

## Test of Nuclear Wave Functions for Pseudospin Symmetry

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Using the fact that pseudospin is an approximate symmetry of the Dirac Hamiltonian with realistic scalar and vector mean fields, we derive the wave functions of the pseudospin partners of eigenstates of a realistic Dirac Hamiltonian and compare these wave functions with the wave functions of the Dirac eigenstates.

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Pseudospin doublets were introduced more than thirty years ago into nuclear physics to accommodate an observed near degeneracy of certain normal-parity shell-model orbitals with nonrelativistic quantum numbers  $(n_r, \ell, j = \ell + 1/2)$  and  $(n_r - 1, \ell + 2, j = \ell + 3/2)$  where  $n_r$ ,  $\ell$ , and  $j$  are the single-nucleon radial, orbital, and total angular momentum quantum numbers, respectively [1,2]. The doublet structure,  $j = \tilde{\ell} \pm \tilde{s}$ , is expressed in terms of a “pseudo” orbital angular momentum  $\tilde{\ell} = \ell + 1$  coupled to a “pseudo” spin,  $\tilde{s} = 1/2$ . This pseudospin “symmetry” has been used to explain features of deformed nuclei [3], including superdeformation [4] and identical bands [5,6]. Although the observed reduction in pseudo spin-orbit splitting follows from nuclear relativistic mean fields [7], only recently has the pseudospin “symmetry” been shown to arise from a relativistic symmetry of the Dirac Hamiltonian [8,9].

The Dirac Hamiltonian,  $H$ , with an external scalar,  $V_S$ , and vector,  $V_V$ , potentials is invariant under a SU(2) algebra for  $V_S = V_V + \text{const}$ , leading to pseudospin symmetry in nuclei [9]. The pseudospin generators,  $\hat{S}_\mu$ , which satisfy  $[H, \hat{S}_\mu] = 0$  in the symmetry limit, are given by

$$\hat{S}_\mu = \begin{pmatrix} \hat{s}_\mu & 0 \\ 0 & \hat{s}_\mu \end{pmatrix} = \begin{pmatrix} U_p \hat{s}_\mu U_p & 0 \\ 0 & \hat{s}_\mu \end{pmatrix}, \quad (1)$$

where  $\hat{s}_\mu = \sigma_\mu/2$  are the usual spin generators,  $\sigma_\mu$  are the Pauli matrices, and  $U_p = \frac{\sigma \cdot \mathbf{p}}{p}$  is the momentum-helicity unitary operator introduced in [7]. If, in addition, the potentials are spherically symmetric,  $V_{S,V}(\mathbf{r}) = V_{S,V}(r)$ , the Dirac Hamiltonian has an additional invariant SU(2) algebra,  $[H, \hat{L}_\mu] = 0$ , with the pseudo-orbital angular momentum operators given by  $\hat{L}_\mu = \begin{pmatrix} \hat{\ell}_\mu & 0 \\ 0 & \hat{\ell}_\mu \end{pmatrix}$ . Here  $\hat{\ell}_\mu = U_p \hat{\ell}_\mu U_p$ ,  $\hat{\ell}_\mu = \mathbf{r} \times \mathbf{p}$ , while  $\hat{j}_\mu = \hat{\ell}_\mu + \hat{s}_\mu = U_p(\hat{\ell}_\mu + \hat{s}_\mu)U_p = \hat{\ell}_\mu + \hat{s}_\mu$ . The eigenfunctions of the Dirac Hamiltonian are also eigenfunctions of the Casimir operator of this algebra,  $\hat{L} \cdot \hat{L} |\tilde{n}_r, \tilde{\ell}, j, m\rangle = \tilde{\ell}(\tilde{\ell} + 1) |\tilde{n}_r, \tilde{\ell}, j, m\rangle$ , where we have used a coupled basis,  $\mathbf{j} = \tilde{\ell} + \tilde{s}$ , and set  $\hbar = c = 1$ . Here  $j$  is the eigenvalue of the total angular momentum operator  $\hat{J}_\mu = \hat{L}_\mu + \hat{S}_\mu$ ,  $\hat{J} \cdot \hat{J} |\tilde{n}_r, \tilde{\ell}, j, m\rangle =$

$j(j + 1) |\tilde{n}_r, \tilde{\ell}, j, m\rangle$ ,  $m$  is the eigenvalue of  $\hat{J}_z$ , and  $\tilde{n}_r$  is the pseudoradial quantum number which we define below.

