Superfluid Density in the d-Density-Wave Scenario

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Recently Chakravarty, Laughlin, Morr, and Nayak [Phys. Rev. B **62**, 4880 (2000)] made an interesting proposal that the cuprate superconductors possess a hidden "d-density-wave" (DDW) order. We study the implication of this proposal for the superfluid density ρ_s. We find that it predicts a temperature gradient $|d\rho_s/dT|_{T=0}$ that is strongly doping dependent near the critical doping at which the superconducting gap vanishes. This demonstrates that the DDW scenario is inconsistent with existing well-established experimental data.

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Recently Tallon and Loram critically examined the existing experimental data on specific heat, photoemission, magnetic susceptibility, and optical conductivity of the cuprate superconductors [1]. They argued that these data are consistent with the existence of two competing energy gaps, a pseudogap and the superconducting gap, both of which disperse according to d-wave symmetry. The pseudogap exists in the normal state of the underdoped cuprates while the superconducting gap opens up in the BCS fashion at temperature T_c. Tallon and Loram also argue that the data indicate the closing of the pseudogap at a doping level $x \approx 19\%$.

This observation motivated Chakravarty, Laughlin, Morr, and Nayak (CLMN) to propose the "d-density-wave" (DDW) scenario [2]. According to this proposal a staggered long-range order in the orbital magnetic moment, as first suggested by Hsu, Marston, and Affleck in another context [3], exists at low temperatures when the doping is less than a critical value $x_c \approx 0.19$. Such order gives rise to a nonsuperconducting gap (the pseudogap) with the same $(\cos k_x - \cos k_y)$ dispersion as the superconducting (SC) gap. Moreover, the DDW order competes with superconductivity and causes $T_c(x)$ to trace out the familiar "superconducting dome" in the doping-temperature phase diagram of the cuprates. So far there is no direct experimental evidence for the DDW order. Neither is there evidence for the expected Ising-like phase transition into such a state. CLMN argue that disorder gets in the way of a sharp phase transition and turns it into a crossover.

In addition to offering a possible new phase for high-T_c cuprates, there are a number of attractive features in CLMN's proposal. Since the DDW metal (i.e., the normal state in the presence of DDW order) is a doped band insulator, one expects the Drude weight in the normal state [4] and the superfluid density in the superconducting state both to be proportional to the doping density [5], as indeed found in the experiments. The closing of the DDW gap at $x = 0.19$ can easily explain the observed kink in the jump of the T-linear specific heat coefficient [6]. In addition, the small hole pockets centered around the nodes of the DDW gap could be the progenitor of the Fermi arcs observed in angle-resolved photoemission [7]. From a technical point of view a doped band insulator has more resemblance to a doped Mott insulator than a large-Fermi-surface metal; hence it is a better starting point for a description of the underdoped cuprates. Finally the CLMN proposal is crisp and in principle falsifiable. For all the above reasons we feel that it is worthwhile to further check the prediction of this proposal against existing experiments.

The theory presented in Ref. [2] is mean field in nature. For such a theory, quasiparticles are sharply defined. Since a much better case can be made for the existence of quasiparticles in the superconducting states [8], we confine our calculations to low temperatures where SC order parameters exist.

Among various superconducting properties we focus on the superfluid density ρ_s because of its rather unconventional doping and temperature dependence, which has been the focus of many theories [5,9]. Experimentally it is established that, for a fairly wide range of doping, $d\rho_s/dT$ is nearly doping independent at low temperatures [10,11]. In contrast the extrapolated zero temperature superfluid density changes significantly with doping [11]. *In this Letter we work out the DDW theory's prediction for $\rho_s(T,x)$ and demonstrate that it is inconsistent with existing experimental data.*

Even if the quasiparticles are well defined in the underdoped superconducting states, the Mott constraint can substantially modify the results of the free, mean-field theory. Therefore along the mean-field prediction we also present the results of the "projected DDW model" where the electron occupation constraint is taken into account. More specifically, the no-double-occupancy constraint is implemented by introducing the slave bosons (holons) plus the gauge fields which couple to both bosons and fermions (spinons). The strict occupation constraint is reflected in the absence of the Maxwell term for the gauge fields, i.e., the coupling constant is infinity. In a recent paper, one of us looked into such a gauge theory where the underlying mean-field vacuum is the d-wave resonant-valance-bond

(RVB) state of Kotliar and Liu [12]. It was shown that the gauge field can be integrated out exactly in the continuum approximation of the lattice theory [13,14]. This continuum theory describes the (correlated) density and current fluctuations of holons and spinons above the length scale of the interholon distance λ_h. The physics below such a length scale is entirely summarized by a few parameters in the effective action.

