Localized Gravity in String Theory

Andreas Karch and Lisa Randall
Center for Theoretical Physics & Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139
(Received 14 May 2001; published 23 July 2001)

We propose a string realization of the AdS_4 brane in AdS_5 that is known to localize gravity. Our theory is M D5 branes in the near horizon geometry of N D3 branes, where M and N are appropriately tuned.

DOI: 10.1103/PhysRevLett.87.061601 PACS numbers: 11.25.Mj, 04.50.+h, 04.65.+e, 11.10.Kk

String theories are consistent only with many additional dimensions. Ever since the notion of extra dimensions was introduced, it was believed that consistency with observed four-dimensional gravity requires that these dimensions are of finite extent, so it has been held that a viable string theory background requires the compactification of the additional dimensions. Gravity localization provides an alternative to this viewpoint; it is necessary only that a bound state graviton mode dominates over the Kaluza-Klein (KK) modes associated with the additional dimensions over the length scales for which we have observed four-dimensional gravity [1]. In [2] an effective five-dimensional Lagrangian was given which exhibits this phenomenon. However, it seemed difficult to realize this possibility in string theory or even a supersymmetric theory [3–8]. One of the major difficulties in the attempted constructions was the attempt to eliminate the infinite volume region of space; that is, geometry had to satisfy a finite "volume" condition; alternatively, there were stringent conditions on the regions of space far from the localizing brane, including the boundary. Attempts to realize the warped geometry in string theory dealt with this through explicit compactification so the world is manifestly four dimensional at long distances [3]. However, it was recently shown that the localization idea is far more general and does not require anything about the regions of space far from the brane [1]. The important point is that gravity needs to be localized only in a region of finite extent; the full space might truly reflect the higher-dimensional geometry. This is a dramatically different picture from compactification, which requires global properties of the space-time. It is also a much weaker requirement on the geometry than the original model [2] seemed to indicate. Furthermore, the supergravity solution is known explicitly only for a very limited class of objects within string theory; it seems quite credible that the required geometric features could be realized in a string background. We initiate an exploration of this possibility in this Letter.

Our proposal is that a setup with N D3 branes (say along 0126) intersecting M D5 branes (say along 012345) over a common three-dimensional world volume can lead to localized gravity in a four-dimensional anti–de Sitter (AdS) space. The fact that this setup preserves supersymmetry (in fact, 16 supersymmetries) guarantees stability.

A special property of the AdS brane is that it lends itself to a very natural holographic interpretation [1,9]. As shown by Cardy [10], $SO(3,2)$ is the subgroup of the 4D conformal group $SO(4,2)$ that preserves a given boundary. This means the symmetry of AdS_4 is preserved.

As we showed in another paper [9] (see also [11]) the $AdS_5 \times S^5$ near horizon geometry setup by the N D3 branes, the 5-branes span an $AdS_4 \times S^2$ world volume. This is very easy to see already from the way $AdS_5 \times S^5$ arises as the D3 near horizon geometry: the dictionary of AdS/CFT (conformal field theory) comes with an identification of flat space embedding coordinates of the D3 brane and the near horizon geometry. When AdS_5 is written in Poincaré patch coordinates, the four Minkowski directions are the four world volume directions of the D3 (0126). The transverse directions are written in spherical coordinates. The radial direction of this flat transverse space becomes the radial direction in AdS_5 and the sphere surrounding the D3 brane is the S^5 of the near horizon geometry. The D5 brane, whose defining equation in the flat embedding space coordinates is

$$x_6 = x_7 = x_8 = x_9 = 0,$$

hence has an $AdS_4 \times S^2$ world volume, both of curvature radius L. The latter is given by $x_7 = x_8 = x_9 = 0$ and the former by $x_6 = 0$.

In this Letter, we show that we expect such a brane to be a viable candidate for localized gravity. One can already argue from the gravity side why such a wrapped 5-brane in the near horizon geometry of the D3 branes should describe localized gravity. Forgetting for the moment the issue of stability, consider a probe brane inside the near horizon geometry of a stack of N D3 branes. The near horizon geometry is $AdS_5 \times S^5$. A 5-brane wrapped over an S^2 inside the S^5 effectively becomes a 3-brane in AdS_5. A 3-brane living in the AdS_5 creates a warped geometry of the sort considered in [1,12] (see also [13]), that is, an AdS_4 brane. This is possible because five of the dimensions in the near horizon region have already been compactified, so a 3-brane is adequate to provide a warped AdS geometry.

We now argue that the corresponding near horizon solution of the intersecting D3-D5 configuration should localize gravity along the lines of [1,12,14,15]. The reasons for this are as follows:

The conformal symmetry.—As shown in [9], the geometry setup by the intersecting D3-D5 configuration has a dual CFT description in terms of a CFT with boundary. This dual makes it clear that SO(3, 2) symmetry is preserved, so our space-time has an AdS_4 slicing.