In the pseudospin symmetry limit, the eigenstates of the Dirac Hamiltonian in the doublet  $j = \tilde{\ell} \pm 1/2$  are degenerate, and are connected by the pseudospin generator  $\hat{S}_\mu$ :

$$\hat{S}_\mu |\tilde{n}_r, \tilde{\ell}, j_i, m_i\rangle = \sum_{j_f, m_f} A_{j_f, m_f, j_i, m_i} |\tilde{n}_r, \tilde{\ell}, j_f, m_f\rangle. \quad (2)$$

Here  $A_{j_f, m_f, j_i, m_i} = (-1)^{1/2 - m_f + \tilde{\ell}} \sqrt{\frac{3(2j_i + 1)(2j_f + 1)}{2}} \times \begin{pmatrix} j_f & 1 & j_i \\ -m_f & \mu & m_i \end{pmatrix} \begin{Bmatrix} \frac{1}{2} & \tilde{\ell} & j_i \\ j_f & 1 & \frac{1}{2} \end{Bmatrix}$ , where the symbols are Wigner 3-j and 6-j symbols, respectively. However, in the exact pseudospin limit,  $V_S = -V_V$ , there are no bound Dirac valence states. For nuclei to exist the pseudospin symmetry must therefore be broken. Nevertheless, realistic mean fields involve an attractive scalar potential and a repulsive vector potential of nearly equal magnitudes,  $V_S \sim -V_V$ , and calculations in a variety of nuclei confirm the existence of an approximate pseudospin symmetry in the energy spectra [10–12]. Since pseudospin symmetry is broken, the pseudospin partner produced by the raising and lowering operators acting on an eigenstate will not necessarily be an eigenstate. The question is how different is the pseudospin partner from the eigenstate with the same quantum numbers? As noted, energy splittings suggest that the breaking of pseudospin symmetry is small, but is the breaking in the eigenfunctions small as well? While previous studies have compared the lower components of the Dirac wave functions of the two states in the doublet [10–12], it is the behavior of the upper components which is of most interest since they dominate the Dirac eigenstates. The relativistic pseudospin symmetry has unique and interesting features in the following sense. First, the pseudospin generators of Eq. (1) intertwine space and spin, and thus lead to an uncommon symmetry structure of doublets with different radial wave functions. Second, since bound Dirac valence states do not exist in the symmetry limit, the pseudospin properties of realistic wave functions can not be determined by perturbation theory. These aspects motivate this paper.

To determine the pseudospin partners we need to expand the Dirac eigenfunction into a spherical basis,

$$\begin{aligned} \langle \mathbf{r} | \tilde{n}_r, \tilde{\ell}, j = \tilde{\ell} + \frac{1}{2}, m \rangle &= \{ g_{\tilde{n}_r-1, \tilde{\ell}, j}(r) [Y^{(\tilde{\ell}+1)}(\hat{r}) \chi_m^{(j)}], \\ &\quad if_{\tilde{n}_r, \tilde{\ell}, j}(r) [Y^{(\tilde{\ell})}(\hat{r}) \chi_m^{(j)}] \} \quad (3a) \\ \langle \mathbf{r} | \tilde{n}_r, \tilde{\ell}, j = \tilde{\ell} - \frac{1}{2}, m \rangle &= \{ g_{\tilde{n}_r, \tilde{\ell}, j}(r) [Y^{(\tilde{\ell}-1)}(\hat{r}) \chi_m^{(j)}], \\ &\quad if_{\tilde{n}_r, \tilde{\ell}, j}(r) [Y^{(\tilde{\ell})}(\hat{r}) \chi_m^{(j)}] \}, \quad (3b) \end{aligned}$$