In the following treatment of the projected DDW model we follow the same path taken in Ref. [13] while replacing the Kotliar-Liu RVB mean-field vacuum by the CLMN mean-field vacuum in the spinon sector. We expect the program carried out in Ref. [13] to work well in the presence of DDW order, because the DDW metal—a doped band insulator—has very little spinon density and current fluctuations below the length scale λ_h. The fluctuations above this length scale are already captured by the analysis of Ref. [13].

Our results are as follows: For $x_l < x < x_u$ and at low temperatures, the mean-field DDW theory predicts a superfluid density that behaves as

$$\rho_s(T,x) = \rho_{\rm DDW}(0,x) - \alpha_{\rm DDW}(x)T\,. \qquad (1)$$

Results for $\rho_{\rm DDW}(0,x)$ are shown in the main panel of Fig. 1(a). Note that the zero-temperature superfluid density is nonzero at $x = x_l, x_u$. An infinitesimal temperature will, however, destroy superfluidity because the pairing gap vanishes there. The temperature gradient of ρ_s is given by $\alpha_{\rm DDW}(x) \sim t/\Delta(x)$, where t is the hopping integral and $\Delta(x)$ is the maximum d-wave superconducting gap. The x dependence of $\alpha_{\rm DDW}$ is shown in the inset of Fig. 1(a). *Such strong doping dependence is not seen experimentally.*

The projected DDW model predicts

$$\rho_s(T,x) = z_j(x)\rho_{\rm DDW}(0,x) - z_j(x)^2\alpha_{\rm DDW}(x)T\,, \qquad (2)$$

where $z_j(x)$ is the current renormalization factor to be explained later. The doping dependence of $z_j(x)\rho_{\rm DDW}(0,x)$

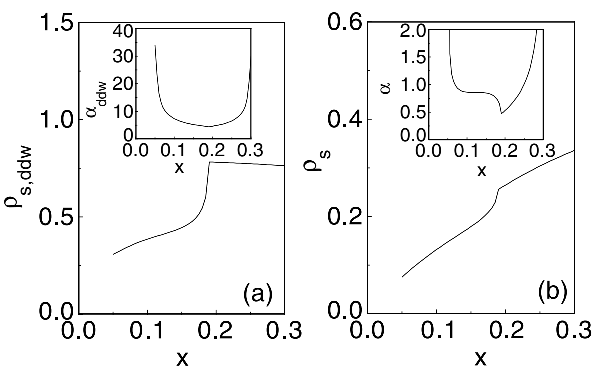

FIG. 1. (a) The doping dependence of $\rho_{s,\rm DDW}(0,x)$ (main panel) and $\alpha_{\rm DDW}(x)$ (inset). (b) The doping dependence of $\rho_s(0,x) = z_j(x)\rho_{\rm DDW}(0,x)$ and $\alpha(x) = z_j(x)^2\alpha_{\rm DDW}(x)$ using the same notation as in (a). The parameters used are $\Delta_0 = 0.2t$, $D_0 = t$, and $t_b = 2t$.

(main panel) and $z_j(x)^2\alpha_{\rm DDW}(x)$ (inset) are shown in Fig. 1(b). Unlike the mean-field result, $d\rho_s/dT$ has a sharp variation near x_c which originates from the kink in the mean-field superfluid density $\rho_{\rm DDW}(x,0)$.

Let us compare the above theoretical results with existing data on $d\rho_s(T,x)/dT$. In the oxygen-depleted $\rm YBa_2Cu_3O_y$ thin films reported in Ref. [11], the transition temperature T_c ranges from 90 K at the optimal doping to 38 K on the underdoped side. Meanwhile, $|d\rho_s/dT|$ decreases by about 15% from its *maximal* value at optimal doping. In contrast, the mean-field DDW theory predicts an increase of $|d\rho_s/dT|$ by 140%, provided T_c scales with $\Delta_0(x)$. After projection, the dependence of $|d\rho_s/dT|$ on x is about 100%. It should be pointed out that in the DDW theory $|d\rho_s/dT|$ is not universal and is subject to quantitative change if the doping and temperature dependences of the pairing amplitude are different from what we assumed in this Letter. There is no question that the experimental data happen to span the plateau regime in Fig. 1(b) (inset). In this case, the present theory predicts a rapid increase of $|d\rho_s/dT|$ when the doping level is decreased even further. In the following we report the details of the calculations.

