Hadronic Loop Corrections to the Muon Anomalous Magnetic Moment

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The dominant theoretical uncertainties in both the anomalous magnetic moment of the muon and the value of the electromagnetic coupling at the Z scale, M_Z, arise from their hadronic contributions. Since these will ultimately dominate the experimental errors, we study the correlation between them, as well as with other fundamental parameters. To this end we present analytical formulas for the QCD contribution from higher energies and from heavy quarks. Including these correlations affects the Higgs boson mass extracted from precision data.

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The magnetic moment of the electron, g_e, provides the best determination of the fine structure constant, but is currently not measured precise enough to give a sensitive probe of electroweak physics. On the other hand, electroweak contributions to the anomalous magnetic moment of the muon [1], $a_\mu = (g_\mu - 2)/2$, are enhanced by a factor $m_\mu^2/m_e^2 \sim 4 \times 10^4$, which renders them sizable enough to be detectable at the ongoing E821 experiment at the AGS at BNL (a_τ has not yet been observed experimentally). E821 already reduced the experimental error to $\pm 1.6 \times 10^{-9}$ [2]. The anticipated final error of about $\pm 0.4 \times 10^{-9}$ will mean a factor of 20 improvement relative to previous results [3]. a_μ therefore provides a good laboratory to test the standard model (SM) and probe theories beyond it [4]. For example, scenarios of low-energy supersymmetry with large $\tan\beta$ and moderately light superparticle masses can give large contributions to a_μ [5].

Unfortunately, the interpretation of a_μ is compromised by a large theoretical uncertainty introduced by hadronic effects. The 2- and 3-loop vacuum diagrams containing light quark loops cannot be calculated reliably in perturbative QCD (PQCD). Instead they are obtained by computing dispersion integrals over measured (at low energies) and theoretical (at higher energies) hadronic cross sections. At two loop [6],

$$a_\mu(\text{had}; 2\text{-loop}) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} \hat{K}(s) R(s), \quad (1)$$

where $R(s)$ is the cross section of $e^+e^- \to$ hadrons, normalized to the tree level cross section of $e^+e^- \to \mu^+\mu^-$,

$$\hat{K}(s) = \int_0^1 dx \, \frac{3x^2(1-x)}{1 - x + x^2 m_\mu^2/s}. \quad (2)$$

The uncertainty introduced by this procedure is significantly larger than the anticipated experimental uncertainty. For example, Ref. [7] quotes an error of $\pm 0.67 \times 10^{-9}$, while other evaluations [8–10] give larger uncertainties. An analogous uncertainty occurs in the QED coupling constant, $\hat{\alpha}(\mu)$, [modified minimal subtraction scheme ($\overline{\text{MS}}$) quantities will be marked by a caret], preventing its precise theoretical computation from the fine structure constant, α, for $\mu \gtrsim 2m_{\pi^0}$. Knowledge of $\hat{\alpha}(M_Z)$ is indispensable for the extraction of the Higgs boson mass, M_H, from the mass of the W boson, M_W, and the weak mixing angle, $\hat{s}_W^2$. Again one must rely on a dispersion integral to be taken over $R(s)$, but using a different kernel function. As a result, these hadronic uncertainties are strongly correlated with each other, and also with other fundamental SM parameters, such as the strong coupling constant and the heavy quark masses.

To address these correlations we obtain compact analytical expressions for $a_\mu(\text{had})$ wherever possible, i.e., for the charm and bottom quarks, as well as for the high-energy integration region (above 1.8 GeV) for light quarks where PQCD appears to be applicable. A similar idea was pursued in Ref. [11], for the calculation of $\hat{\alpha}(M_Z)$.

To lowest order in QED, the anomalous magnetic moments are lepton universal and mass independent [12], $a_\ell^{\text{1-loop}} = \alpha/2\pi$. To 2-loop order, one has

$$a_\mu^{\text{2-loop}} = a_\mu^\mu + a_\mu^e + a_\mu^\tau + a_\mu^b + a_\mu^c + a_\mu^{uds} + a_\mu^{\text{had}}, \quad (3)$$

where the lepton contributions [13] amount to $a_\mu^\mu + a_\mu^e + a_\mu^\tau = 4132.18 \times 10^{-9}$. a_μ^b and a_μ^c are heavy quark contributions. To proceed, we used the expansion

$$\hat{K}(s) = 1 + k_1(\sqrt{s}) \frac{m_\mu^2}{s} + k_2(\sqrt{s}) \frac{m_\mu^4}{s^2} + \mathcal{O}\left(\frac{m_\mu^6}{s^3}\right),$$

where

$$k_1(x) = \frac{25}{4} + 3 \ln\frac{m_\mu^2}{x^2}, \qquad k_2(x) = \frac{291}{10} + 18 \ln\frac{m_\mu^2}{x^2}.$$