where  $Y_{m_i}^{(\tilde{\ell})}(\hat{r})$  is the spherical harmonic and  $\chi$  is the spin function. From this expansion we see that the pseudoradial quantum number,  $\tilde{n}_r$ , is the radial quantum number of the lower component of the Dirac eigenfunction [13] as well as the radial quantum number of the upper component of the eigenstate with  $j = \tilde{\ell} - 1/2$ . Because the pseudospin generators  $\hat{S}_\mu$  depend on the unit momentum vector  $\hat{p}$ , we transform the eigenfunctions to momentum space in order to calculate the effect of the pseudospin generators on the eigenfunctions:

$$\begin{aligned} \langle \mathbf{p} | \tilde{n}_r, \tilde{\ell}, j = \tilde{\ell} + \frac{1}{2}, m \rangle &= \{ \tilde{g}_{\tilde{n}_r-1, \tilde{\ell}, j}(p) [Y^{(\tilde{\ell}+1)}(\hat{p}) \chi_m^{(j)}], \\ &\quad if_{\tilde{n}_r, \tilde{\ell}, j}(p) [Y^{(\tilde{\ell})}(\hat{p}) \chi_m^{(j)}] \}, \quad (4a) \\ \langle \mathbf{p} | \tilde{n}_r, \tilde{\ell}, j = \tilde{\ell} - \frac{1}{2}, m \rangle &= \{ \tilde{g}_{\tilde{n}_r, \tilde{\ell}, j}(p) [Y^{(\tilde{\ell}-1)}(\hat{p}) \chi_m^{(j)}], \\ &\quad if_{\tilde{n}_r, \tilde{\ell}, j}(p) [Y^{(\tilde{\ell})}(\hat{p}) \chi_m^{(j)}] \}. \quad (4b) \end{aligned}$$

The corresponding spherical Bessel transforms of the radial wave functions are given by

$$\begin{aligned} \tilde{g}_{\tilde{n}_r-1, \tilde{\ell}, j}(p) &= (-i)^{\tilde{\ell}+1} \sqrt{\frac{2}{\pi}} \int_0^\infty j_{\tilde{\ell}+1}(pr) g_{\tilde{n}_r-1, \tilde{\ell}, j}(r) r^2 dr, \\ j &= \tilde{\ell} + \frac{1}{2}, \quad (5a) \end{aligned}$$

$$\begin{aligned} \tilde{g}_{\tilde{n}_r, \tilde{\ell}, j}(p) &= (-i)^{\tilde{\ell}-1} \sqrt{\frac{2}{\pi}} \int_0^\infty j_{\tilde{\ell}-1}(pr) g_{\tilde{n}_r, \tilde{\ell}, j}(r) r^2 dr, \\ j &= \tilde{\ell} - \frac{1}{2}, \quad (5b) \end{aligned}$$

$$\begin{aligned} \tilde{f}_{\tilde{n}_r, \tilde{\ell}, j}(p) &= (-i)^{\tilde{\ell}} \sqrt{\frac{2}{\pi}} \int_0^\infty j_{\tilde{\ell}}(pr) f_{\tilde{n}_r, \tilde{\ell}, j}(r) r^2 dr, \\ j &= \tilde{\ell} \pm \frac{1}{2}. \quad (5c) \end{aligned}$$

We then are able to derive

$$\begin{aligned} \hat{S}_\mu | \tilde{n}_r, \tilde{\ell}, j_i, m_i \rangle &= A_{j_i, m_i, j_i, m_i} | \tilde{n}_r, \tilde{\ell}, j_i, m_i \rangle \\ &\quad + A_{j_f, m_f, j_i, m_i} | \tilde{n}_r, \tilde{\ell}, j_f, m_f \rangle^{psp}. \quad (6) \end{aligned}$$

