Quantum Enhancement of Momentum Diffusion in the Delta-Kicked Rotor

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We present detailed observations of the quantum delta-kicked rotor in the vicinity of a quantum resonance. Our experiment consists of an ensemble of cold cesium atoms subject to a pulsed off-resonant standing wave of light. We measure the mean energy and show clearly that at the quantum resonance it is a local maximum. We also examine the effect of noise on the system and find that the greatest sensitivity to this occurs at the resonances. This makes these regions ideal for examining quantum-classical correspondence. A picture based on diffraction is developed which allows the experiments to be readily understood.

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The delta-kicked rotor is one of the most extensively investigated systems in the field of classical chaos theory [1]. It consists of a particle which is periodically given kicks whose size and direction depend upon the position of the particle. Such a system displays a wealth of phenomena ranging from regular motion to the anomalous diffusion associated with accelerator modes. It is also extremely interesting from the perspective of quantum chaos because of the simplicity of the equations of motion, the existence of a readily obtainable experimental realization [2,3], and the fact that it is a promising candidate for investigating quantum-classical correspondence. In the quantum case the classical particle is replaced with a de Broglie wave and the delta kicks are provided by a pulsed corrugated potential.

Perhaps one of the most striking aspects of the quantum delta-kicked rotor's (QDKR) behavior is the existence of quantum resonances [4]. The primary quantum resonances occur for kicking intervals which allow plane waves with certain initial momenta in the kicking direction to experience no overall evolution in the time between the kicks. The momentum distribution of the wave function under these conditions should exhibit ballistic growth in its width as the number of kicking pulses is increased. Contrary to expectation, the first experiments on the QDKR near to a quantum resonance showed no ballistic growth in the momentum [2]. Although subsequent work [5] has demonstrated that this observation was due to a lack of temporal resolution, there still remains much to be done to fully understand this resonant behavior.

In this Letter we experimentally examine the behavior of the QDKR in the vicinity of the quantum resonances using an ensemble of laser cooled cesium atoms exposed to a pulsed off-resonant standing wave of light [6]. Because of a gradient in the potential formed by the ac-stark shift, the atoms experience kicks of a size which depends on their position in the standing wave. Figure 1 shows momentum distributions of the ensemble measured using a time-of-flight technique for different pulse separations. Note how the distributions become much wider for a small range of times around the quantum resonances at 66, 133, and 200 μs. These features are also well resolved in the mean kinetic energy (henceforth referred to as the mean energy) plots of Fig. 2(a).

In addition we subject the system to a source of noise (in effect a "shaking" of the potential) so that we can investigate what happens when we move towards a more classical regime [3]. Figure 2 shows the mean energy as a function of the time between pulses when we introduce this shaking by causing the atoms to spontaneously emit a photon with various degrees of probability. In this situation we find that the mean energy at the quantum resonances increases significantly even for small amounts of spontaneous emission. On the other hand, at pulse intervals slightly above and below the resonance, the mean energy increases at a much slower rate until the mean number of added spontaneous emissions per pulse is greater than 0.2. As we

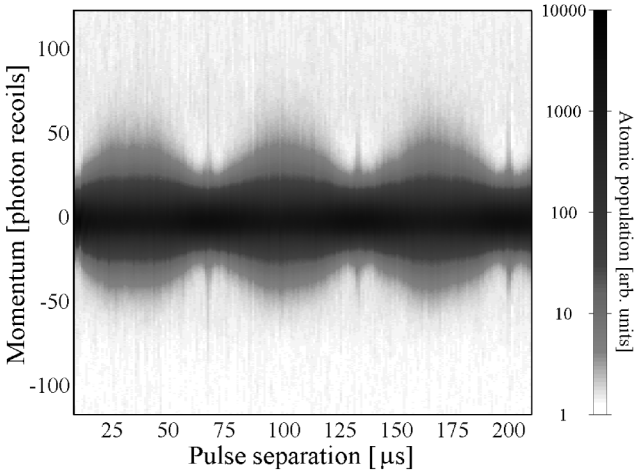

FIG. 1. Experimental momentum distributions of an ensemble of cold cesium atoms subjected to 30 kicks from an off-resonant standing light wave. The horizontal axis represents the time between the kicks. Note the sharp resonances seen at 66, 133, and 200 μs.

