, while still only requiring one classical bit. To achieve this, we first (following a suggestion of Ambainis) divide the  $n$  states to be transmitted into subblocks of size  $s$ ;  $s \rightarrow \infty$  as  $n \rightarrow \infty$ , but  $2^s/n \rightarrow 0$ . Within each subblock the basic scheme described above is followed. But instead of performing a separate von Neumann measurement on her half of each of the ebits, Alice does a less intrusive measurement: for each set of  $s$  ebits constituting a column in her table, she performs a two-outcome incomplete von Neumann measurement. The "1" outcome, obtained with probability  $2^{-s}$ , signals that *all* Bob's particles are in the desired state  $\prod_{j=1}^s |\psi_j\rangle$ ; the "0" outcome signals all other possibilities. The joint state remaining between Alice and Bob when "0" is obtained,  $\rho_0$ , is still highly entangled, and pure entanglement can be recovered from it by distillation. From Bob's viewpoint the state  $\rho_0$  is mixed, because he does not know the bases of Alice's measurements. Averaging over all such bases, the diagonal elements of  $\rho_0$  in the generalized Bell basis are

$$\langle B|\rho_0|B\rangle = \frac{2^s - 2}{2^s - 1} \delta_{sr} + \left(\frac{1}{3}\right)^{s-r} \frac{1}{2^s(2^s - 1)}, \quad (1)$$

where  $B$  is any tensor product of Bell states  $\{\Phi^\pm = |00\rangle \pm |11\rangle, \Psi^\pm = |01\rangle \pm |10\rangle\}$  containing  $r$  instances of  $\Phi^+$  and  $s - r$  instances of the other Bell states. Alice and Bob collect all these  $\rho_0$  states until  $s'$  RSPs have been performed, with  $s \ll s' \ll n$ ; at this point they have about  $c = s'2^s/s$  copies of  $\rho_0$ . They then perform an entanglement distillation procedure. After dephasing in the Bell basis (which can be accomplished by a twirling [4] performed by Alice and Bob), the state  $\rho_0^{\otimes c}$  can be approxi-

mated, using a typical subset, by an equal mixture of  $2^{cs}$  different products of Bell states. Here the von Neumann entropy of the twirled  $\rho_0$  is  $S = s2^{-s}(2 + \frac{1}{2}\log 3)$ . By the random-hashing technique [4],  $c(s - S)$  pure singlets can be distilled from this mixture with the help of back communication from Bob to Alice. Counting also the one pair consumed when the successful "1" outcome is obtained, the number of ebits consumed per state transmitted becomes  $e_0 = 1 + cS/s' = 3 + \frac{1}{2}\log 3 \approx 3.79$ . This point  $(e_0, b = 1)$  is labeled **R** in Fig. 1.

*More restricted protocols and Lo's conjecture.*—For any set of  $n$  states to be remotely prepared, the above protocols are *exactly faithful*, i.e., always work, reproducing exactly the desired output even for finite  $n$ , but only *asymptotically efficient*, since the expected classical communication approaches one bit per qubit only in the limit of large  $n$ , while for any finite  $n$ , there is some chance that the classical communication cost will exceed that required for teleportation. We know of no exactly faithful RSP protocol for finite  $n$  that always uses less classical communication than would be required by teleportation. In this sense Lo's conjecture still stands.

In a more restricted setting we can prove Lo's conjecture. Suppose Alice wants to remotely prepare a *single* quantum state  $\psi$  in a  $d$ -dimensional Hilbert space (for simplicity  $d$  is a power of 2) for Bob. As in teleportation, we restrict Bob to performing a unitary transformation on some system in his lab determined by the classical data he receives from Alice. Also, as in teleportation, we require that the probability that Alice sends message  $i$  to Bob not depend on the state that she is remotely preparing. If such a protocol is exactly faithful, we can show that it must use at least  $2 \log d$  classical bits of communication from Alice to Bob, as in teleportation. The argument is as follows. Let  $k$  be the number of classical

