Long-Range Order and Low-Energy Spectrum of Diluted 2D Quantum Antiferromagnet

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The problem of a diluted two-dimensional quantum antiferromagnet on a square lattice is studied using spin-wave theory. The influence of impurities on static and dynamic properties is investigated and a good agreement with experiments and Monte Carlo data is found. The hydrodynamic description of spin waves breaks down at characteristic wavelengths $\Lambda \gtrsim \exp(\mathrm{const}/x)$, x being an impurity concentration, while the order parameter is free from anomalies. We argue that this dichotomy originates from strong scattering of the low-energy excitations in two dimensions.

The interest in magnetic properties of high-T_c compounds has been a major driving force of intensive studies of low-dimensional magnetic systems during the last decade [1]. One such system is the $La_2Cu_{1-x}Zn(Mg)_xO_4$, a parental compound doped with *static* vacancies, which shows greater stability of the antiferromagnet (AF) order against doping than its mobile hole doped counterpart $La_{2-x}Sr_xCuO_4$ [2,3]. It represents a fine example of a diluted 2D quantum $S = 1/2$ AF whose properties are the subject of this study. The problem of diluted spin systems has attracted much attention in the past [4,5] and their physics is well understood. The general understanding is that the low-energy excitations of these systems are weakly damped spin waves which belong to the infinite cluster [5] and are well defined up to the percolation threshold. In this work we show that quantum effects and low dimensionality prevent the excitation spectrum to be defined in these hydrodynamic terms [6] at arbitrarily small doping and lead to a paradoxical situation where the long-range order is preserved but the long-wavelength spectrum is not ballistic.

The site-diluted AF on a square lattice is described by

$$\mathcal{H} = J \sum_{\langle ij \rangle} p_i p_j \mathbf{S}_i \cdot \mathbf{S}_j, \qquad (1)$$

with $p_i = 1(0)$ for the magnetic (nonmagnetic) site. The pure system at long wavelengths can be described by the nonlinear σ model [1]. Its applicability in the presence of a quenched disorder has been questioned [1] since impurities destroy the Lorentz invariance of the model. Some generalizations of the σ model have, nevertheless, been proposed with parameters modified according to Monte Carlo (MC) data [7] and the classical percolation theory [8].

In this work we study the problem of disorder using the t-matrix approach combined with a configurational average over the random positions of impurities. It also

leads to description of the system in terms of an effective medium with renormalized parameters. In an earlier work Harris and Kirkpatrik [4] have shown that in 2D a non-hydrodynamic term appears in the spin-wave self-energy which explicitly violates Lorentz invariance: $\Sigma_{\mathbf{k}}(\omega) \propto xk(\ln|\omega| - i\pi/2)$. A recent study by Wan *et al.* [9] has confirmed this result. One can show that the spectrum is overdamped at wavelengths $\Lambda \gtrsim \exp(\pi/4x)$ which implies the existence of a new length scale in the system. The absence of a well-defined long-wavelength mode also suggests that the true order should be unstable under an infinitesimal impurity doping, or that $d = 2$ is the upper critical dimension for this problem [4].

In this work we study the effect of impurities on the spectrum, staggered magnetization, $M(x)$, and Néel temperature, $T_N(x)$, to clarify the problem of stability of a (quasi-)2D AF order. We show that neither $M(x)$ nor $T_N(x)$ possesses anomalous contributions which would imply an instability. Thus, we have a somewhat paradoxical situation when the spectrum is ill defined while the order parameter is not affected. Such a dichotomy comes from the strong influence of disorder in the low-energy excitations in 2D. The averaging procedure, which effectively restores translational invariance, fails to recover the long-wavelength excitation spectrum of this effective medium [10]. The low-frequency modes do exist in some form but they cannot be classified in terms of an effective wave vector and thus the long-wavelength propagation is entirely diffusive. This scheme implies an existence of a disorder-induced energy scale which restricts the applicability of the continuum approach.