Asymptotics.—Far away along the 6 direction the geometry is still that of the D3 branes alone. So the geometry asymptotes to $AdS_5 \times S^5$ for r going to $\pm\infty$, where r is the transverse coordinate appearing in the warp factor.

Discrete symmetry.—The setup with the intersecting branes has a discrete Z_2 symmetry, $x_6 \rightarrow -x_6$. Hence the warp factor will have to have the same symmetry, $r \rightarrow -r$.

Positive tension.—Since the energy density on the 5-brane is positive, the jump in extrinsic curvature has to be reflected by a jump in the warp factor that has the standard "up-down" shape.

At the moment the full localized supergravity solution is not known even though the results from [9] should help to construct one. Instead, we consider the backreaction from a five-dimensional effective theory point of view. In the 5D string frame the Einstein-Hilbert term is given by

$$\frac{L^5}{g_s^2} R ,$$

while the tension of M D5 branes wrapping the S^2 of radius L^2 is given by

$$4\pi T_{D5} M L^2,$$

where we need to recall that $T_{D5} \sim \frac{1}{g_s}$. So the jump in extrinsic curvature is proportional to

$$M g_s .$$

First note that for $g_s \rightarrow 0$, $g_s N$ fixed, but $g_s M \rightarrow 0$, the backreaction can be neglected. It was in this limit that the probe calculation of [9] established that the D5 brane world volume is $AdS_4 \times S^2$. Clearly we need to get away from this limit to get a significant backreaction and a localized graviton.

Figure 1 summarizes the essential features of the solution for finite $g_s M$. As we increase the tension by increasing $g_s M$, l, the curvature scale of the four-dimensional AdS space, increases. The critical value at which the 4D cosmological constant vanishes in this approximation is

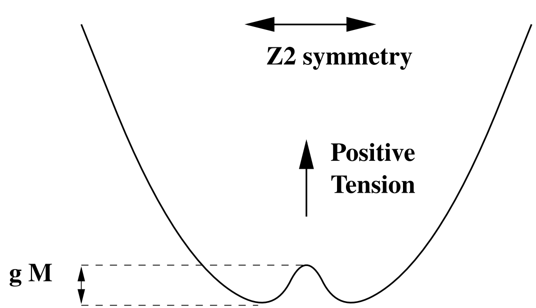

FIG. 1. Turning on $g_s M$ at $q = 0$.

$$T_{\text{crit}} = \frac{3}{4\pi G_N L},$$

where

$$G_N = (2\pi)^3 g_s^2 l_s^8 / L^5$$

is the 5D Newton's constant. Since $L \sim (g_s N)^{1/4}$, we see that the critical tension is reached when $L^2 g_s M$ is of order $L^4 \sim g_s N$, that is, when $g_s M$ is of order $(g_s N)^{1/2}$. This is the point at which the size of the 5-brane throat $r_{\text{throat}} \sim (g_s M)^{1/2}$ (as computed in flat space with an unwrapped 5-brane) becomes comparable to the AdS_5 curvature radius L. We expect that at this point one can no longer treat the wrapped 5-brane as a thin 3-brane brane in AdS_5. In order to get an analytic expression for l in terms of $g_s M$ one would need the corresponding gravity solution. We expect as we approach the critical value, the effective field theory approach where we neglect the effect of the 5-branes on the sphere will break down, so one requires the full ten-dimensional solution. However, we anticipate the essential properties of the solution will remain. It is reassuring that the point at which a very flat brane happens, that is, the number of 5-branes is sufficient to generate the critical tension, is when the analysis we have done is just breaking down, so there should be a value of $g_s M$ for which $l \gg L$. We therefore expect that the setup of intersecting D3 and D5 branes is a string theory realization of localized gravity for sufficiently many 5-branes.

We can also argue for the viability of the theory as a theory of localized gravity from the point of view of the boundary CFT. The 5D gravitons give rise to the $\frac{1}{x^8} TT$ correlators of the 4D CFT. A localized 4D mode of the graviton corresponds to $\frac{1}{x^6}$ correlators on the boundary of the CFT. This means from the dual point of view there should exist a three-dimensional field theory on the boundary. The existence of the three-dimensional theory is a necessary (but not sufficient) condition for gravity localization. This was not satisfied, for example, in a theory where q $D3$ branes end on a single $D5$ brane.

