Off-Diagonal Geometric Phase in a Neutron Interferometer Experiment

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Off-diagonal geometric phases acquired by an evolution of a 1/2-spin system have been observed by means of a polarized neutron interferometer. We have successfully measured the off-diagonal phase for noncyclic evolutions even when the diagonal geometric phase is undefined. Our data confirm theoretical predictions and the results illustrate the significance of the off-diagonal phase.

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When a quantum system is transported through a curve C in parameter space, a geometric phase component develops which is Hamiltonian independent and depends solely upon curve C. This geometric phase factor was first recognized by Pancharatnam [1] and reformulated for adiabatic transport in a closed loop by Berry [2], which aroused considerable interest [3]. The concept of geometric phase by Berry was generalized for nonadiabatic transport [4]. Many experiments to determine the geometric phase were reported [3]: early neutron experiments were made with regard to spinor evolution of neutrons [5,6]. Quite recently, off-diagonal matrix elements, $\langle\Psi_j\,|\,\Psi_k\rangle$, of nondegenerate eingenstates, $|\Psi_i\rangle$, were investigated and found to carry the geometrical phase information even when the conventional diagonal geometric phase is undefined [7]. The formalism was applied to the deformed microwave resonator experiment [8].

Neutron interferometer experiments have made an impact in fundamental quantum and neutron physics for more than two decades [9–12]. In particular, those with a polarized incident beam served as an almost ideal tool to investigate properties of a 1/2-spin system in a superior way [13]: for instance, spinor superposition [14,15], double resonance flipper experiments [16], and geometric phase measurements [17,18]. A neutron interferometer with coupled interference loops was used to create and to measure the spin-independent geometric phase in kind of a double-slit experiment [19].

In this Letter, we describe the neutron interferometric experiment that explicitly observes the off-diagonal geometric phase acquired by the noncyclic spinor evolution of a 1/2-spin system. An incident polarized neutron beam splits into two beam paths and each spinor is rotated to induce appropriate spinor evolutions. With the help of a polarization analysis, the off-diagonal geometric phases appear as the phase shifts of the intensity modulations.

We consider here the time evolution of a spinor $|\Psi(t)\rangle$, describing a 1/2-spin system, namely, a neutron. As one of the simplest but nevertheless the most important examples, we treat here the unitary evolution, $U(t)$, of a spinor lying in the $\hat{\mathbf{x}}$-$\hat{\mathbf{y}}$ plane by a magnetic field parallel to $\hat{\mathbf{z}}$. The initial spinor is assumed to be $|\Psi^+(0)\rangle$; the orthogonal spinor is given by $|\Psi^-(0)\rangle$. The initial spinor, $|\Psi^+(0)\rangle$, evolves to $|\Psi^+(t)\rangle = U(t)\,|\Psi^+(0)\rangle$. This is a parallel-transport process [20] with invariant phase and therefore $|\Psi^{+\parallel}(t)\rangle = |\Psi^+(t)\rangle$ in our particular circumstances. The off-diagonal geometric phase, γ_{+-} [7] is expressed as

$$\begin{aligned}\gamma_{+-} &\equiv \Phi\big(\langle\Psi^{+\parallel}(0)\,|\,\Psi^{-\parallel}(t)\rangle\cdot\langle\Psi^{-\parallel}(0)\,|\,\Psi^{+\parallel}(t)\rangle\big) = \Phi\big(\langle\Psi^{+}(0)\,|U(t)|\Psi^{-}(0)\rangle\cdot\langle\Psi^{-}(0)\,|U(t)|\Psi^{+}(0)\rangle\big)\\ &= \Phi\big(\langle\Psi^{+}(0)\,|U(t)\cdot P\big(|\Psi^{-}(0)\rangle\big)\cdot P\big(|\Psi^{-}(0)\rangle\big)\cdot U(t)\,|\Psi^{+}(0)\rangle\big),\end{aligned} \tag{1}$$

where $\Phi(z) = z/|z|$ for a complex quantity $z \neq 0$, and $P(|\Psi\rangle) \equiv |\Psi\rangle\langle\Psi|$, which is a projection operator to the state $|\Psi\rangle$ which will be achieved in an experiment by the use of a polarizer.