The mean-field DDW theory.—Following CLMN, we adopt the following mean-field Hamiltonian:

$$H_{\rm DDW} = \sum_{k\sigma}(X_k - \mu)c^\dagger_{k\sigma}c_{k\sigma} + iD_k c^\dagger_{k+Q\sigma}c_{k\sigma} - \sum_k \Delta_k(c_{k\uparrow}c_{-k\downarrow} + \text{H.c.})\,. \qquad (3)$$

Here $Q = (\pi,\pi)$, and $X_k = -2t(\cos k_x + \cos k_y)$ is the nearest-neighbor tight-binding dispersion relation, $D_k = D(x)(\cos k_x - \cos k_y)$ is the momentum space DDW order parameter, and $\Delta_k = \Delta(x)(\cos k_x - \cos k_y)$ is the d-wave superconducting gap function. In the above and in the rest of the Letter we assume that the lattice constant is unity.

Because of the breaking of translation symmetry by the DDW order, the Brillouin zone is half of its original size. For $\Delta(x) = 0$ there are two bands given by $\varepsilon_{k\pm} = \pm\sqrt{X_k^2 + D_k^2} - \mu$ [15]. In the presence of pairing, the quasiparticle dispersion becomes $E_{k\pm} = \sqrt{\varepsilon_{k\pm}^2 + |\Delta_k|^2}$.

In order to study the doping dependence of the superfluid density it is necessary to specify the dependence of $D(x)$ and $\Delta(x)$ on x. Following CLMN, we use the solution of $\partial E(D,\Delta)/\partial D = 0$ and $\partial E(D,\Delta)/\partial\Delta = 0$ to parametrize $D(x)$ and $\Delta(x)$ where

$$E(D,\Delta) = a_D(x - x_d)D^2 + a_\Delta(x - x_u)\Delta^2 + b_D D^4 + b_\Delta\Delta^4 + wD^2\Delta^2. \qquad (4)$$

Here a_D, a_Δ, b_D, b_Δ, and w are (positive) material-dependent constants, while x_d (x_u) is the doping level below which the DDW (SC) order parameter becomes nonzero in the absence of the other. In the presence of pairing the onset of DDW happens at $x_c < x_d$, while the SC order parameter begins its decline precisely at x_c and vanishes at x_l. The functional form for the order parameters are given by

$$D(x) = D_0\sqrt{(x_c - x)/x_c}\,,$$
$$\Delta(x) = \Delta_0\sqrt{(x - x_l)/(x_c - x_l)}; \qquad x_l < x < x_c\,, \quad (5)$$
$$\Delta(x) = \Delta_0\sqrt{(x_u - x)/(x_u - x_c)}; \qquad x_c < x < x_u\,.$$

Here D_0 (Δ_0) is the overall scale of D (Δ). In the following we set $x_l = 0.05$, $x_c = 0.19$, $x_d = 0.2$, and $x_u = 0.3$.

The superfluid density measures the free energy increase caused by the phase twist of the superconducting order parameter [16]. After some lengthy but straightforward algebra we obtain

$$\rho_s^{ab} = \frac{1}{L^2}\sum_{k,\nu=\pm}\left(1 - \frac{\varepsilon_{k\nu}}{E_{k\nu}}\tanh\frac{\beta E_{k\nu}}{2}\right)\partial_a\partial_b\varepsilon_{k\nu} - \frac{\beta}{2L^2}\sum_{k,\nu=\pm}\partial_a\varepsilon_{k\nu}\partial_b\varepsilon_{k\nu}\,\mathrm{sech}^2\frac{\beta E_{k\nu}}{2} + \frac{4}{L^2}\sum_{k,\nu=\pm}\frac{\nu|\Delta_k|^2}{E_{k+}^2 - E_{k-}^2}\frac{\tanh(\beta E_{k\nu}/2)}{E_{k\nu}}\frac{(X_k\partial_a D_k - D_k\partial_a X_k)(X_k\partial_b D_k - D_k\partial_b X_k)}{X_k^2 + D_k^2}\,, \qquad (6)$$