Here the superscript *psp* on the second term denotes the pseudospin partner with  $j_f \neq j_i$ . Even with pseudospin breaking, the pseudospin generators do not change  $\tilde{\ell}$ . In addition, from Eq. (6) we see that the first term with  $j_f = j_i$  is exactly equal to the original eigenstate, independent of the amount of pseudospin breaking. This follows from the orthogonality of the spherical Bessel functions,  $\frac{2}{\pi} \int_0^\infty p^2 dp j_{\tilde{\ell}}(pr) j_{\tilde{\ell}}(px) = \frac{\delta(r-x)}{r^2}$ . For the partner with  $j_f \neq j_i$ , the wave function in coordinate space reads

$$\begin{aligned} \langle \mathbf{r} | \tilde{n}_r, \tilde{\ell}, j = \tilde{\ell} + \frac{1}{2}, m \rangle^{psp} &= \{ g_{\tilde{n}'_r-1, \tilde{\ell}, j}(r) [Y^{(\tilde{\ell}+1)}(\hat{r}) \chi_m^{(j)}], \\ &\quad if_{\tilde{n}'_r, \tilde{\ell}, j}(r) [Y^{(\tilde{\ell})}(\hat{r}) \chi_m^{(j)}] \}, \quad (7a) \\ \langle \mathbf{r} | \tilde{n}_r, \tilde{\ell}, j = \tilde{\ell} - \frac{1}{2}, m \rangle^{psp} &= \{ g_{\tilde{n}'_r, \tilde{\ell}, j}(r) [Y^{(\tilde{\ell}-1)}(\hat{r}) \chi_m^{(j)}], \\ &\quad if_{\tilde{n}'_r, \tilde{\ell}, j}(r) [Y^{(\tilde{\ell})}(\hat{r}) \chi_m^{(j)}] \}, \quad (7b) \end{aligned}$$

where, in general,  $\tilde{n}'_r$  can differ from  $\tilde{n}_r$  since the states with  $j_f \neq j_i$  in Eq. (6) are not Dirac eigenstates. The expressions for  $g_{\tilde{n}'_r, \tilde{\ell}, j-1}(r)$  with  $j = \tilde{\ell} + \frac{1}{2}$  and  $g_{\tilde{n}'_r-1, \tilde{\ell}, j+1}(r)$  with  $j = \tilde{\ell} - \frac{1}{2}$  involve a double integral  $-\frac{2}{\pi} \int_0^\infty p^2 dp \int_0^\infty x^2 dx$  over  $j_{\tilde{\ell}-1}(pr) j_{\tilde{\ell}+1}(px) \times g_{\tilde{n}_r-1, \tilde{\ell}, j}(x)$  and  $j_{\tilde{\ell}+1}(pr) j_{\tilde{\ell}-1}(px) g_{\tilde{n}_r, \tilde{\ell}, j}(x)$ , respectively. The  $p$  integration can be carried out, and altogether we find [14]

$$\begin{aligned} g_{\tilde{n}'_r, \tilde{\ell}, j-1}(r) &= g_{\tilde{n}_r-1, \tilde{\ell}, j}(r) - (2\ell + 1)r^{\tilde{\ell}-1} \\ &\quad \times \int_r^\infty \frac{dx}{x^{\tilde{\ell}}} g_{\tilde{n}_r-1, \tilde{\ell}, j}(x) \quad j = \tilde{\ell} + \frac{1}{2}, \quad (8a) \end{aligned}$$

$$\begin{aligned} g_{\tilde{n}'_r-1, \tilde{\ell}, j+1}(r) &= g_{\tilde{n}_r, \tilde{\ell}, j}(r) - \frac{(2\tilde{\ell} + 1)}{r^{\tilde{\ell}+2}} \\ &\quad \times \int_0^r dx x^{\tilde{\ell}+1} g_{\tilde{n}_r, \tilde{\ell}, j}(x) \\ j &= \tilde{\ell} - \frac{1}{2}, \quad (8b) \end{aligned}$$

$$f_{\tilde{n}'_r, \tilde{\ell}, j\pm 1}(r) = f_{\tilde{n}_r, \tilde{\ell}, j}(r), \quad j = \tilde{\ell} \pm \frac{1}{2}. \quad (8c)$$