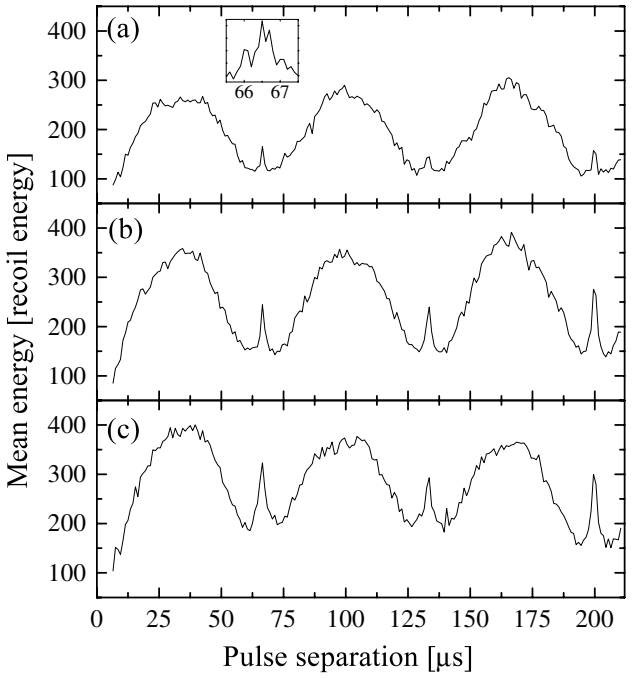

FIG. 2. Experimental data of the mean energy as a function of the time between pulses, T, after 30 pulses. Graphs (a), (b), and (c) have a mean number of added spontaneous emissions per pulse of 0, 0.1, and 0.2, respectively, and a phase modulation depth of $\phi_d \simeq \pi$. Note the periodicity of the mean energy and the dips with central sharp peaks at 66, 133, and 200 μs. These are the quantum resonances. Inset of (a) shows a higher resolution scan around the resonance at 66 μs.

will show, the pulse intervals at which the wave function is most sensitive to the shaking correspond to where the wave function is the most delocalized, that is, in a sense the most quantum.

The pulsed standing wave of off-resonant laser light used to create the corrugated potential had the form $U(x,t) = \frac{U_{\max}}{2}[1 + \cos(Gx)]\sum_{N_p=1}^{N} F(t - N_p T)$, where x is the position in the direction of the standing wave, N is the total number of pulses, $G = 2k_L$ (k_L is the light wave vector), and $U_{\max} = \frac{\hbar\Omega^2}{4\Delta}$ is the modulation depth of the potential (Ω is the Rabi frequency of the transition, Δ is the detuning from resonance). The summation denotes a series of rectangular pulses F, of duration t_p, separated in time by T. We define the depth of the phase modulation induced by the potential after one pulse as $\phi_d = \frac{U_{\max} t_p}{2\hbar}$.

One picture which can be employed to understand the structures of Fig. 2 is that of diffraction. The interaction of a de Broglie wave with a periodic potential is equivalent to diffraction from a phase grating. Since the pulse length is very short, the evolution of the atomic wave function during the interaction is negligible and we are in the Raman-Nath regime of a thin grating. For an initial plane de Broglie wave the amplitude for diffraction into the nth order $\exp[i\frac{p_n}{\hbar}x]$ is $(-i)^n J_n(\phi_d)$, where $p_n = p_i + n\hbar G$ and p_i is the momentum of the initial plane wave along the standing wave. We can represent the standing wave interaction and the free propagation during time t by the unitary transformations $U_{mn}^{\text{int}} = (-i)^{n-m} J_{n-m}(\phi_d)\,|m\rangle\langle n|$ and $U_{mn}^{\text{free}} = \exp[i\phi_n^{\text{free}}(t)]\delta_{mn}|m\rangle\langle n|$, where m and n are the final and initial diffraction orders, and $\phi_n^{\text{free}}(t) = \frac{1}{\hbar}\frac{p_n^2}{2M}t$ (M is the mass of the atom). These two transformations applied in the appropriate sequence enable us to model numerically the effect of any number of pulses on a particular incident atomic plane wave. In order to find the overall momentum distribution, this calculation is performed for each of the plane waves present in the initial distribution. The final result is the weighted sum of all the output momentum distributions.