In Eq. (1) the integration is over the imaginary part of the photon polarization function which is related to R by $R(s) = 12\pi \,\text{Im}\Pi(q^2 + i\epsilon)$ and is taken along the real axis from the quarkonium threshold to infinity. Analytic continuation allows us to integrate instead over the full function, $\Pi(s)$, along a circle of radius $s = \hat{m}_q^2(\hat{m}_q^2)$ counterclockwise around the origin. This avoids a complicated integration over resonances, i.e., a region where perturbative QCD is not applicable, at the expense of introducing a new uncertainty from the imperfect knowledge of the $(\overline{\text{MS}})$ quark mass (see the discussion in the next paragraph). With $\Pi(s)$ known up to $\mathcal{O}(\alpha_s^2)$ [14], we find, for a quark of charge Q_q,

$$a_\mu^q = \frac{Q_q^2}{4}\frac{\alpha^2}{\pi^2}\left\{\frac{m_\mu^2}{4\hat{m}_q^2}\left[\frac{16}{15} + \frac{3104}{1215}a_s + a_s^2\left(0.5099 + \frac{2414}{3645}n_l\right)\right]\right.$$
$$\left. + \frac{m_\mu^4}{16\hat{m}_q^4}\left[\frac{108}{1225} - 0.1943a_s + 3\ln\frac{m_\mu^2}{\hat{m}_q^2}\left(\frac{16}{35} + \frac{15728}{14175}a_s + 1.4123a_s^2 + \frac{290179}{637875}n_l a_s^2\right)\right]\right\}, \quad (4)$$

where $a_s = \alpha_s(\hat{m}_q)/\pi$, $n_l = 3$ for charm, and $n_l = 4$ for bottom. Higher orders in $m_\mu^2/\hat{m}_q^2$ can safely be neglected. Terms of $\mathcal{O}(\alpha_s^2 m_\mu^4)$ can also be dropped unless they are logarithmically enhanced. Nonperturbative effects in the operator product expansion were also computed and found to be negligible. Using (here and in the following) $\alpha_s(M_Z) = 0.120$, $\hat{m}_c = 1.31$ GeV, and $\hat{m}_b = 4.24$ GeV, we find $a_\mu^c = 1.39 \times 10^{-9}$ and $a_\mu^b = 0.03 \times 10^{-9}$.

In contrast to the numerical integration over resonances, Eq. (4) is a simple and transparent representation of the heavy quark contribution. More importantly, the uncertainty implied by Eq. (4) is smaller. To see this, notice first the excellent behavior of its α_s expansion, implying a small truncation error which we evaluated to be $\pm 0.05 \times 10^{-9}$. The dominant uncertainty is induced via the $\overline{\text{MS}}$ charm mass, but it is *necessarily* smaller than in the conventional approach *regardless* of the available data in the resonance region: if one compares the two treatments—integration over the real axis heavily relying on experimental results and integration over the circle contour relying only on PQCD—one can *determine* the heavy quark masses by demanding consistency. This effectively amounts to deriving a *specific* QCD sum rule. The point we are making here is that this is only one of a large number of possible sum rules, but not the one which uses the available information most efficiently. The charm mass extracted from the most efficient sum rule [15] (and other precise quark mass determinations) can then be used for Eq. (4). An unweighted average of various determinations yields [11] $\Delta\hat{m}_c = \pm 0.07$ GeV and $\Delta a_\mu^c = \pm 0.16 \times 10^{-9}$. In the near future these uncertainties are likely to reduce to about ± 0.04 GeV and $\pm 0.09 \times 10^{-9}$, respectively. Similarly, $\Delta\hat{m}_b = \pm 0.11$ GeV, but the induced error in a_μ is negligible.