An interferometric measurement with polarized neutrons allows us to observe this phase factor γ_{+-} as the phase shift of the oscillations, when the intensity modulation to be measured is given by

$$\begin{aligned} I = |&e^{i\chi}\cdot P^\dagger\big(|\Psi^-(0)\rangle\big)\cdot U^\dagger(t)\cdot|\Psi^+(0)\rangle\\ &+ P\big(|\Psi^-(0)\rangle\big)\cdot U(t)\cdot|\Psi^+(0)\rangle|^2, \end{aligned} \tag{2}$$

where χ is the phase shift induced by an auxiliary phase shifter. Taking $P^\dagger(|\Psi\rangle) = P(|\Psi\rangle)$ and $U^\dagger(t) = U^{-1}(t)$, Eq. (2) is rewritten with abbreviating $|\Psi^\pm(0)\rangle$ to $|\Psi^\pm\rangle$ as

$$I = |P(|\Psi^-\rangle)\cdot\{e^{i\chi}\cdot U^{-1}(t) + U(t)\}\cdot|\Psi^+\rangle|^2. \tag{3}$$

This describes the intensity of the beam obtained from the polarized incident spinor (the third factor), split and affected by appropriate spinor rotations with the phase shift (the middle factor), and finally measured with the spinor analysis of the orthogonal spinor to the incident (the first factor).

Writing down the incident spinor $|\Psi^+\rangle = \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$ (the corresponding orthogonal spinor is given by $|\Psi^-\rangle = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$) in the $|\pm y\rangle$ spinor basis, the unitary evolution by the magnetic field in the $+\hat{\mathbf{z}}$ direction is given by

$$U(t) = \begin{bmatrix}\cos(\omega_L t/2) & -\sin(\omega_L t/2)\\ \sin(\omega_L t/2) & \cos(\omega_L t/2)\end{bmatrix} \tag{4}$$

with the Larmor frequency ω_L. Equation (3) now reduces to $I = \sin^2(\omega_L t/2)\{1 + \cos(\chi + \pi)\}/2$. Thus, the off-diagonal geometric phase is expected to be π and is independent of the spinor rotation angle, $\omega_L t$.

The experiment was carried out at the perfect crystal interferometer beam line S18 at the high flux reactor at the Institut Laue Langevin (ILL) [21]. A schematic view of the experimental setup is shown in Fig. 1. The neutron beam was monochromatized to have a mean wavelength of $\lambda_0 = 1.92$ Å $(\Delta\lambda/\lambda_0 \sim 0.02)$ by the 220 Bragg reflection of a Si perfect crystal monochromator placed in the thermal neutron guide H25. This incident beam was polarized vertically, i.e., perpendicular to the beam trajectory defining $\hat{\mathbf{z}}$, by magnetic-prism refractions. Its cross section was confined to 8×8 mm^2 and the beam passed through a spin rotator F, used for tuning the polar angle of the incident spinor, and entered a skew-symmetric triple-Laue interferometer. This interferometer was adjusted to give 220 reflection in a nondispersive arrangement relative to the monochromator. A pair of water-cooled Helmholtz coils produced a fairly uniform magnetic guide field, $B_0\hat{\mathbf{z}}$, around the interferometer. These coils were enclosed by an isothermal box to achieve reasonable thermal environmental isolation. A magnetically saturated Heusler crystal together with a Larmor accelerator and a rectangular $\pi/2$-spin turner enabled the selection of neutrons with certain polarization directions.

In order to estimate depolarization, the efficiency of the spinor rotations in the spin analyzing part, and other imperfections of the setup, the interferometer was adjusted to accept only spin-up polarized neutrons, i.e., $|+z\rangle$, with the spin rotator F turned off. The fraction of the $|+z\rangle$ component was measured to be 0.92 with the use of the spin turner. Since the present measurement demands the polarization of the incident beam to lie in the $\hat{\mathbf{x}}$-$\hat{\mathbf{y}}$ plane, the spin rotator F was turned on to rotate the $|+z\rangle$ spinor to the $|+y\rangle$ direction. The magnetic guide field was optimized to $B_0 = 18$ G to avoid additional depolarization of the spinor in the $\hat{\mathbf{x}}$-$\hat{\mathbf{y}}$ plane. the measurement with the use of the Larmor accelerator revealed that the fraction of the desired component was reduced to 0.87, which was still high enough to accomplish the measurements. Nonessential spinor precessions are induced by the guide field all through the beam trajectories. Nevertheless, the commutability between the operators for the essential spinor rotations [U and U^{-1} in Eq. (3)] and the nonessential rotations (guide field) allows one to compensate for these nonessential precessions behind the interferometer.