We start with the Hamiltonian (1) in the spin-wave approximation $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{imp}^A + \mathcal{H}_{imp}^B$:

$$\mathcal{H}_0 \simeq 4SJ \sum_{\mathbf{k}} \omega_{\mathbf{k}}(\alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}} + \beta_{\mathbf{k}}^\dagger \beta_{\mathbf{k}}), \qquad (2)$$

$$\mathcal{H}_{imp}^A \simeq -4SJ \sum_{l \in A, \mathbf{k}, \mathbf{k}'} e^{i(\mathbf{k}-\mathbf{k}')\mathbf{R}_l}[V_{A,\mathbf{k}\mathbf{k}'}^{\alpha\alpha}\alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}'} + V_{A,\mathbf{k}\mathbf{k}'}^{\beta\beta}\beta_{\mathbf{k}}^\dagger\beta_{\mathbf{k}'} + V_{A,\mathbf{k}\mathbf{k}'}^{\alpha\beta}(\alpha_{\mathbf{k}}^\dagger\beta_{-\mathbf{k}'}^\dagger + \mathrm{H.c.})], \qquad (3)$$

where the quadratic part of the pure host Hamiltonian $\mathcal{H}_0$ is diagonalized using Bogolyubov transformation:

$S_{\mathbf{k},A}^{\dagger}/\sqrt{2S} \simeq a_{\mathbf{k}} = u_{\mathbf{k}}\alpha_{\mathbf{k}} + v_{\mathbf{k}}\beta_{-\mathbf{k}}^{\dagger}$, $S_{\mathbf{k},B}^{\dagger}/\sqrt{2S} \simeq b_{\mathbf{k}}^{\dagger} = u_{\mathbf{k}}\beta_{\mathbf{k}}^{\dagger} + v_{\mathbf{k}}\alpha_{-\mathbf{k}}$, A and B denote the sublattices, with $u_{\mathbf{k}}^2 - v_{\mathbf{k}}^2 = 1$, $2u_{\mathbf{k}}v_{\mathbf{k}} = -\gamma_{\mathbf{k}}/\omega_{\mathbf{k}}$, $\gamma_{\mathbf{k}} = [\cos(k_x) + \cos(k_y)]/2$, and bare spin-wave frequency $\omega_{\mathbf{k}} = \sqrt{1 - \gamma_{\mathbf{k}}^2}$. All momenta belong to the magnetic Brillouin zone $k_x + k_y \leq \pi$. $\mathcal{H}_{\mathrm{imp}}^{B} = \mathcal{H}_{\mathrm{imp}}^{A}(A \to B)$, interactions are given by $V_A^{\alpha\alpha} = V_B^{\beta\beta} = u_1 u_2 + \gamma_{1-2}v_1 v_2 + \gamma_1 v_1 u_2 + \gamma_2 v_2 u_1$, $V_A^{\beta\beta} = V_B^{\alpha\alpha} = v_1 v_2 + \gamma_{1-2}u_1 u_2 + \gamma_1 u_1 v_2 + \gamma_2 u_2 v_1$, and $V_{A,1,2}^{\alpha\beta} = V_{B,2,1}^{\alpha\beta} = u_1 v_2 + \gamma_{1-2}v_1 u_2 + \gamma_1 v_1 v_2 + \gamma_2 u_2 u_1$, l runs over the impurity sites. As a first step we solve the scattering problem for the single impurity. Since the impurity potential is short ranged one uses the t-matrix approach. The advantage of this method is that the single impurity problem can be solved *exactly* (within the spin-wave approximation) [11]. Therefore, the results of this approach become exact in the dilute limit $x \to 0$.