The remaining terms in Eq. (3) are due to u, d, and s quark effects, which we separated into the contributions from $\sqrt{s} \geq \mu_0 = 1.8$ GeV (a_μ^{uds}) and $\sqrt{s} \leq \mu_0$ (a_μ^{had}). a_μ^{uds} can be written as an expansion in m_μ^2/μ_0^2. For the leading contribution, we find

$$a_\mu^{uds;\text{LO}} = \frac{2}{9}\frac{\alpha^2}{\pi^2}\frac{m_\mu^2}{\mu_0^2}\left(1 + B_1 + \sum_{n=2}^{\infty} d_n B_n\right), \quad (5)$$

where $d_2 = 299/24 - 9\zeta(3)$ and $d_3 = 58057/288 - 779\zeta(3)/4 + 75\zeta(5)/2$ are coefficients of the Adler D-function, and

$$B_n = \frac{1}{2\pi i}\oint_{|s|=\mu_0^2}\frac{ds}{s}\left(1 - \frac{\mu_0^2}{s}\right)\left[\frac{\alpha_s(-s)}{\pi}\right]^n, \quad (6)$$

which we compute with a 4-loop renormalization group improvement. For a representative value $\alpha_s(\mu_0)/\pi = 0.1$,

$$B_1 = 7.069 \times 10^{-2}, \qquad B_2 = 4.514 \times 10^{-3},$$
$$B_3 = 2.562 \times 10^{-4}, \qquad B_4 = 1.243 \times 10^{-5}. \quad (7)$$

Notice that B_4 is small enough that even with the fourth order coefficient, d_4, unknown this treatment keeps the truncation error at a negligible level. (For an estimate of the uncertainty of d_4, see Ref. [16].) Denoting $k_1' = k_1(\mu_0) - \frac{3}{2}$ and $k_2' = k_2(\mu_0) - 6$, we find for the subleading contributions,

$$\frac{\pi^2}{\alpha^2}a_\mu^{uds;\text{rem}} = \frac{2}{9}a_s\frac{m_\mu^2\hat{m}_s^2(\mu_0^2)}{\mu_0^4} + a_s^2\frac{m_\mu^2}{\mu_0^2}G\left(\frac{\mu_0^2}{\hat{m}_c^2(\hat{m}_c^2)}\right) + \left\{(1 + a_s)\frac{k_1'}{9} + a_s^2\left[\left(\frac{34}{27} - \zeta(3)\right)k_1' + \frac{3}{16}\right]\right\}\frac{m_\mu^4}{\mu_0^4}$$
$$+ \frac{2}{3}\left\{(1 + a_s)\frac{k_2'}{9} + a_s^2\left[\left(\frac{281}{216} - \zeta(3)\right)k_2' + \frac{1}{2}\right]\right\}\frac{m_\mu^6}{\mu_0^6}, \quad (8)$$

where in $\mathcal{O}(\hat{\alpha}_s)$ we kept the small s quark mass effect ($\sim 3 \times 10^{-12}$). $G(x)$ arises from virtual charm quark effects inside a light quark loop (double bubble diagram). Even below threshold it can be well approximated as an expansion in x despite the fact that $\hat{m}_c^2 < \mu_0^2$,

$$G(x) \approx \frac{x}{1215}\left[\frac{3503}{75} - \frac{2\pi^2}{3} - \frac{88}{5}\ln x + 2\ln^2 x\right] + \frac{x^2}{11340}\left[\frac{1723}{420} - \ln x\right]. \quad (9)$$

$G(x)$ also applies to b quarks, but this contribution can be safely neglected. We find $a_\mu^{uds} = 4.38 \times 10^{-9}$ and the leading order resummation in Eq. (5) renders the truncation error negligible.

Leading order nonperturbative contributions due to gluon and light quark condensates are suppressed by two powers of α_s and they change a_μ by less than 10^{-12}, an effect completely negligible. Effects from up- and down-quark condensates are suppressed by a further factor of m_π^2/m_K^2, and quartic mass terms are tiny as well. Uncertainties from nonperturbative effects not accounted for by the operator product expansion, which are due to the transition from the data region to PQCD at μ_0, were estimated in Ref. [7] which quotes $\pm 0.024 \times 10^{-9}$.

We take the low-energy contribution from Ref. [7],

$$a_\mu^{\text{had}} = (63.43 \pm 0.60) \times 10^{-9}, \tag{10}$$

which includes a QCD sum rule improvement, PQCD down to a relatively low $\mu_0 = 1.8$ GeV, as well as additional information from τ decays. The quoted error is not uncontroversial [4,10] and needs to be confirmed. Note, however, that inclusion of the τ-decay data decreases the difference between the SM prediction and the current experimental result [2]. Our reason for using Ref. [7] is that it quantifies the correlation (69%) with the corresponding result on $\Delta\alpha_{\text{had}} = (56.53 \pm 0.83) \times 10^{-4}$. We also account for the almost perfect anticorrelation with higher order hadronic uncertainties, as will be discussed below. The total 2-loop quark contribution is $a_\mu^b + a_\mu^c + a_\mu^{uds} + a_\mu^{\text{had}} = 69.23 \times 10^{-9}$.