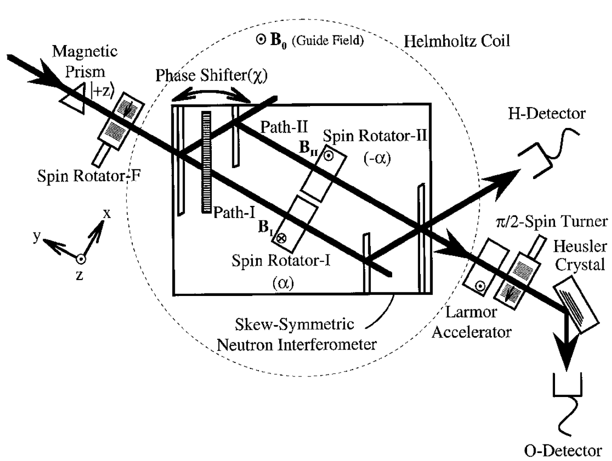

FIG. 1. Schematic view of the whole experimental setup to observe the off-diagonal geometric phase. An incident polarized neutron beam splits into two beam paths and each spinor is rotated to induce appropriate spinor evolutions. An appropriate spin analysis enables one to observe the off-diagonal geometric phase.

The unitary evolutions, $U(t)$ and $U^{-1}(t)$, were realized in the experiment by setting the magnetic field in the two identical spin rotators in an antiparallel direction. In practice, a pair of spin rotators -I and -II was inserted in the split beam paths. These spin rotators were identical dc coils except for the current directions, i.e., the directions of the induced magnetic fields. They were connected in series to give the same strengths of the magnetic fields in the antiparallel direction. Each coil was made of a 8 mm wide and 0.5 mm thick Al band, specially D doped to avoid small angle scattering in the insulator, wound onto a water-cooled Cu frame. These coils induced 180° spinor rotations when operated at 9.8 A dissipating about 0.6 W each. The rotation angle was tuned by adjusting the current, if necessary, accompanied by changing their polarities. Special attention was paid to eliminate both heat and vibration transmission to the interferometer.

Interferograms were obtained in two detectors by rotating a 5 mm thick parallel-sided Al plate as a phase shifter. The detectors were placed downstream of the interfering O beam in the forward direction behind a spin analysis system, and directly in the H beam in the reflected direction. Two intensities, one with spinor rotations and the other without them, were accumulated for each phase shifter position: typically one with 10 sec measurement times with the spin rotators -I and -II turned on, after measuring the other for 20 sec with them turned off. This procedure gave simultaneously two interferograms with and without spinor rotations to avoid instabilities producing undesired phase shifts between the two measurements. The contrast of the interference oscillations, obtained by the O detector, was typically 64% for the empty interferometer and was reduced to about 40% after inserting the dual spin rotators. This contrast dropped in some situations to about 30%, mainly due to thermal disturbances. We repeated the same measurements at least three times to ensure that the results were reliable.

The spin rotators -I and -II were tuned to give rotations, $\alpha = 45°, \pm 67.5°, \pm 90°, \pm 180°$, and $\pm 220°$. The Larmor accelerator was adjusted so that the 180° precession was induced to analyze the spinor orthogonal to the incident one. In addition, the nonessential precession by the guide field was compensated. Figure 2 depicts typical interferograms

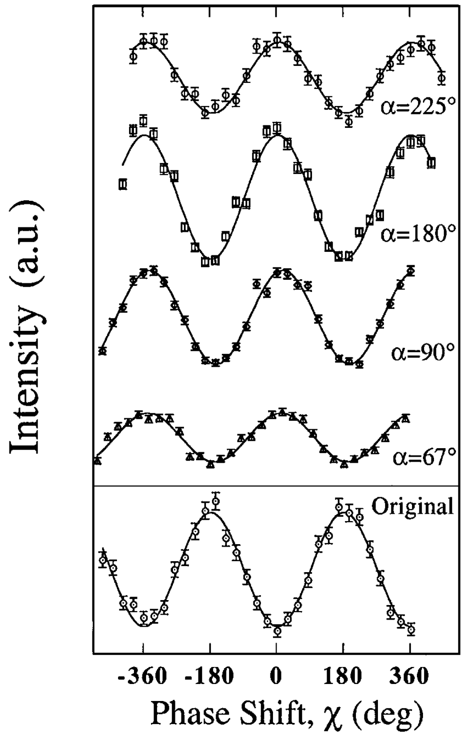

FIG. 2. Typical interferograms with various spin rotation angles, α, in comparison with the unaffected beam (without spin rotations). The patterns with spinor rotations show the phase shift of 180° to the beam without spinor rotations.

for several spinor rotation angles, α, together with the original interferogram ($\alpha = 0°$) as a reference [22]. Here, the phase shift, χ, is determined to have the same value at the same position, when the coils are not activated. The curves are the least squares fits. All patterns with spinor rotations are reflected, which confirms a phase shift of 180°, as expected. In addition, one sees that the amplitude of the oscillations has its maximum at $\alpha = 180°$ and that it falls gradually when α departs from 180°. This arises from the fact that, with the increase of the spinor rotation angle, the orthogonal component to the incident spinor increases until $\alpha = 180°$ and then decreases, as described by the unitary evolution, $U(t)$. The observed off-diagonal geometric phases at various spinor rotation angles, α, are plotted in Fig. 3 together with the straight line from the theory. The experimental results showed good agreements with the theory.