One particularly important quantity is the difference in the free propagation phase acquired by the nth and $(n-1)$th diffraction orders between consecutive kicks, $\phi_n^{\text{free}}(T) - \phi_{n-1}^{\text{free}}(T) = [(2n-1) + 2\frac{p_i}{\hbar G}]\frac{\hbar G^2}{2M}T$ [6]. When the nonbracketed component of this equation is equal to 2π, the time between the pulses is of special significance. This is the Talbot time and is given by $T_{\text{Talbot}} = \pi\frac{4M}{\hbar G^2}$, which for cesium is 133 μs. Quantum resonances will occur whenever it is possible for the above phase difference to be a multiple of 2π. At the Talbot time this condition can be fulfilled by all momenta which are a multiple or half-multiple of $\hbar G$. At $\frac{T_{\text{Talbot}}}{2}$ it is only half-multiples of $\hbar G$ which will work. For these special combinations of time and momentum the free evolution leaves the wave function unchanged. Thus N standing wave pulses have the effect of one large pulse with a phase modulation depth of $N\phi_d$. The properties of the Bessel functions imply that the diffraction orders populated with the greatest probability have a momentum $\pm N\phi_d\hbar G$, so that the mean energy is proportional to N^2. The last observation is the one which is most often associated with a quantum resonance [4] and explains the sharp peaks at 66, 133, and 200 μs in Fig. 2(a). This is quantum enhancement of momentum diffusion caused by the relatively small fraction of atoms which have one of the special momenta mentioned above.

Why do these peaks grow relative to their surroundings as the amount of spontaneous emission is increased? To answer this question we consider the case where the pulse interval is T_{Talbot} (note that similar arguments could be applied to any resonance time). We also take the initial momentum to have the form $p_i = r/q\hbar G$, where r and q are integers. The evolution is clearer if we move to the reference frame in which the grating is moving with velocity $v_{\text{grat}} = -\frac{p_i}{M}$. Thus we can ignore the free evolution of the wave field and instead take the potential to have been spatially offset by an amount $-\frac{p_i}{M}T$ during the time between the pulses. After q pulses the sum of the offset standing wave pulses has no spatial dependence and hence can produce no diffraction [7]. In other words after q pulses the atomic de Broglie wave is just the input wave to within some constant phase factor. Thus, an incident plane wave with momentum $r/q\hbar G$ will have an oscillatory mean energy with minima every q pulses. Since any incident momentum can be approximated to a number with such a

rational form we expect that mean energy growth should be dramatically inhibited by these oscillations. Increasing the amount of spontaneous emission will destroy the special rephasing conditions which allow the mean energy to oscillate. Thus even a small amount of spontaneous emission should produce a significant increase in mean energy.

The other notable feature of the mean energy plots of Fig. 2 is the way in which the minima on either side of the quantum resonance peak show only small increases in mean energy as spontaneous emission is applied. This can be explained by considering Fig. 3, the theoretical wave field plot after one pulse for an incident plane wave with no momentum parallel to the standing wave. It can be seen that at the times corresponding to each quantum resonance the amplitude has no spatial variation (there is only phase modulation), while to either side the wave field is periodically localized. For plane waves with other incident momenta the whole wave field will be offset by an amount $\frac{p_i}{M}t$, where t is the time since the light pulse. In this general situation, the region of localization will no longer be where the standing wave gradient is zero, however due to the localization of the wave packet it effectively experiences a single value of the standing wave gradient. When the next pulse is applied it will be deflected in a way which allows it to largely maintain its form. If a spontaneous emission occurs, this offsets the wave function to a point where there is a different phase gradient. Thus although the wave packet will not be deflected by the same amount at the next pulse, it will keep the same shape and should exhibit the diffusive behavior which is more characteristic of a classical particle.