where ∂_a denotes the momentum derivative in the a direction and L is the linear dimension of the system. By symmetry, the tensor is diagonal and direction independent. The last term is nonzero only if the DDW and superconducting order coexist. It is a consequence of the time-reversal symmetry breaking. A similar expression, without the last term, has been derived in Ref. [15]. For the entire doping range considered, the third term has a negligible contribution to the superfluid density compared to others. For self-consistency, one can check that the superfluid density vanishes in the absence of pairing due to the fact that (i) the third term vanishes identically when $\Delta(x) = 0$. (ii) The first two terms combine to give $\sum_{k\nu}\partial_a\{[1 - \tanh(\beta\varepsilon_{k\nu}/2)]\partial_b\varepsilon_{k\nu}\} = 0$. At temperatures much smaller than the maximum superconducting gap, the suppression of superfluid density comes from the thermal excitation of nodal quasiparticles. The second term in Eq. (6) gives a T-linear suppression, while all the other terms contribute higher order temperature corrections. As a result, the superfluid density takes the form of Eq. (1).

We find $\alpha_{\rm DDW}(x) = (4\ln 2)t/\pi\Delta(x)$, independent of the DDW order parameter. As to $\rho_{\rm DDW}(0,x)$ in Eq. (1), we were able to compute it only numerically, and the result is shown in Fig. 1(a).

The general trend of the doping dependence of $\rho_{\rm DDW}(0,x)$ shown in Fig. 1(a) is consistent with experimental data. However, the same cannot be said about $\alpha_{\rm DDW}(x)$. Because of the competition between DDW order and superconductivity in the DDW theory, $\Delta(x)$ is suppressed by the emergence of the DDW order, forming a dome. Thus $\Delta(x) \to 0$ as $x \to x_l, x_u$ which implies that $\alpha_{\rm DDW}(x)$ diverges as $x \to x_l, x_u$. Such x-dependent $d\rho_s/dT$ has not been observed experimentally.

The projected DDW model.—The lattice action for the projected DDW model is given by $L = L_b + L_{\rm DDW} - i\sum_i a_{0i}$, where

$$L_b = \sum_i[\bar{b}_i(\partial_0 + ia_{i0} - iA_{i0} - \mu_b)b_i] - t_b\sum_{\langle ij\rangle}[e^{i(a_{ij}-A_{ij})}\bar{b}_i b_j + \text{H.c.}] + U_b\sum_{\langle ij\rangle}\bar{b}_i b_i\bar{b}_j b_j\,,$$
$$L_{\rm DDW} = \sum_i[\bar{f}_{i\sigma}(\partial_0 + ia_{i0} - \mu_f)f_{i\sigma}] - \sum_{\langle ij\rangle}[(t + iD_{ij})e^{ia_{ij}}\bar{f}_{i\sigma}f_{j\sigma} + \text{H.c.}] - \sum_{\langle ij\rangle}[\Delta_{ij}e^{i\phi_{ij}}\epsilon_{\sigma\sigma'}\bar{f}_{i\sigma}\bar{f}_{j\sigma'} + \text{H.c.}]. \qquad (7)$$

In the above b_i and $f_{i\sigma}$ are the holon and spinon fields, respectively. With ϕ_{ij} set to zero, $L_{\rm DDW}$ is the real space equivalent of Eq. (3). In Eq. (7) a_μ is the gauge field that enforces the constraint $b_i^\dagger b_i + f_{i\sigma}^\dagger f_{i\sigma} = 1$. The DDW mean-field theory describes a doped band insulator; consequently, spinon density fluctuations occur only above the length scale λ_h. Obviously the same is true for the holon density fluctuation. As a result it should be adequate to project out the spinon and holon density fluctuations above the length scale λ_h.

The effective action above such cutoff length is

$$\mathcal{L} = \mathcal{L}_b + \mathcal{L}_{fp} + \mathcal{L}_{\rm Dirac} + \mathcal{L}_j\,,$$
$$\mathcal{L}_b = \frac{K_b}{2}|\phi_b^*(\nabla + i\mathbf{a} - i\mathbf{A})\phi_b|^2 + \frac{U_b}{2}\delta\rho_b^2,$$
$$\mathcal{L}_{sp} = \frac{K_{sp}}{2}|\phi_{sp}^*(\nabla + 2i\mathbf{a})\phi_{sp}|^2 + \frac{1}{2U_{sp}}(\phi_{sp}^*\partial_0\phi_{sp} + 2ia_0)^2, \qquad (8)$$
$$\mathcal{L}_j = i\delta\rho_b(\phi_b^*\partial_0\phi_b + ia_0 - iA_0) + J_\mu^{qp}(\phi_{sp}^*\partial_\mu\phi_{sp} + 2ia_\mu) + \bar{\rho}\left(\phi_b^*\partial_0\phi_b - \frac{1}{2}\phi_{sp}^*\partial_0\phi_{sp}\right) - i\bar{\rho}A_0 - i\mathbf{j}_0\cdot\mathbf{A}\,.$$