This completes the discussion of the terms appearing in Eq. (3). The leading contribution at $\mathcal{O}(\alpha^3)$ is from the light-by-light diagram containing an electron loop [17]. The contribution due to $a_\mu^{e;\text{lbl}} = 20.9479\alpha^3/\pi^3 = 262.54 \times 10^{-9}$ is almost 4 times larger than the entire hadronic contribution at $\mathcal{O}(\alpha^2)$. The corresponding contribution involving a muon loop is $a_\mu^{\mu;\text{lbl}} = 0.3710\alpha^3/\pi^3 = 4.65 \times 10^{-9}$, and the τ contributes [17], $a_\mu^{\tau;\text{lbl}} = 0.03 \times 10^{-9}$. An evaluation of the hadronic light-by-light contribution yields [18]

$$a_\mu^{\text{had;lbl}} = (-0.792 \pm 0.154) \times 10^{-9}, \tag{11}$$

which is consistent with the finding in Ref. [19]. Notice that hadrons and leptons contribute with opposite signs. Since these results are based on model calculations, an independent confirmation would be desirable.

The purely leptonic 3-loop vacuum polarization contribution is again dominated by electron loops and given by $a_\mu^{\ell;\text{vpol}} = 2.7294\alpha^3/\pi^3 = 34.21 \times 10^{-9}$. On the other hand, the contribution from two hadronic loops is suppressed by a factor $m_\mu^4/16m_{\pi^\pm}^4$ and therefore small $a_\mu^{\text{had;vpol}} = (0.027 \pm 0.001) \times 10^{-9}$ [9]. Similar to our strategy at two loops, we separated the mixed leptonic-hadronic contribution into heavy quarks, light quarks ($\sqrt{s} \geq \mu_0$), and light hadrons ($\sqrt{s} \leq \mu_0$). The τ-hadronic contribution is of the order of 10^{-12} and negligible. As for the other leptons, we use the kernel functions from Ref. [20] and obtain, for charm and bottom quarks,

$$a_\mu^{\ell-q;\text{vpol}} = \frac{\alpha^3}{\pi^3} Q_q^2 \frac{m_\mu^2}{4\hat{m}_q^2}\left[\left(\frac{8}{5} + \frac{1552}{405} a_s\right)\left(\frac{110}{27} - \frac{\pi^2}{3} + \frac{23}{36}\ln\frac{m_\mu^2}{\hat{m}_q^2} + \frac{1}{9}\ln\frac{m_\mu^2}{m_e^2}\right) - \frac{1771}{675}\right], \tag{12}$$

which amounts to -0.05×10^{-9} and -0.002×10^{-9}, respectively. For the light quarks, we obtain,

$$\begin{aligned} a_\mu^{\ell-uds;\text{vpol}} = {} & \frac{\alpha^3}{\pi^3}\frac{1 + a_s}{9}\left[\frac{m_\mu^2}{\mu_0^2}\left(\frac{371}{9} - 4\pi^2 + \frac{23}{3}\ln\frac{m_\mu^2}{\mu_0^2} + \frac{4}{3}\ln\frac{m_\mu^2}{m_e^2}\right)\right. \\ & \left. + \frac{m_\mu^4}{\mu_0^4}\left(\frac{20359}{576} - \frac{103}{24}\pi^2 + \frac{509}{72}\ln\frac{m_\mu^2}{\mu_0^2} + \frac{19}{6}\ln\frac{m_\mu^2}{m_e^2} - \frac{5}{24}\ln^2\frac{m_\mu^2}{\mu_0^2} - 2\ln\frac{m_\mu^2}{\mu_0^2}\ln\frac{m_\mu^2}{m_e^2}\right)\right] \\ = {} & -0.15 \times 10^{-9}. \end{aligned} \tag{13}$$

We found the vacuum polarization contribution arising from an electron in one loop and light hadrons in the other one, $a_\mu^{e-\text{had;vpol}} = 0.97 \times 10^{-9}$, by constructing a simplified function $R(s)$ which reproduces the results in Ref. [7]. $a_\mu^{e-\text{had;vpol}}$ is almost completely (>99.9%) correlated with a_μ^{had} in Eq. (10). On the other hand, $a_\mu^{\mu-\text{had;vpol}} = -1.80 \times 10^{-9}$ is very strongly (≈−97%) anticorrelated with a_μ^{had}. The small uncorrelated error contributions are clearly negligible, and the correlated ones can be added (subtracted) linearly, slightly reducing the error in Eq. (10) to $\pm 0.59 \times 10^{-9}$. By including the uncertainty from the light-by-light contribution in Eq. (11), we obtain $\pm 0.61 \times 10^{-9}$ as the total hadronic error, excluding parametric uncertainties.