In our case the square lattice site defect scatters only in s-, p_x-, p_y-, and d-symmetric channels, and one needs to solve the t matrix for each harmonic. The t-matrix equations are

$$\Gamma_{A,i}^{\alpha\alpha} = -V_{A,i}^{\alpha\alpha} - V_{A,i}^{\alpha\alpha} G_{\alpha}^0 \Gamma_{A,i}^{\alpha\alpha} - V_{A,i}^{\alpha\beta} G_{\beta}^0 \Gamma_{A,i}^{\alpha\beta},$$
$$\Gamma_{A,i}^{\alpha\beta} = -V_{A,i}^{\alpha\beta} - V_{A,i}^{\beta\beta} G_{\beta}^0 \Gamma_{A,i}^{\alpha\beta} - V_{A,i}^{\alpha\alpha} G_{\alpha}^0 \Gamma_{A,i}^{\alpha\alpha}, \quad (4)$$

where V_i's are the $i = s, p_x, p_y, d$ components of V's (3). We suppress $\mathbf{k}$ and ω in vertex functions $\Gamma_i(\mathbf{k}, \mathbf{k}', \omega)$, bare Green's functions $G_{\alpha}^0(\mathbf{k}, \omega) = G_{\beta}^0(\mathbf{k}, -\omega) = \{\omega - \omega_{\mathbf{k}} + i0\}^{-1}$, and interactions $V_{\mathbf{k}\mathbf{k}'}$. Products $VG\Gamma$ involve summation over the internal momenta, all energies are in units of $4SJ$. An equivalent set of equations gives the vertices $\Gamma_{A,i}^{\beta\beta}$. For the impurity in B sublattice $\Gamma_{B,i} \equiv \Gamma_{A,i}(\alpha \leftrightarrow \beta)$. Since the impurity potential in each partial wave is separable: $V_{A,s}^{\alpha\alpha} = u_{\mathbf{k}}\omega_{\mathbf{k}}u_{\mathbf{k}'}\omega_{\mathbf{k}'}$, $V_{A,p_\sigma}^{\alpha\alpha} = \phi_{k_\sigma}v_{\mathbf{k}}\phi_{k_\sigma'}v_{\mathbf{k}'}/2$, $V_{A,d}^{\alpha\alpha} = \gamma_{\mathbf{k}}^- v_{\mathbf{k}}\gamma_{\mathbf{k}'}^- v_{\mathbf{k}'}$, $V_{A,i}^{\beta\beta} = V_{A,i}^{\alpha\alpha}(u \leftrightarrow v)$, $V_{A,s}^{\alpha\beta} = -u_{\mathbf{k}}\omega_{\mathbf{k}}v_{\mathbf{k}'}\omega_{\mathbf{k}'}$, $V_{A,p_\sigma}^{\alpha\beta} = \phi_{k_\sigma}v_{\mathbf{k}}\phi_{k_\sigma'}u_{\mathbf{k}'}/2$, $V_{A,d}^{\alpha\beta} = \gamma_{\mathbf{k}}^- v_{\mathbf{k}}\gamma_{\mathbf{k}'}^- u_{\mathbf{k}'}$, where $\phi_{k_\sigma} = \sin(k_\sigma)$, $\gamma_{\mathbf{k}}^- = [\cos(k_x) - \cos(k_y)]/2$, Eqs. (4) can be readily solved: $\Gamma_{A,i}^{\sigma\sigma'}(\mathbf{k}, \mathbf{k}', \omega) = -V_{A,i}^{\sigma\sigma'}(\mathbf{k}, \mathbf{k}')\tilde{\Gamma}_i(\omega)$, where

$$\tilde{\Gamma}_s(\omega) = -1/\omega - (1 + \omega)\rho(\omega)/[1 - \omega(1 + \omega)\rho(\omega)],$$
$$\tilde{\Gamma}_p(\omega) = 2/\{1 + \omega + (1 - \omega)[\omega^2\rho(\omega) - \rho_d(\omega)]\}, \quad (5)$$
$$\tilde{\Gamma}_d(\omega) = 1/[1 + (1 - \omega)\rho_d(\omega)],$$