Here $\bar{\rho}$ is the doping density, $\mathbf{j}_0$ is the transverse ground-state current produced by the DDW order, and ϕ_b and ϕ_{sp} are the U(1) phase factors associated with the holon field and the spinon pair field, respectively. In addition, $\mathcal{L}_{\rm Dirac}$ is the Dirac action for the spinon quasiparticles near the d-wave gap nodes, and $J^{qp} = \frac{1}{2}(\sum_n\psi_{n\sigma}^\dagger\tau_z\psi_{n\sigma}, iv_F\psi_{1\sigma}^\dagger\psi_{1\sigma}, iv_F\psi_{2\sigma}^\dagger\psi_{2\sigma})$ is their 3-current (τ_z is the third component of the Pauli matrices, and $\psi_{n\sigma}$ is the spinon Nambu spinor associated with the nth d-wave gap node). $K_b = t_b x$ is the holon superfluid density, and $4K_{sp}$ is the spinon zero-temperature superfluid density given by Eq. (1). U_b and U_{sp} also depend on the parameters in Eq. (7); however, their values are not important for the following discussion.

Given Eq. (8) the gauge field a_μ can be integrated out straightforwardly to yield the final effective action of a correlated DDW superconductor:

$$\mathcal{L} = \frac{K}{2}|\phi^*\nabla\phi - 2i\mathbf{A}|^2 + \frac{1}{2U}(\phi^*\partial_0\phi - 2iA_0)^2 - z_j\mathbf{J}^{qp}\cdot(\phi^*\nabla\phi - 2i\mathbf{A}) - z_\rho\rho^{qp}(\phi^*\partial_0\phi - 2iA_0) + \frac{\bar{\rho}}{2}\phi^*\partial_0\phi - i\bar{\rho}A_0 - i\mathbf{j}_0\cdot\mathbf{A} + \mathcal{L}'_{\text{Dirac}}. \quad (9)$$

In the above $\phi \equiv \phi_{sp}^*\phi_b^2$, and

$$\begin{aligned} K &\equiv K_{sp}K_b/(K_b + 4K_{sp}), \\ U &\equiv U_{sp} + 4U_b, \\ z_j &\equiv K_b/(K_b + 4K_{sp}), \\ z_\rho &\equiv U_{sp}/(U_{sp} + 4U_b), \end{aligned} \quad (10)$$

and $\mathcal{L}'_{\text{Dirac}} \equiv \mathcal{L}_{\text{Dirac}} + 2|\mathbf{J}^{qp}|^2/(K_b + 4K_{sp}) + 2(\rho^{qp})^2/(U_b^{-1} + 4U_{sp}^{-1})$. Combining the above result with Eq. (1) we obtain Eq. (2) as the superfluid density prediction of the projected DDW theory where

$$z_j(x) = t_b x/[t_b x + \rho_{\text{DDW}}(0,x)]. \quad (11)$$

In Fig. 1(b) we plot the x dependence of the zero-temperature superfluid density in the main panel and $d\rho_s/dT$ in the inset. For $x > x_l$ the zero-temperature superfluid density varies with x in roughly linear fashion. As in the mean-field DDW theory, $d\rho_s/dT$ has a strong x dependence.

We emphasize that the general conclusion reached in this Letter is insensitive to the specific parametrization used in Eq. (5), as long as the Fermi pockets exist in the normal state. One might argue that the divergence of $|d\rho_s/dT|$ could be avoided by keeping $\Delta(x)$ finite but invoking, e.g., the phase fluctuation, which however lies outside the underlying premises of the CLMN proposal. One might also argue that by pinning the chemical potential at zero for all doping, one can avoid the divergence of $d\rho_s/dT$, because in that case $|d\rho_s/dT|_{T\to 0} = a(2t/D + D/2t)$ to leading order in Δ/D, where a is of order unity. However, this pinning removes an attractive feature of the DDW theory—the presence of Fermi pockets in the normal state, without which the DDW scenario can no longer account for the kink in the specific heat jump at 19% doping and the Fermi arc in angle-resolved photoemission spectroscopy. Finally in comparing with experiment it is important to bear in mind that several mechanisms can cause the temperature dependence of the penetration depth to become T^2 at very low temperatures [17,18].

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