In the pseudospin limit ( $V_S + V_V = \text{const}$ ),

$$| \tilde{n}_r, \tilde{\ell}, j, m \rangle^{psp} = | \tilde{n}_r, \tilde{\ell}, j, m \rangle. \quad (9)$$

Since pseudospin symmetry is slightly broken in nuclei, the pseudospin partner can differ from the Dirac eigenstate and it is of interest to examine the deviations from the condition of Eq. (9). Dirac bound states satisfy  $g_{\tilde{n}_r, \tilde{\ell}, j=\tilde{\ell}\pm 1/2} \sim r^{\tilde{\ell}\pm 1}$  for small  $r$ , and fall off exponentially,  $\sim \exp(-\sqrt{M^2 - E^2} r)$ , for large  $r$  [13], where  $M$  is the nucleon mass and  $E$  is the Dirac eigenenergy. Consequently, as seen from Eq. (8a), for the Dirac eigenstate with  $j = \tilde{\ell} + 1/2$ , its pseudospin partner  $g_{\tilde{n}'_r, \tilde{\ell}, j-1}$  has the expected behavior for small and large  $r$ . On the other hand, as seen from Eq. (8b), for the Dirac eigenstate with  $j = \tilde{\ell} - 1/2$ , its pseudospin partner  $g_{\tilde{n}'_r-1, \tilde{\ell}, j+1} \sim r^{\tilde{\ell}-1}$  for small  $r$ , and falls off as a power law  $r^{-(\tilde{\ell}+2)}$  for large  $r$ . As such it has a behavior which is very different from that of a Dirac bound state with  $j = \tilde{\ell} + 1/2$ . This asymmetry in the behavior of the pseudospin partners of  $j = \tilde{\ell} + 1/2$  or  $j = \tilde{\ell} - 1/2$  Dirac eigenstates is evident in the analysis of nuclear wave functions presented below. These realistic wave functions were obtained in a relativistic point coupling

model, and we refer the reader to [10] for details on the parametrization of the potentials, and the data used to fix it.

We first examine Dirac eigenstates with  $j = \tilde{\ell} + 1/2$  and wave functions as in Eq. (3a). Their partners with  $j' = j - 1$  are obtained from Eqs. (8a) and (8c). As an example, we consider the realistic relativistic mean field Dirac eigenstates  $0d_{3/2}$  and  $1d_{3/2}$  ( $\tilde{\ell} = 1, j = 3/2$ ) for  $^{208}\text{Pb}$  [10]. In Fig. 1 we compare the spatial wave functions of these pseudospin partners,  $[P(0d_{3/2})]s_{1/2}$  and  $[P(1d_{3/2})]s_{1/2}$ , with the eigenstates,  $1s_{1/2}$  and  $2s_{1/2}$ . (The symbol  $P$  means the  $s_{1/2}$  partner of the  $0d_{3/2}$  or  $1d_{3/2}$  eigenstates.) The lower components agree very well, which was noted previously [10–12], except for some disagreement on the surface. For the upper components the agreement is not as good in the magnitude but the shapes agree well, with the number of radial nodes being the same [ $\tilde{n}'_r = \tilde{n}_r$  in Eq. (7a)]. The agreement improves as the radial quantum number increases, which is consistent with the observed decrease in the energy splitting between the doublets [8,10]. As another example in the same category ( $j = \tilde{\ell} + 1/2$ ), we compare in Fig. 2 the  $[P(0h_{9/2})]f_{7/2}$  partner of the  $0h_{9/2}$  eigenstate ( $\tilde{\ell} = 4, j = 9/2$ ) with the  $1f_{7/2}$  eigenstate. The radial wave functions have the same number of radial quantum numbers and, again, for the lower components the agreement is better.