with $\rho = \sum_{\mathbf{p}} 1/(\omega^2 - \omega_{\mathbf{p}}^2)$ and $\rho_d = \sum_{\mathbf{p}}(\gamma_{\mathbf{p}}^-)^2/(\omega^2 - \omega_{\mathbf{p}}^2)$, which can be expressed through the complete elliptic integrals [12]. For the impurity in B sublattice $\Gamma_{B,i}^{\sigma\sigma'} = -V_{B,i}^{\sigma\sigma'}\tilde{\Gamma}_i(-\omega)$. The s-wave scattering (5) reveals a zero-frequency mode which originates from the oscillations of the fictitious degrees of freedom at the impurity site. Roughly speaking, since in the spin-wave approximation spins are quantized through bosons even if S' at the impurity site is set to zero there is still $a_0^{\dagger}a_0$ left in

S_0^z (for discussion, see Ref. [9]). This gives rise to the unphysical zero-frequency mode which has to be projected out. We do so by introducing magnetic fields at the impurity sites (similar to Refs. [13,14]) $\Delta\mathcal{H} = H_z \sum_l a_i^{\dagger} a_i$. Within this approach after straightforward algebra [12] in the limit $H_z \to \infty$ one obtains $\Gamma_{A,s}^{\sigma\sigma'}(\mathbf{k}, \mathbf{k}', \omega) = -V_{A,s}^{\sigma\sigma'}(\mathbf{k}, \mathbf{k}')\tilde{\Gamma}_s(\omega) + \Delta\Gamma_{A,s}^{\sigma\sigma'}(\mathbf{k}, \mathbf{k}', \omega)$,

$$\tilde{\Gamma}_s(\omega) = -(1 + \omega)\rho(\omega)/[1 - \omega(1 + \omega)\rho(\omega)], \quad (6)$$

which is free from the zero-frequency pole. $\Delta\Gamma_{A,s}^{\alpha\alpha} = u_{\mathbf{k}}u_{\mathbf{k}'}[\omega_{\mathbf{k}} + \omega_{\mathbf{k}'} - \omega]$, $\Delta\Gamma_{A,s}^{\beta\beta} = -v_{\mathbf{k}}v_{\mathbf{k}'}[\omega_{\mathbf{k}} + \omega_{\mathbf{k}'} + \omega]$, $\Delta\Gamma_{A,s}^{\alpha\beta} = -u_{\mathbf{k}}v_{\mathbf{k}'}[\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} + \omega]$. $\Gamma_B = \Gamma_A \times (-\omega)\{\alpha \leftrightarrow \beta\}$ as before, p and d waves are not affected by the projection.

The next step is the averaging procedure which restores translational invariance. Assuming random distribution of impurities one readily transforms scattering vertices into the spin-wave self-energies: $\Sigma_{\mathbf{k}}^{\sigma\sigma'}(\omega) = \sum_i \Sigma_{i,\mathbf{k}}^{\sigma\sigma'}(\omega)$, with $\Sigma_{i,\mathbf{k}}^{\sigma\sigma'}(\omega) = x\delta_{\mathbf{k}-\mathbf{k}'} \times [\Gamma_{A,i}^{\sigma\sigma'}(\mathbf{k}, \mathbf{k}', \omega) + \Gamma_{B,i}^{\sigma\sigma'}(\mathbf{k}, \mathbf{k}', \omega)]$

$$\Sigma_{s,\mathbf{k}}^{\alpha\alpha}(\omega)/x = -\omega_{\mathbf{k}}^2[u_{\mathbf{k}}^2\tilde{\Gamma}_s(\omega) + v_{\mathbf{k}}^2\tilde{\Gamma}_s(-\omega)] + 2\omega_{\mathbf{k}} - \omega,$$
$$\Sigma_{p(d),\mathbf{k}}^{\alpha\alpha}(\omega) = xA_{\mathbf{k}}^{p(d)}[v_{\mathbf{k}}^2\tilde{\Gamma}_{p(d)}(\omega) + u_{\mathbf{k}}^2\tilde{\Gamma}_{p(d)}(-\omega)], \quad (7)$$
$$\Sigma_{i,\mathbf{k}}^{\alpha\beta}(\omega) = xB_{\mathbf{k}}^i[\tilde{\Gamma}_i(\omega) + \tilde{\Gamma}_i(-\omega)],$$
$$\Sigma_{i,\mathbf{k}}^{\beta\beta}(\omega) = \Sigma_{i,\mathbf{k}}^{\alpha\alpha}(-\omega),$$