Figure 4 shows experimental and theoretical mean energy plots for a whole ensemble of initial momenta as a function of the number of pulses with no additional spontaneous emission. Several significant features can be identified. First, dynamical localization [8], where the mean energy initially increases linearly and then saturates, is observed at $T = 61$ μs, the time where the deep minima near the quantum resonance of Fig. 2 exist. At $T = 20$ μs, dynamical localization should also occur, although this will take many more pulses than we were able to apply in the experiment.

Interestingly the 66 μs case, at one of the quantum resonances, undergoes a type of damped oscillation. As might be expected from earlier discussions, this shape is characteristic of a function which consists of many oscillating curves all with different frequencies. Although there is reasonable qualitative agreement between the theory and the experiment, there are still significant differences. This is especially true at 66 μs, where the discrepancies are caused primarily by limits on signal to noise. Here a large part of the mean energy is the result of the small amount of population at high momentum. With a limited signal-to-noise ratio it is not possible to detect all of this population and consequently the mean energy determined by the experiment is lower.

In Fig. 5 the effect of small amounts of added spontaneous emission is examined. This additional noise greatly modifies the behavior at the quantum resonance of $T = 66$ μs, yet has a much smaller effect on the 61 μs data

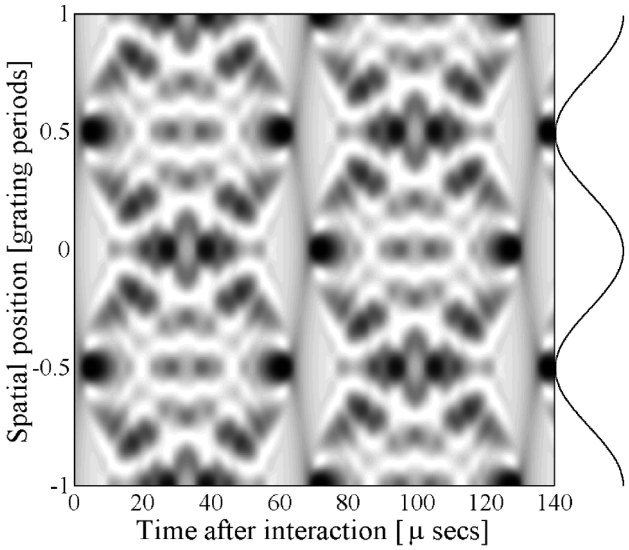

FIG. 3. The theoretical atomic wave field probability (probability proportional to darkness of shading) as a function of time after one pulse of a periodic potential with $\phi_d = \pi$. The incident wave packet was a plane wave with no momentum parallel to the standing light wave. Note the lack of any amplitude variation where the quantum resonances are located at 0, 66, and 133 μs. On either side of these regions a pronounced localization of the wave function occurs. The position of the standing light wave is also plotted.

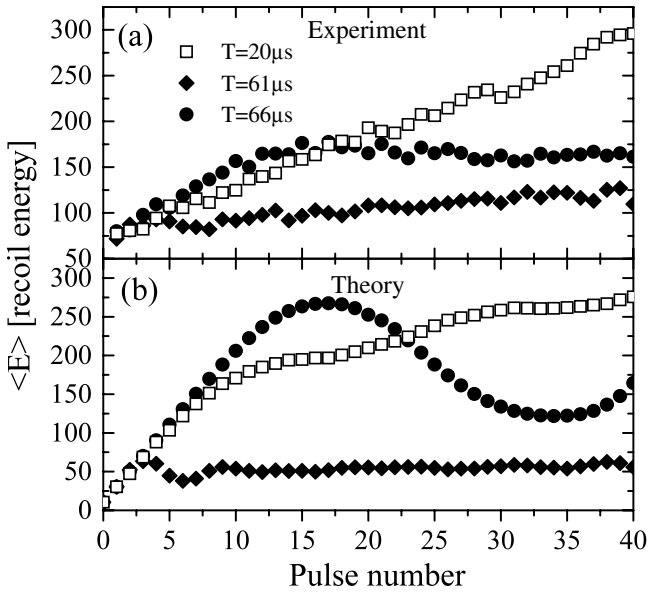

FIG. 4. The experimental (a) and the theoretical (b) mean energy of the ensemble as a function of the number of standing wave pulses with negligible spontaneous emission. Strength of the potential was $\phi_d = \pi$, and the data in each of the curves were obtained with a different value of the pulse repetition time T.