Next we examine the other category of Dirac eigenstates with  $j = \tilde{\ell} - 1/2$  and wave functions as in Eq. (3b). Their partners with  $j' = j + 1$  are obtained from Eqs. (8b) and (8c). As an example, we consider the realis-

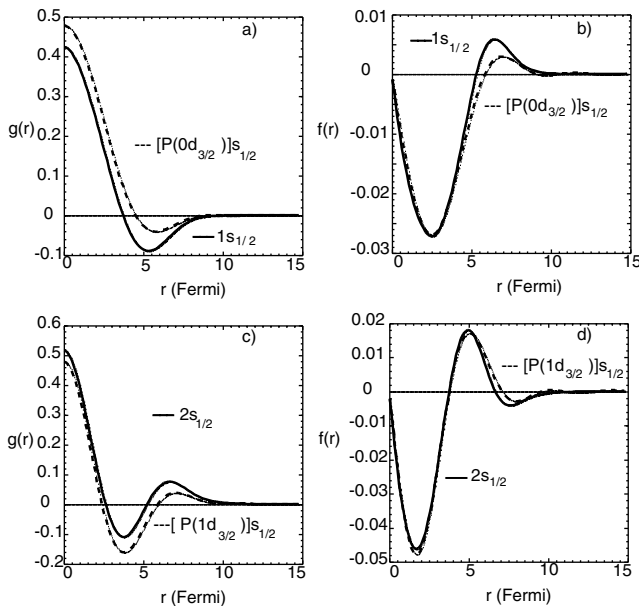


FIG. 1. (a) The upper component  $[g(r)]$  and (b) the lower component  $[f(r)]$  in  $(\text{Fermi})^{-3/2}$  of the  $[P(0d_{3/2})]s_{1/2}$  partner of the  $0d_{3/2}$  eigenstate compared to the  $1s_{1/2}$  eigenstate. (c) The upper component and (d) the lower component of the  $[P(1d_{3/2})]s_{1/2}$  partner of the  $1d_{3/2}$  eigenstate compared to the  $2s_{1/2}$  eigenstate for  $^{208}\text{Pb}$  [10].

tic relativistic mean field eigenstates  $0s_{1/2}$ ,  $1s_{1/2}$ , and  $2s_{1/2}$  ( $\tilde{\ell} = 1, j = 1/2$ ) for  $^{208}\text{Pb}$  [10]. The  $0s_{1/2}$  eigenstate will have a partner which we denote by  $[P(0s_{1/2})]d_{3/2}$ , but there is no  $d_{3/2}$  eigenstate at approximately the same energy as the  $0s_{1/2}$  eigenstate, so there is no eigenfunction with which to compare. On the other hand, the  $1s_{1/2}$  and  $2s_{1/2}$  eigenstates are almost degenerate with the  $0d_{3/2}$  and  $1d_{3/2}$  eigenstates, respectively. In Fig. 3 we compare the spatial wave functions of the pseudospin partners  $[P(1s_{1/2})]d_{3/2}$  and  $[P(2s_{1/2})]d_{3/2}$  with the respective  $0d_{3/2}$  and  $1d_{3/2}$  eigenstates. These partners agree well with the eigenfunctions in the interior but not on the nuclear surface. In fact, the partners do not have the same number of nodes and do not fall off exponentially but inversely as the cubic power,  $r^{-3}$ , in agreement with the  $r^{-(\tilde{\ell}+2)}$  behavior reported in Eq. (8b).