where $A_{\mathbf{k}}^p = -\omega_{\mathbf{k}}^2 + (\gamma_{\mathbf{k}}^-)^2$, $A_{\mathbf{k}}^d = -(\gamma_{\mathbf{k}}^-)^2$, $B_{\mathbf{k}}^s = \omega_{\mathbf{k}}^2 u_{\mathbf{k}}v_{\mathbf{k}}$, $B_{\mathbf{k}}^p = A_{\mathbf{k}}^p u_{\mathbf{k}}v_{\mathbf{k}}$, $B_{\mathbf{k}}^d = A_{\mathbf{k}}^d u_{\mathbf{k}}v_{\mathbf{k}}$. It is interesting that "on shell" ($\omega = \omega_{\mathbf{k}}$) Eqs. (5) and (6) yield identical $\Sigma_{s,\mathbf{k}}(\omega_{\mathbf{k}})$.

At low energies $\omega, \omega_{\mathbf{k}} \to 0$ self-energies are given by

$$\Sigma_{\mathbf{k}}^{\alpha\alpha}(\omega) \simeq x\omega_{\mathbf{k}}[\rho(\omega) + 2 - \pi/2] - x\omega + \mathcal{O}(\omega_{\mathbf{k}}\omega^2\rho^3),$$
$$\Sigma_{\mathbf{k}}^{\alpha\beta}(\omega) \simeq x\omega_{\mathbf{k}}[\rho(\omega) + \pi/2] + \mathcal{O}(\omega_{\mathbf{k}}\omega^2\rho^3), \quad (8)$$

with $\rho(\omega) \simeq (2/\pi)\ln(|\omega|/4) - i$,

which includes contributions from s- and p-wave scattering, $\Sigma_d \simeq \mathcal{O}(\omega_{\mathbf{k}}^3)$. One can see that besides the "normal" softening of the long-wavelength mode and the damping proportional to the higher power of k ($\sim k^3$) one acquires a nonlinear dispersion term with the damping $\tilde{\gamma}_{\mathbf{k}}/\omega_{\mathbf{k}} \sim x$ having only parametric smallness with respect to the bare spectrum. Renormalization of the real part of the spectrum is dominated by the $\ln(|\omega|)$ term at low frequencies. A naive "on-shell" pole equation $\tilde{\omega}_{\mathbf{k}}' = \omega_{\mathbf{k}} + \mathrm{Re}\Sigma_{\mathbf{k}}^{\alpha\alpha}(\omega_{\mathbf{k}})$ suggests a vanishing of the spectrum [9] and an instability of the ground state. Consideration of the diagrammatic series for the Green's functions shown in Fig. 1 with self-energies defined in Eq. (7) and $G_{\mathbf{k}}^{11}(\omega) = \{\omega - \omega_{\mathbf{k}} - \Sigma_{\mathbf{k}}^{\alpha\alpha}(\omega)\}^{-1}$ gives
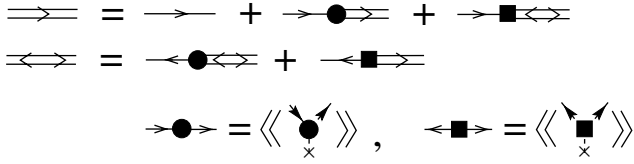
FIG. 1.   Belyaev diagram series for the diagonal, $G^{\alpha\alpha}$, and off-diagonal, $G^{\alpha\beta}$, Green's functions. Self-energies $\Sigma^{\alpha\alpha}$ (circle) and $\Sigma^{\alpha\beta}$ (square) are the configurational averages of $\Gamma^{\alpha\alpha}$ and $\Gamma^{\alpha\beta}$, respectively.