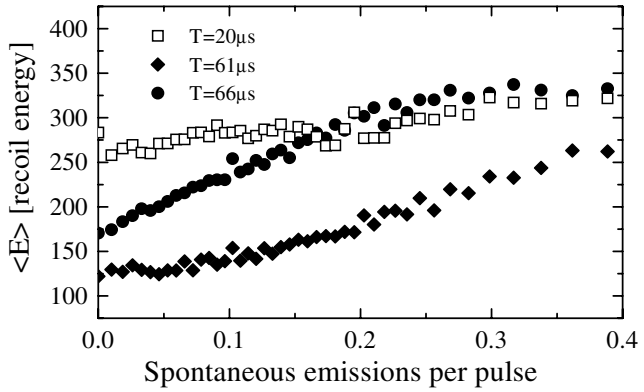

FIG. 5. The experimentally determined mean energy of the ensemble after 30 pulses as a function of the amount of added spontaneous emission. Strength of the potential was $\phi_d \simeq \pi$, and each of the curves shows data for different values of the pulse repetition time T.

until more than 0.2 added spontaneous emissions per kick are present. This is consistent with the picture that at 61 μs the wave field is well localized and consequently relative movements between the atoms and the grating have less of an effect. Conversely, at the resonance the position of the grating is absolutely critical if the rephasing necessary to reduce the mean energy is to occur.

The basis of our experiment has been described elsewhere [6], so here we give only a brief description. Cesium atoms from a magneto-optic trap were cooled in optical molasses and then released to fall under gravity. After a few milliseconds, standing wave pulses from a Ti:sapphire laser were applied to the cold atomic ensemble. The standing wave was orientated vertically, so that there was a net gravitational acceleration between the atoms and the standing wave. This acceleration was removed by an electro-optic phase shifter which was placed between the atomic cloud and the retroreflection mirror used to form the standing wave. After each $t_p = 0.5$ μs pulse of the standing light wave, the voltage on the phase shifter was adjusted in order to compensate for the movement of the atoms caused by gravity. The Ti:sapphire laser was tuned 30 GHz below the D1 $(6S_{1/2}, F = 4) \rightarrow (6P_{1/2}, F = 3)$ transition, with a power in the standing wave of $\sim$120 mW and a beam waist of roughly 1 mm. This allowed us to produce standing waves with $\phi_d \simeq \pi$ having a residual spontaneous emission rate which was approximately 10^{-3} per pulse. The controlled addition of spontaneous emission was achieved by switching on a fraction of the power in four of the molasses beams immediately following each standing wave pulse.

The atomic momentum distribution was determined using a time-of-flight technique. As the atoms fell through a resonant probe laser beam located 50 cm below the trap, the amount of absorption in the beam was measured. Each shot was repeated 3 times so that the overall signal-to-noise ratio for a given momentum distribution was approximately 500:1. This is an extremely important point when considering experiments looking at the mean energy, since noise at high atomic momentum has a disproportionate effect. To ensure that we were not unduly affected by noise in the wings of the momentum distribution it was necessary to filter the time-of-flight measurements by using a Fourier technique.

Our results have shown that quantum resonances are characterized by a local maximum in mean energy. For certain initial momenta there is a critical phase relationship between the momentum states populated by the delta kicks. This leads to enhanced momentum diffusion. When additional spontaneous emission is introduced the mean energy at the resonances grows relative to that at neighboring pulse intervals. Since the introduction of noise makes the system in some sense more classical, the pronounced response to it at the resonances makes them ideal regions for studying quantum-classical correspondence.

Adjacent to the maxima associated with the resonances exist minima which correspond to the pulse interval for which the wave field is periodically localized. In this region the strongly localized wave function sees very little sinusoidal variation in the potential and can only diffuse to a small extent. Even with moderately high levels of spontaneous emission the minima in mean energy adjacent to the quantum resonances are largely unaffected. This lack of sensitivity to processes which destroy coherence can be interpreted as being due to the fact that the wave field has a well-localized form which in some respects resembles a classical particle.

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