The Dirac eigenstates with  $\tilde{n}_r = 0$  and  $j = \tilde{\ell} - 1/2$  are special, because no eigenstates exist with the quantum numbers of their partners,  $\tilde{n}_r = 0$  and  $j = \tilde{\ell} + 1/2$ . An example is given in Figs. 3a and 3b for  $\tilde{\ell} = 1$  and  $j = 1/2$ . For heavy nuclei these states with large  $j$  are the “intruder” states. Before the SU(2) algebra of pseudospin was discovered, these states were discarded from the pseudospin scheme. However, that is clearly not a valid procedure if pseudospin symmetry is a symmetry of the Dirac Hamiltonian. As another example, we show in Figs. 4a and 4b the radial wave function of the  $[P(0f_{7/2})]h_{9/2}$  partner of the  $0f_{7/2}$  intruder state ( $\tilde{\ell} = 4, j = 7/2$ ). There is no quasidegenerate  $h_{9/2}$  eigenstate with which to compare. The upper component has the  $r^{-6}$  falloff alluded to above. Although both components have zero radial quantum number, they do not compare well with the  $0h_{9/2}$  eigenstate shown in Figs. 4c and 4d. In Figs. 4c and 4d we also show the radial wave function of the  $[P(1f_{7/2})]h_{9/2}$  partner of the  $1f_{7/2}$  state ( $\tilde{\ell} = 4, j = 7/2$ ) and compare it to the  $0h_{9/2}$  eigenstate. The upper component again has the  $r^{-6}$  falloff and therefore does not compare well on the surface. Also the number of radial quantum numbers differ. The lower components agree better.

In summary, we have shown that the radial wave functions of the upper components of the  $j = \tilde{\ell} - 1/2$

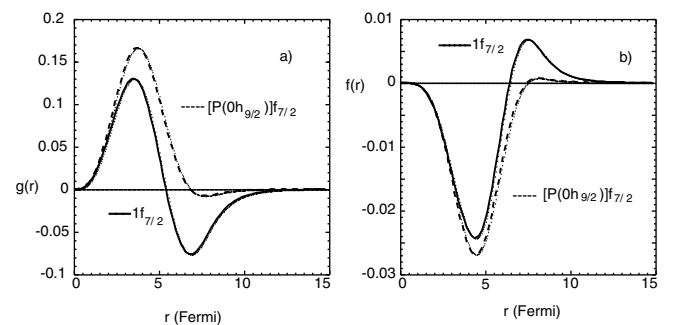


FIG. 2. (a) The upper component  $[g(r)]$  and (b) the lower component  $[f(r)]$  in  $(\text{Fermi})^{-3/2}$  of the  $[P(0h_{9/2})]f_{7/2}$  partner of the  $0h_{9/2}$  eigenstate compared to the  $1f_{7/2}$  eigenstate for  $^{208}\text{Pb}$  [10].