$$G_{\mathbf{k}}^{\alpha\alpha}(\omega) = \frac{G_{\mathbf{k}}^{11}(-\omega)^{-1}}{[G_{\mathbf{k}}^{11}(\omega)G_{\mathbf{k}}^{11}(-\omega)]^{-1} - [\Sigma_{\mathbf{k}}^{\alpha\beta}(\omega)]^2},$$
$$G_{\mathbf{k}}^{\alpha\beta}(\omega) = \frac{\Sigma_{\mathbf{k}}^{\alpha\beta}(\omega)}{[G_{\mathbf{k}}^{11}(\omega)G_{\mathbf{k}}^{11}(-\omega)]^{-1} - [\Sigma_{\mathbf{k}}^{\alpha\beta}(\omega)]^2}. \tag{9}$$

At $k \gg \omega_0 \sim \exp(-\pi/4x)$ spectral function reveals a damped quasiparticle peak at $\omega \lesssim k$ plus some structure at low frequencies $\omega \sim \omega_0$. At low $k \sim \omega_0$ the quasiparticle peak merges with this structure and disappears. The spectrum is completely overdamped at $k \lesssim \omega_0$. Beyond this scale no hydrodynamic description is possible.

A potential concern is whether an anomaly in Eq. (8) can be fixed by the higher order terms. General discussion of this problem in terms of the susceptibility (Ref. [4])

concluded that the divergency should be unrelated to the expansion in x. One can also show that the summation of the Green's function series (9) and dressing of the inner lines in self-energies using self-consistent equation on $\rho(\omega)$ and $G^{\sigma\sigma}$ actually enhance the anomaly.

In a complementary problem of a quasi-2D AF a small interplane coupling $\alpha = J_\perp/J$ cuts off the log singularity: $\rho_{3D}(\omega) \simeq (1/\pi)\ln(\alpha/32) + i\mathcal{O}(\omega)$ at $\omega < \sqrt{2\alpha}$. Therefore a "safe" range of concentrations $x < x^* \sim \ln^{-1}(1/\alpha)$ can be found where the long-wavelength quasiparticles are still well defined deep in the 3D region of the $\mathbf{k}$ space ($\Lambda^{-1} \ll \sqrt{\alpha}$). However, one should be able to observe a nonlinearity of the spectrum and an abnormal damping of the spin waves in the 2D long-wavelength region ($\sqrt{\alpha} < \Lambda^{-1} \ll 1$) similar to the quasi-1D problem [15]. For the real materials $\alpha \sim 10^{-4}$ giving $x^* \sim 20\%$. Above the concentration x^* all the low-energy excitations are incoherent because the 2D disorder-induced scale ω_0^{-1} is shorter than the 3D length $1/\sqrt{\alpha}$ so the spin waves lose coherence before they can propagate in 3D. The same consideration applies to the case of small anisotropies introducing gaps in the spectrum with a modified $\alpha = \alpha_{\text{eff}}$ accumulating the total effect of the gaps and 3D coupling.

Now we proceed with quantities whose expansion in x can be shown to be reliable. Staggered magnetization at the magnetic site $M(x) = M_0 - \delta M(x)$, $M_0 = S - \delta\lambda$,

$$\delta M(x) = \sum_{\mathbf{k}} [\langle\alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}}\rangle - \gamma_{\mathbf{k}}\langle\alpha_{\mathbf{k}}^\dagger \beta_{\mathbf{k}}^\dagger\rangle]/\omega_{\mathbf{k}} = -\sum_{\mathbf{k}} \int_{-\infty}^{\infty} \frac{n(\omega)\,d\omega}{\pi\omega_{\mathbf{k}}} \text{Im}[G_{R,\mathbf{k}}^{\alpha\alpha}(\omega) - \gamma_{\mathbf{k}} G_{R,\mathbf{k}}^{\alpha\beta}(\omega)], \tag{10}$$