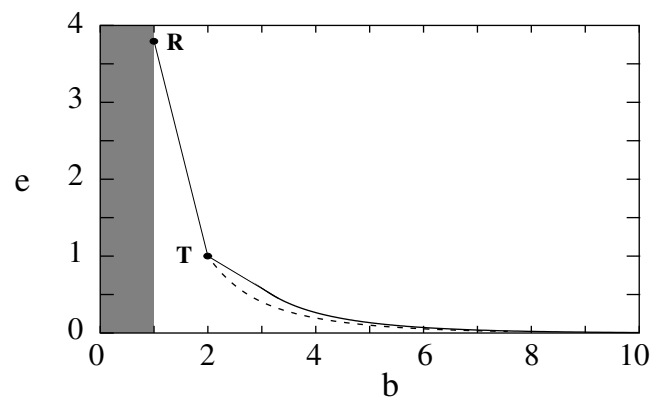


FIG. 1. Entanglement ( $e$ ) and forward classical communication ( $b$ ) costs of remotely preparing qubit states in various ways, including teleportation (**T**), our high-entanglement method with entanglement recycling (**R**), and convex combinations (solid line between **T** and **R**). The shaded region  $b < 1$  is inaccessible because it would violate causality. Solid curve below and to the right of **T** is our low-entanglement method and convex combinations with teleportation. Dashed curve is Devetak-Berger method.

bits that Alice sends to remotely prepare  $\psi$ . We will have Bob guess this data. He infers from the protocol that he will get message  $i$  ( $i = 1, \dots, 2^k$ ) with probability  $p_i$  ( $\sum_{i=1}^{2^k} p_i = 1$ ). Thus he flips a coin with bias  $p_i$  and he implements the corresponding unitary transformation in his lab. Since the protocol only allows him to carry out unitary transformations, guessing wrong means that instead of getting  $|\psi\rangle$  he will obtain  $U|\psi\rangle$  where  $U$  is some unitary transformation. The total probability  $p$  of Bob guessing correctly is given by the sum over  $i$  of the probability that Alice sends  $i$  and Bob correctly guesses  $i$ , which is  $\sum_i p_i^2 \geq 2^{-k} \sum_i p_i = 2^{-k}$ . Alice and Bob have thus created a channel  $S$  which acts upon the state  $\psi$  in Alice's lab and outputs the state  $\rho = S(|\psi\rangle\langle\psi|) = p|\psi\rangle\langle\psi| + (1-p)S_{\text{wrong}}(|\psi\rangle\langle\psi|)$  where  $p \geq 2^{-k}$ . Since Bob used zero communication to make this state, it must be that

$$f(S) \equiv \frac{1}{\text{Vol}(\psi)} \int d\psi \langle\psi|S(|\psi\rangle\langle\psi|)|\psi\rangle \leq \frac{1}{d}. \quad (2)$$

If not, Alice and Bob would have created a superluminal channel. We can use a result by the Horodeckis [5] which relates  $f(S)$  to the maximally entangled fraction  $F(S) \equiv \langle\Phi_d^+|(\mathbf{1} \otimes S)(|\Phi_d^+\rangle\langle\Phi_d^+|)|\Phi_d^+\rangle$ , i.e.,  $f(S) = [F(S)d + 1]/(d + 1)$ . Since  $S$  is the identity operator with probability larger than or equal to  $2^{-k}$  we have  $f(S) \geq (2^{-k}d + 1)/(d + 1) > 1/d$  for  $k < 2 \log d$  in contradiction to (2). Thus in a very restricted "teleportation" type of RSP, Lo's conjecture still holds. Besides being exactly faithful, this restricted protocol is *oblivious*; Bob receives no additional information about  $\psi$  other than the state  $\psi$  itself. This is due to the fact that the probability with which Alice sends a classical message does not depend on the state  $\psi$ . In the high-entanglement RSP protocol, by contrast, Bob can gain some additional information about  $\psi$  by measuring the singlets in the unsuccessful columns instead of recycling them. Perhaps Lo's conjecture holds for all oblivious, exactly faithful protocols.