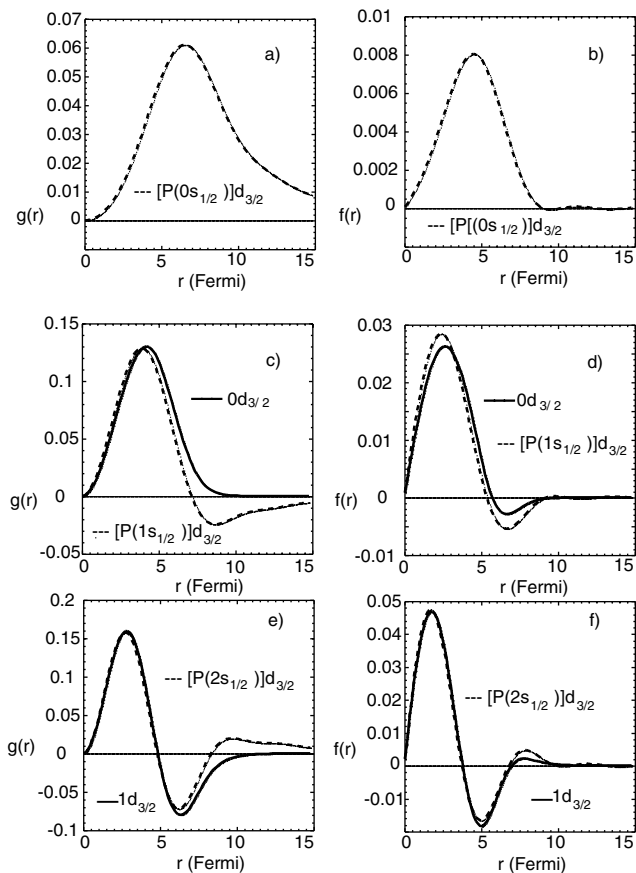


FIG. 3. (a) The upper component  $[g(r)]$  and (b) the lower component  $[f(r)]$  in  $(\text{Fermi})^{-3/2}$  of the  $[P(0s_{1/2})]d_{3/2}$  partner of the  $0s_{1/2}$  eigenstate. (c) The upper component and (d) the lower component of the  $[P(1s_{1/2})]d_{3/2}$  partner of the  $1s_{1/2}$  eigenstate compared to the  $0d_{3/2}$  eigenstate. (e) The upper component and (f) the lower component of the  $[P(2s_{1/2})]d_{3/2}$  partner of the  $2s_{1/2}$  eigenstate compared to the  $1d_{3/2}$  eigenstate for  $^{208}\text{Pb}$  [10].

pseudospin partner of the eigenstate with  $j = \tilde{\ell} + 1/2$  is similar in shape to the  $j = \tilde{\ell} - 1/2$  eigenstate but there is a difference in magnitude. However, the  $\tilde{n}_r \neq 0$  radial wave functions of the upper components of the  $j = \tilde{\ell} + 1/2$  pseudospin partner of the eigenstate with  $j = \tilde{\ell} - 1/2$  is not similar in shape to the  $j = \tilde{\ell} + 1/2$  eigenstate. In fact these wave functions approach  $r^{\tilde{\ell}-1}$  rather than  $r^{\tilde{\ell}+1}$  for  $r$  small, do not have the same number of radial nodes as the eigenstates, and do not fall off exponentially as do the eigenstates, but rather fall off as  $r^{-(\tilde{\ell}+2)}$ . Furthermore, the pseudospin partners of the “intruder” eigenstates,  $\tilde{n}_r = 0$ , also fall off as  $r^{-(\tilde{\ell}+2)}$ . We have confirmed that the radial wave functions of the lower components of the pseudospin partners of eigenstates of the Dirac Hamiltonian for  $j = \tilde{\ell} \pm 1/2$  are very similar to the eigenstates with the same quantum numbers, except for some differences on the surface.

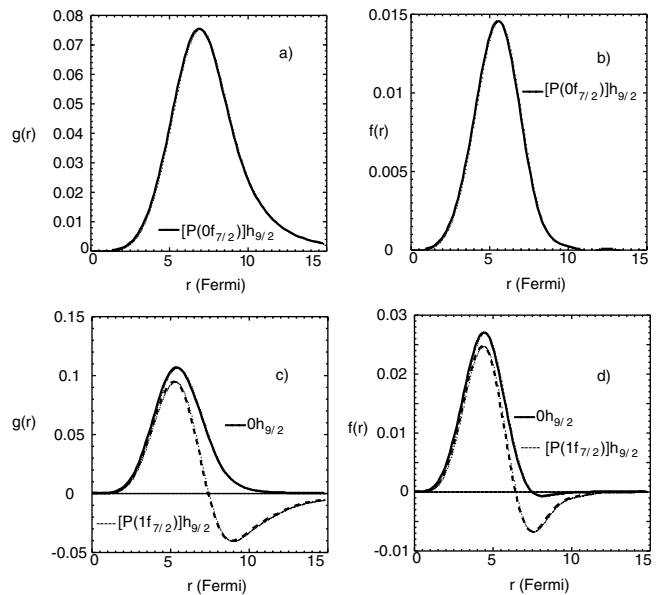


FIG. 4. (a) The upper component  $[g(r)]$  and (b) the lower component  $[f(r)]$  in  $(\text{Fermi})^{-3/2}$  of the  $[P(0f_{7/2})]h_{9/2}$  partner of the  $0f_{7/2}$  eigenstate. (c) The upper component and (d) the lower component of the  $[P(1f_{7/2})]h_{9/2}$  partner of the  $1f_{7/2}$  eigenstate compared to the  $0h_{9/2}$  eigenstate for  $^{208}\text{Pb}$  [10].

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