where $\delta\lambda = \sum_{\mathbf{k}} v_{\mathbf{k}}^2 \simeq 0.1966$ is from zero-point fluctuations, $n(\omega) = [e^{\omega/T} - 1]^{-1}$ is the Bose occupation number, index R denotes retarded. At $T = 0$ $n(\omega) = -\theta(-\omega)$ and integrals in (10) with $G^{\sigma\sigma'}$ from (9) can be taken numerically. It can be shown [12] that the expansion in x for $G^{\sigma\sigma'}$ can be performed before the integration $G^{\alpha\alpha} \simeq G_\alpha^0 + G_\alpha^0 \Sigma^{\alpha\alpha} G_\alpha^0$, $G^{\alpha\beta} \simeq G_\beta^0 \Sigma^{\alpha\alpha} G_\alpha^0$ and that all integrals for the linear in x term are convergent giving $\delta M(x) = 0.209(8)x + \mathcal{O}(x^2)$. For $S = 1/2$ $M(x)/M_0 \simeq 1 - 0.691(5)x$. This linear slope together with the results of calculations using $G^{\sigma\sigma'}$ from Eq. (9), MC [16], and NMR data [17] are shown in Fig. 2. Note that Fig. 2 and Eq. (10) show the reduction of the magnetic moment by the quantum fluctuations induced by impurities. In order to extract the same quantity from the MC data [16], which are averaged over all sites, one needs to deduct a probability of finding a spin-occupied site from them. One can see a very good agreement of our results with numerical data up to high concentrations. The oxidation of the crystals can be the reason of a faster decrease of $M(x)$ in NMR data. The absolute value of $\delta M(x)$ is independent of S in the linear spin-wave approximation similar to the quantum reduction of S by $\delta\lambda$. We plot $\delta M(x)$ in Fig. 2 (right axes) in order to emphasize the agreement with the MC data for $S = 1/2$ and $S = 1$, which show only weak S dependence.

The quantity which one would expect to exhibit a different behavior is T_N for quasi-2D problem. The thermal corrections to the staggered magnetization $\delta M^T \sim T\ln(T/\omega)|_{\omega=\sqrt{\alpha}}$ yield a finite $T_N \sim \ln(1/\alpha)^{-1}$ for a pure system. Since the original spectrum is bent from the linear
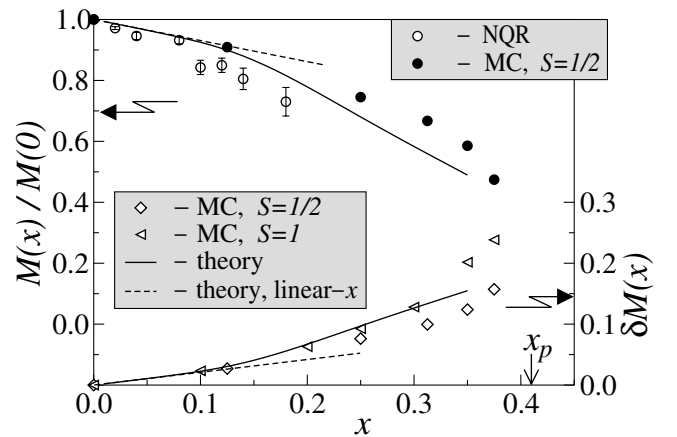