Taking the other theoretical uncertainties mentioned earlier to be 100% correlated with the corresponding uncertainties in $\Delta\alpha_{\text{had}}$ (which is a fit parameter), we obtain a residual correlation of 55% or $\pm 0.34 \times 10^{-9}$ (the uncorrelated error is $\pm 0.51 \times 10^{-9}$).

The 4-loop contribution [21], $a_\mu^{4\text{-loop}} = 126.04\alpha^4/\pi^4 = 3.67 \times 10^{-9}$, and the 5-loop estimate [22], $a_\mu^{5\text{-loop}} = 930\alpha^5/\pi^5 = 0.06 \times 10^{-9}$, are also included.

The 1-loop electroweak corrections due to W and Z boson loops are given by [23]

$$a_\mu^{\text{EW;1-loop}} = \frac{G_F m_\mu^2}{24\sqrt{2}\,\pi^2}[5 + (1 - 4s_W^2)^2]. \tag{14}$$

The 2-loop corrections to Eq. (14) are significant due to large logarithmic contributions [24–26],

$$\frac{\alpha G_F m_\mu^2}{24\sqrt{2}\,\pi^3}\left[41 - \frac{124}{3}\,s_W^2(1 - 2s_W^2)\right]\ln\frac{M_Z^2}{m_\mu^2}\,. \qquad (15)$$

The fermionic 2-loop result, including some additional logarithms and subleading contributions, was obtained in Ref. [25] for small and large values of M_H. We constructed an interpolation formula for other values of M_H which reads, in units of $\alpha(m_\mu)G_F m_\mu^2/(3\sqrt{2}\,\pi^3)$,

$$\frac{13 + 6\ln x}{9}(1 - \omega) + x\left[3 + \frac{\pi^2}{3} + (\ln x + 1)^2\right]\omega\,, \qquad (16)$$

where $x = \hat{m}_t^2/M_H^2$ and $\omega = e^{-3.31849x}$. Equation (16) reproduces the exact result [25] for $M_H = \hat{m}_t$. The bosonic 2-loop corrections were obtained only for large values of M_H and as an expansion in s_W^2 [26]. We take the leading nonlogarithmic contribution in the $M_H \to \infty$ limit (a_{-2} in Ref. [26]) as the uncertainty induced by subleading bosonic 2-loop effects. The leading logarithms to 3-loop order have also been computed [27] and included in our analysis. We obtain $a_\mu^{\rm EW} = (1.52 \pm 0.03) \times 10^{-9}$.

Summation of the various contributions gives $(g_\mu - 2 - \alpha/\pi)/2 = 4506.28 \times 10^{-9}$. Including our evaluation of a_μ into a global analysis of electroweak data yields,

$$\frac{1}{2}\left(g_\mu - 2 - \frac{\alpha}{\pi}\right) = (4506.35 \pm 0.37 \pm 0.51) \times 10^{-9}, \qquad (17)$$

where the first error includes $\Delta\alpha_{\rm had}$ and all other parametric uncertainties, such as the ± 0.028 error in $\alpha_s(\mu_0)$ obtained from current global fits. The extracted Higgs mass from this fit is $M_H = 88^{+49}_{-33}$ GeV. The current experimental world average, $(4510.55 \pm 1.51) \times 10^{-9}$ [which differs by 2.6σ from Eq. (17)], needs to be included into the analysis as well. This is achieved by combining the experimental error with the second error in Eq. (17) in quadrature (for a total of 1.59×10^{-9}). The global fit then yields 4506.52 ± 0.36 (parametric) with a pull of 2.5. The crucial observation here is that now $M_H = 83^{+47}_{-31}$ GeV, i.e., both the central value and the uncertainty of M_H decrease. The increase in the precision in M_H gained by properly correlating it to $\Delta\alpha_{\rm had}$ (and assuming the SM) is almost identical to that provided by the M_W measurement at the Tevatron run I.

To summarize, we carefully examined the error correlations between a_μ and other quantities entering electroweak tests, especially $\Delta\alpha(M_Z)$, α_s, and the quark masses. We derived new analytical results for the hadronic contributions and showed that the proper treatment discussed in this article has a significant effect on the extraction of M_H, which could (depending on future experimental findings) become even more dramatic.

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