For the next two sections we relax the requirement of exact fidelity, requiring only that protocols be *asymptotically faithful*, i.e., for any set of  $n$  input states, they should produce an approximation to the desired output  $\psi_1 \otimes \psi_2 \otimes \dots \otimes \psi_n$  whose fidelity approaches 1 in the limit of large  $n$ . This definition has the advantage of allowing RSP to be composed with other asymptotically faithful processes such as Schumacher compression [6].

*Low-entanglement RSP.*—Here we bound the forward classical communication  $b$  needed to remotely prepare qubit states using entanglement  $e < 1$  ebit per qubit. To do so, Alice sends Bob some classical information about the states  $\psi_1 \dots \psi_n$ , so as to reduce their posterior von Neumann entropy from his viewpoint and allow her to teleport them using  $< 1$  ebit per qubit. For example, a qubit uniformly distributed over a circular cap  $C_\theta$  of radius  $\theta < \pi$  and area  $A(\theta) = 2\pi(1 - \cos\theta)$  centered on the north pole has von Neumann entropy  $S(\theta) = H_2[(1 - \cos\theta)/4] < 1$  and can be teleported at an asymptotic cost of  $2S(\theta)$  bits and  $S(\theta)$  ebits.

First assume the states  $\psi_1 \dots \psi_n$  are uniformly distributed (a restriction we later remove). For each block length  $n$  and cap radius  $\theta$ , suppose Alice and Bob have agreed on an  $n$  by  $m = n[4\pi/A(\theta)]^n$  array of random rotations  $R(i, j)$ ,  $i = 1 \dots n$ ,  $j = 1 \dots m$ . Then, given the states  $\psi_1 \dots \psi_n$ , Alice constructs a success/failure table where a success,  $T(i, j) = 1$ , is counted if and only if the rotated state  $R(i, j)\psi_i$  falls within the standard cap  $C_\theta$ . As before she looks for an all-successful column, and uses an expected  $S'(\theta) + o(1)$  bits per state, where  $S'(\theta) = \log_2[4\pi/A(\theta)]$ , to tell Bob its index  $j$ . Finally, she Schumacher compresses the states in the successful column and teleports them, at an additional asymptotic cost of  $2S(\theta)$  bits and  $S(\theta)$  ebits per state, to Bob, who rotates them back into their original positions. If there is no successful column, Alice teleports the states directly, without compression; but this happens so rarely as to not increase the asymptotic entanglement and communication costs,  $e = S(\theta)$  and  $b = S'(\theta) + 2S(\theta)$ . The  $R$  rotations need not actually be random: for each  $n$  and  $\theta$ , there always exists a deterministic set of rotations which performs no worse than average on uniformly distributed  $\psi_i$ . We use  $D(i, j)$  to denote these deterministic rotations.

To make the protocol work on arbitrary sequences of states, even ones maliciously chosen to avoid successes with the particular rotations  $\{D(i, j)\}$  Alice and Bob are using, Alice divides the states into subblocks of size  $s \approx \sqrt{n}$ , and applies the above protocol separately to each subblock, but before doing so applies a set of  $s$  random prerotations  $r_1 \dots r_s$  which Bob removes afterward, to the states in each subblock. Then, even if the original states  $\psi_i$  are awkwardly located, the randomized states  $r_{i \bmod s} \psi_i$  will be random within each subblock. Reusing the prerotations causes the deviations of the actual mixed-state output from the ideal  $\psi_1 \dots \psi_n$  to be correlated between subblocks, but because of the exponentially fast convergence of Schumacher compression with increasing subblock size, the full  $n$ -fold fidelity still approaches unity in the limit  $n \rightarrow \infty$ , for any sequence  $\psi_1 \dots \psi_n$  of states to be remotely prepared. Of course Alice must tell Bob the prerotations  $r_1 \dots r_s$  so he can remove them at the end. If the prerotations are described with precision, say,  $\sqrt{s}$  bits, the finite-precision errors will vanish exponentially rapidly, while keeping the communication overhead sublinear in  $n$ .