FIG. 2.   Left axes: reduction of the local magnetic moment by the quantum fluctuations induced by impurities $M(x)/M(0)$ from NMR ($\bigcirc$, Ref. [17]) and MC ($\bullet$, Ref. [16]) studies. Lines show our results from Eq. (10). Right axes: the absolute value of $\delta M(x)$ from Eq. (10) (lines) and MC data, Ref. [16], for $S = 1/2$ ($\diamond$) and $S = 1$ ($\triangleleft$).
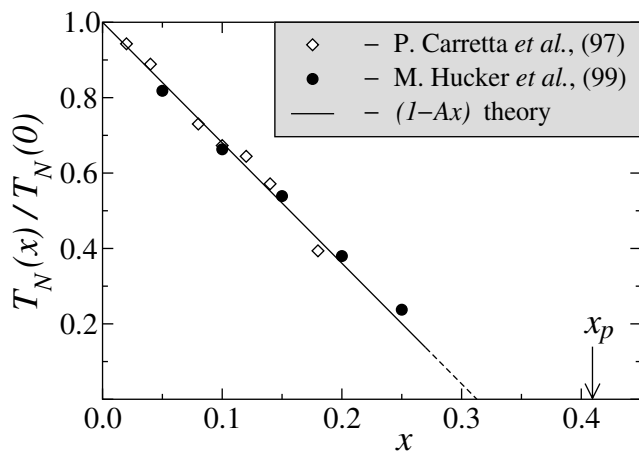
FIG. 3. Néel temperature vs x from μSR and susceptibility measurements Refs. [19,20] for the $La_2Cu_{1-x}Zn(Mg)_xO_4$, and the linear slope from our theory.

form this should manifest itself in the finite temperature part of Eq. (10) as a stronger divergency. One indeed finds such contributions in Eq. (10) ($\sim xT \ln^2|\omega|$) already in a perturbative limit ($x \ln|\omega| \ll 1$). However, these anomalous terms from diagonal and off-diagonal parts cancel each other. From the mean-field equation $M^{T_N}(x) = 0$ after some algebra one obtains (at $\alpha \rightarrow 0$): $T_N(x)/T_N(0) \simeq 1 - xA$ with $A = \pi - 2/\pi + 0.209(8)/M_0$. For $S = 1/2$, $A = 3.196(5)$ [18], which fits very well experimental data [19,20]; see Fig. 3.

Thus the order is preserved up to a high x value [3,16] while the long-wavelength spectrum is not well defined at any x. Experimentally one would need to probe directly the spin-wave spectrum close to the AF ordering vector and look for anomalous broadening and nonlinearity. The observed deviation from the simple exponential behavior of the correlation length $\xi(T, x)$ vs $1/T$ [3] might be related to the character of the low-energy spectrum we discuss.

Another aspect relates our problem to the other problems of disorder in 2D. We observe that some weight from every mode is transferred to the low energies $\omega \sim \omega_0$ which can be understood as some sort of localization, though the localization criteria for this problem is unclear. Altogether these low-energy states result in a peak in the density of states. From the x-expanded form of the Green's function $G^{\alpha\alpha} \simeq G_\alpha^0 + G_\alpha^0 \Sigma^{\alpha\alpha} G_\alpha^0$ one readily gets $N(\omega) = 2\omega/\pi + xC + \mathcal{O}(x\omega \ln|\omega|)$, which implies a finite density of states at $\omega = 0$. With $G^{\alpha\alpha}$ from Eq. (9) one obtains $N(\omega) \approx 2\omega/\pi + xC/[(1 + 4x \ln|\omega|/\pi)^2 + 4x^2]$ which is zero strictly at $\omega = 0$ but has a sharp peak $\sim 1/x$ at $\omega = \omega_0$. This is very close to the dispute over $N(\omega)$ for the certain types of disorder in 2D systems with linear excitation spectrum [10]. Note that in our case there is no "weak" limit for disorder since the perturbation ($\sim JSz$) is of the same strength as the pure coupling [21].

In conclusion, we have provided evidence for the stability of the long-range order in the 2D and quasi-2D AF doped with static vacancies. At the same time the

long-wavelength excitation spectrum is shown to be diffusive for any value of doping, restricting a hydrodynamic approach to the problem from low energies.

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