In the language of the boundary CFT, for $M \geq 2$ there exists an interacting 3D CFT on the boundary whose degrees of freedom grow like MN^2. To see this, replace for a moment the D5 branes with NS5 branes and keep them at finite separation. The gauge theory realized by this brane configuration is 4D SU(N) with a "fat" defect which has a 3D $U(N)^{M-1}$ gauge theory with bifundamentals living on it, the A-type quiver, as one can determine by the usual rules [16]; see Fig. 2. This theory flows to an interacting CFT at the origin of its moduli space, when we take the couplings to infinity, that is, take the NS5 branes to coincide. D5 branes give a mirror realization of the same CFT. The stress energy tensor of this 3D CFT should couple to a mode of the 5D graviton localized on the brane. This

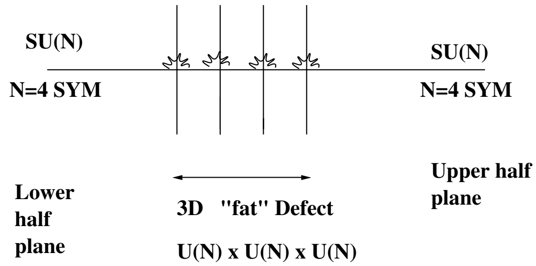

FIG. 2. The fat domain wall giving rise to the CFT associated with a U$(M)^3$ gauge theory for $M = 4$ NS5 branes.

serves as a further check that this theory is a reasonable candidate for gravity localization.

Now we can ask about going beyond the near horizon region. It is important that localization is a local phenomenon; even if asymptotically the three branes give flat space, one does not lose the graviton and its associated four-dimensional gravitational effects. All that happens is that in the regions where we do not expect four-dimensional gravity, the KK contribution is not suppressed relative to the graviton contribution. This is similar to what happens for a positive tension brane in flat space; there still is a graviton, but the KK modes are not suppressed since there is no barrier in the volcano potential. What is important is that there is *some* region with a barrier; the probe calculation argues this is the case. This suffices for gravity for length scales that are not too large.

The localized gravity setup is still relatively young and we have yet to realize the full range of possibilities. What is clear is that one can localize gravity in such a way that only local regions see the localized graviton as dominating the gravitational force. This means one can have a setup where four-dimensional gravity applies only in specific regions of space. In some ways, this is a rather compelling picture in that one does not need to assume radical global properties of the full higher-dimensional space. What we have demonstrated in this Letter is that intersecting branes are sufficient to realize four-dimensional gravity.

It is nice that this construction involves multiple branes, so that there is a realistic possibility that one can realize gauge configurations, and in particular, the standard model, in such a setup.

Of course, the setup as it now stands seems to rely on the fact that we have AdS space, while current evidence is that we live in dS space with small cosmological constant. However, bear in mind that the setup as described is completely supersymmetric. It is not clear how the geometry will be modified with new sources of energy on the brane.

We have argued from an effective theory perspective that we expect a range of M where gravity localization is valid. What is really required to verify this is the full supergravity solution. The fact that this is difficult should not be seen as an argument against localization. After all, we have rigorously demonstrated the AdS nature of the brane, so there must exist *some* supergravity solution. It is an important and challenging problem to find it.

We conclude that localized gravity is most likely a viable alternative to compactification. Because all curvature scales are set by parameters of the theory (gauge charge and tension), many of the difficult moduli problems should not be present. Of course, there is a multiplicity of possible vacua corresponding to the many possible brane setups. Perhaps it is hopeful that a four-dimensional world can exist inside many possible brane structures, since we are sensitive only to the local geometry.

We thank Andy Srominger for many useful suggestions and observations. We also thank Amanda Peet, Emil Martinec, Rob Myers, and Joe Polchinski.

[1] A. Karch and L. Randall, hep-th/0011156.
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[3] H. Verlinde, Nucl. Phys. **B580**, 264–274 (2000).
[4] M. Cvetic *et al.,* hep-th/0011167.
[5] M. J. Duff, J. T. Liu, and K. S. Stelle, hep-th/0007120.
[6] K. Behrndt and M. Cvetic, Phys. Rev. D **61**, 101 901 (2000).
[7] R. Kallosh and A. Linde, J. High Energy Phys. **02**, 005 (2000).
[8] J. Maldacena and C. Nunez, hep-th/0007018.
[9] A. Karch and L. Randall, hep-th/0105132.
[10] J. L. Cardy, Nucl. Phys. **B240**, 514–532 (1984).
[11] C. Bachas and M. Petropoulos, hep-th/0012234.
[12] I. I. Kogan, S. Mouslopoulos, and A. Papazoglou, Phys. Lett. B **501**, 140–149 (2001).
[13] A. Gorsky and K. Selivanov, Phys. Lett. B **485**, 271–277 (2000).
[14] A. Miemiec, hep-th/0011160.
[15] M. D. Schwartz, Phys. Lett. B **502**, 223–228 (2001).
[16] A. Hanany and E. Witten, Nucl. Phys. **B492**, 152–190 (1997).