Recently Devetak and Berger [7] introduced an improved protocol which they prove optimal among low-entanglement RSP methods that use a classical message followed by teleportation to remotely prepare states uniformly distributed on the Bloch sphere. Shown as the dashed curve in Fig. 1, their method is like our low-entanglement methods, but instead of the index of the first successful column, Alice tells Bob the index of the column whose states, viewed as a finite ensemble, have least entropy.

*RSP capacities of quantum channels.*—Our results suggest new kinds of capacity for a general noisy quantum channel, expressing its asymptotic ability to send known

states, with or without the help of prior shared entanglement. For any channel  $\mathcal{N}$  we define the RSP capacity (which might depend on the dimension  $d$  of the Hilbert space  $\mathcal{H}_d$ ) as

$$R^{(d)}(\mathcal{N}) = \lim_{\epsilon \rightarrow 0} \limsup_{m \rightarrow \infty} \left\{ \frac{n \log d}{m} : \exists \mathcal{D}_{mn} \quad \forall \psi_1, \dots, \psi_n \in \mathcal{H}_d \quad \exists \mathcal{E}_{mn} F(\psi_1 \otimes \dots \otimes \psi_n, \mathcal{D}_{mn} \mathcal{N}^{\otimes m} \mathcal{E}_{mn}) > 1 - \epsilon \right\}, \quad (3)$$

where  $\mathcal{E}_{mn}$  denotes a possible block encoder used by Alice, using  $n$  classically described states  $\psi_1, \dots, \psi_n$  to prepare an input to the quantum channel  $\mathcal{N}^{\otimes m}$  (i.e.,  $m$  parallel instances of  $\mathcal{N}$ ); similarly  $\mathcal{D}_{mn}$  denotes a possible block decoder used by Bob, mapping the  $m$  channel outputs to some approximation of the state to be remotely prepared; and  $F(\psi_1 \otimes \dots \otimes \psi_n, \mathcal{D}_{mn} \mathcal{N}^{\otimes m} \mathcal{E}_{mn})$  denotes the fidelity of this approximation [the fidelity of a pure state  $\Psi$  under linear map  $\mathcal{M}$  is naturally defined as  $F(\Psi, \mathcal{M}) = \text{Tr} \Psi \mathcal{M}(\Psi)$ ]. The entanglement-assisted RSP capacity  $R_E(\mathcal{N})$  is defined similarly, except that the encoder and decoder share unlimited prior entanglement.

Clearly, for any channel,  $R^{(d)} \leq C$ , since the classical capacity  $C$  may be viewed as the channel's ability to remotely prepare classical states (i.e., orthogonal states in some basis). On the other hand,  $R^{(d)} \geq Q$ , the quantum capacity, since the efficiency of transmitting known states must be at least that of transmitting unknown states.

In the entanglement-assisted setting, we can show that  $R_E$  is independent of  $d$  and equal to  $C_E$ , the channel's entanglement-assisted classical capacity [8]. This follows from the fact that  $\log d$  bits of classical communication are asymptotically both necessary and sufficient to remotely prepare a general  $d$ -dimensional state.

Without entanglement, there are channels for which  $R^{(2)} > Q$ , for example, a strongly dephasing qubit channel with  $C = 1$  and  $0 < Q \ll 1$ . Given any point  $(e, b)$  on the dashed curve in Fig. 1, such a channel can be used  $n$  times to share  $\approx Qn$  ebits and another  $n$  times to transmit  $n$  classical bits, giving  $R^{(2)} \geq \min\{Q/2e, 1/2b\}$  asymptotically; hence  $R^{(2)}/Q \geq 1/2e$  for small enough  $Q$ . On the other hand,  $R^{(d)} = 0$  for any purely classical channel (i.e., one with  $Q = 0$ ), by causality.