The off-diagonal geometric phase has its greatest importance when the diagonal geometric phase is undefined, i.e., with the 180° spinor rotation. In our setup, the H detector without the spin analysis allowed us to observe the diagonal part of the geometric phase, only if the spinor rotation angle is doubled, i.e., 2α. This is because the difference of the acquired spinor rotation angles between the two beams in the interferometer is 2α. Figure 4 shows the intensity modulations in the measurements of the diagonal and off-diagonal geometric phases when the associated spinor rotation angle is 180°. The modulation of the

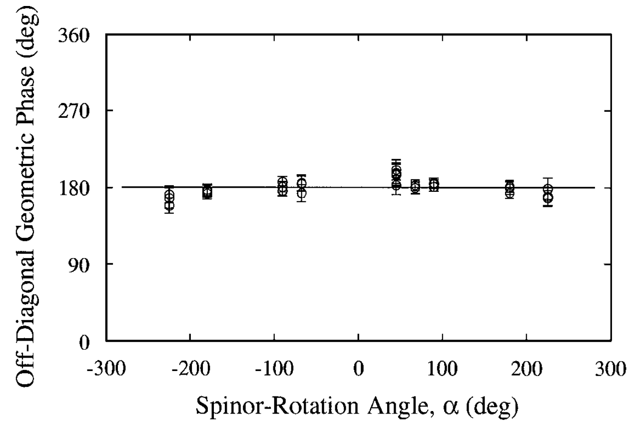

FIG. 3. Off-diagonal geometric phase induced by spinor rotations by α together with the straight line from the theory.

diagonal one is obtained by the H detector with $\alpha = 90°$ as well as that of the off-diagonal one is by the O detector with the $\alpha = 180°$. It is clearly seen that off-diagonal geometric phase is defined even when the diagonal one is undefined due to no oscillation.

As shown above, both off-diagonal and diagonal geometric phases were recorded with the use of the O detector with the spin analysis and the H detector without spin analysis in our experiment. In respect to the wave-particle duality, this spin analysis can modify the path information, resulting in recovering the interference fringes. We observed such phenomena of quantum eraser [23] in our experiment, especially when setting $\alpha = 90°$. In addition, Eq. (1) allows us to investigate the off-diagonal phase factor in more detail for an arbitrary unitary evolution of two-level systems. This demands $\gamma_{ij} = \Phi(U_{ij}^{-1} \cdot U_{ij})$

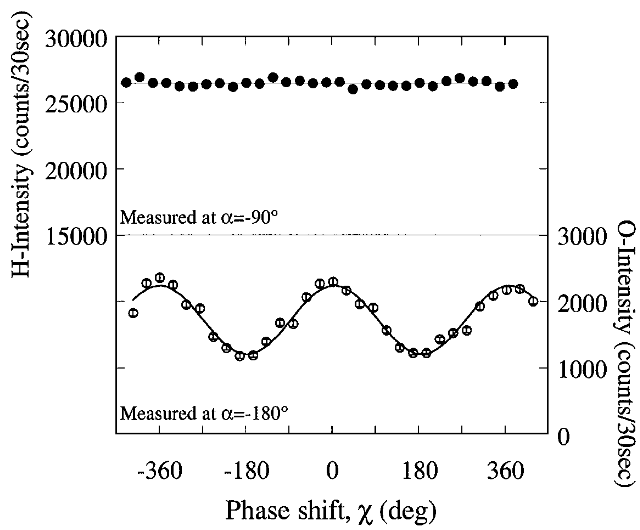

FIG. 4. Intensity modulations obtained by the O and H detectors illustrate the significance of the off-diagonal geometric phase for 180° spinor rotation. While the H detector shows no interference oscillations due to the undefined diagonal geometric phase, the O detector shows a remarkable oscillation since the off-diagonal geometric phase is defined.

$(i, j = 1, 2)$, which reveals that this factor can be only -1 or undefined in this case. Further arguments as well as observations of the geometric phases will be given in a forthcoming paper [24].

In summary, we have utilized the neutron interferometer with the polarized incident beam to observe the off-diagonal geometric phase. There, the essential spinor evolutions were realized by the use of two spinor rotators setting the directions of the magnetic fields antiparallel. Final counts with and without the polarization analysis enabled us to explicity observe both off-diagonal and diagonal geometric phases in two detectors for noncyclic evolutions. Our data confirmed theoretical predictions and our results illustrate the significance of the off-diagonal phase.

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