*Remote Preparation of Entangled States.*—Like teleportation, RSP can be applied not only to pure states, but also to parts of entangled states. However, unlike teleportation, RSP requires *less* classical communication to prepare an entangled state in  $\mathcal{H}_A \otimes \mathcal{H}_B$ , where  $\mathcal{H}_A$  remains in Alice's lab, than to prepare a pure state in  $\mathcal{H}_B$ . To take an extreme example, the standard maximally entangled state  $\Phi_d^+$  in  $d \times d$  dimensions can be converted into any other maximally entangled state in  $d \times d$  dimensions with no classical communication at all, because maximally entangled states are interconvertible by local unitary operations of Alice. Suppose more generally that Alice and Bob share an unlimited supply of ebits, and that Alice wants to prepare a state  $\psi \in \mathcal{H}_A \otimes \mathcal{H}_B$ , which is known to her. We assume both Hilbert spaces have dimension  $d$ ; if necessary the smaller can be extended to make this so. Any state  $\psi \in H_A \otimes H_B$  can be written in Schmidt form as  $\psi = \sum_{i=1}^d \sqrt{\lambda_i} |a_i\rangle \otimes |b_i\rangle$ , where some of the  $\lambda_i$  may be

zero. We give a probabilistic procedure by which Alice can convert the standard state  $\Phi_d^+$  into the desired  $\psi$  with success probability  $1/d$  if  $\psi$  is separable and greater than  $1/d$  if  $\psi$  is entangled.

Alice begins by bringing the standard state to the form  $U_A |\Phi_d^+\rangle = |\phi\rangle = \frac{1}{\sqrt{\Lambda}} \sum_{i=1}^d |a_i\rangle \otimes |b_i\rangle$  by means of a local unitary transformation  $U_A$ . She then performs a local filtering operation on it, which can be described by a positive-operator-valued measure with two elements,  $\Pi_1$  (success) and  $\Pi_0$  (failure), the resulting state in each case being  $(\sqrt{\Pi_j} \otimes I) |\phi\rangle$ . Here we take  $\Pi_1 = \frac{1}{\Lambda} \sum_{i=1}^d \lambda_i \times |a_i\rangle \langle a_i|$  and  $\Pi_0 = I - \Pi_1$ , where  $\Lambda = \max\{\lambda_i\}$ . Success, which leaves the system in the desired state  $\psi$ , occurs with probability  $|(\sqrt{\Pi_1} \otimes I) |\phi\rangle|^2 = 1/(\Lambda d)$ , which is greater than  $1/d$  if  $\psi$  is entangled. This procedure is exactly faithful and asymptotically efficient in the sense that for any sequence of states  $\psi_1 \dots \psi_n \in \mathcal{H}_A \otimes \mathcal{H}_B$  the expected classical cost is  $\sum_j \log(\Lambda_j d) + O(1)$  bits.

As with unentangled states, causality sets a lower bound on the classical cost of RSP for entangled states. The cost of RSP for a set of states  $\psi_1 \dots \psi_n$  must be at least  $S(\bar{\rho}) - \frac{1}{n} \sum_{i=1}^n S(\rho_i)$  bits, where  $\rho_i = \text{tr}_A |\psi_i\rangle \langle \psi_i|$  and  $\bar{\rho} = \frac{1}{n} \sum_{i=1}^n \rho_i$ , because the states could be asymptotically used to encode that much classical information [9]. We are investigating how closely this bound can be approached.

RSP can be generalized to multiparty scenarios. For example, one may ask whether Alice, using prior entanglement shared separately with Bob and Charlie, can remotely prepare an arbitrary tripartite state by sending  $\leq \log d_B$  bits to Bob and  $\leq \log d_C$  bits to Charlie.

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