# Neoclassical Radial Electric Field and Transport with Finite Orbits

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Neoclassical transport in a toroidal plasma with finite ion orbits is studied, including for the first time the self-consistent radial electric field. Using a low-noise $\delta f$ particle simulation, we demonstrate that a deep electric-field well develops in a region with a steep density gradient, because of the self-collision–driven ion flux. We find that the electric field agrees with the standard neoclassical expression, when the toroidal rotation is zero, even for a steep density gradient. Ion thermal transport is modified by the electric-field well in a way which is consistent with the orbit squeezing effect, but smoothed by the finite orbits.

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In magnetic confinement fusion experiments with improved confinement regimes (H mode and internal transport barrier) [1] where turbulence is suppressed and collisional transport is dominant at least for the ion channel, standard neoclassical theory [2] is routinely compared with experimental results. However, this theory assumes that orbit widths are small compared with plasma gradient scale lengths, and is not valid in tokamak plasmas with internal transport barriers. A theory of collisional plasma transport in toroidal magnetic confinement devices, which extends the standard neoclassical theory to allow finite orbit widths, does not yet exist. Although some work has been done on this problem [3], the self-consistent radial electric field $E_r$ has not been included. Determining $E_r$ is a fundamental issue in understanding the physics of plasma confinement. Without a self-consistent radial electric field, neoclassical particle transport is not ambipolar because of self-collision–driven ion flux; ambipolarity requires the development of a neoclassical radial electric field. The electric field in turn affects the collisional transport rates through finite orbit effects, even though the neoclassical transport rates do not depend on the radial electric field in the limit of the small orbit width.

In this Letter, the neoclassical electric field and collisional transport under this self-consistent electric field in toroidal plasma are investigated for the first time using a low-noise $\delta f$ simulation. Our emphasis is on the conditions of steep plasma gradients which characterize the enhanced confinement plasmas in current tokamak experiments. We neglect transport effects due to turbulence. The main results are as follows. Ambipolar neoclassical transport with vanishing self-collision–driven ion flux is demonstrated as the self-consistent electric field $E_r$ develops. The equilibrium $E_r$ is found, regardless of the steepness of the density profile, to be consistent with the standard neoclassical expression for $E_r$ when the ion parallel flow is small (for uniform temperature, the expression reduces to a Boltzmann relation). A deep electric-field well (with strong shear) is found in a region with steep density gradient. Ion thermal transport is reduced on the side of the well with negative $E_r$ shear and increased on the outer side with positive shear, relative to the standard neoclassical level. This is consistent with the orbit squeezing effect of the $E_r$ [4] and is obtained for the first time in numerical simulation. Also, the ion heat flux near the magnetic axis is reduced by the effects of finite orbits and the electric field.

The basic equation governing the radial electric field is, from Poisson's equation and the continuity equations, $\partial E_r/\partial t = -4\pi j_r$, where the radial current $j_r$ is the sum of the classical ion polarization current, $j_p^c = (n_i m_i c^2/B^2)\partial E_r/\partial t$, and the ion guiding-center current (the electron current is neglected because it is smaller than the ion current by a factor of mass ratio $m_e/m_i$). In terms of a general magnetic surface geometry the equation for the radial electric field becomes

$$\left[\langle|\nabla\psi|^2\rangle + 4\pi n_i m_i c^2\left\langle\frac{|\nabla\psi|^2}{B^2}\right\rangle\right]\frac{\partial^2\Phi}{\partial t\partial\psi} = 4\pi e\Gamma_i\,, \quad (1)$$

where $\Phi$ is the potential, $\psi$ is the poloidal magnetic flux, $e$ and $n_i$ are the ion charge and density, respectively, $c$ is the light speed, and the angular brackets denote the flux surface average. Here the flux of the ion guiding centers $\Gamma_i$ is the sum of neoclassical diffusion and polarization currents: $\langle(\vec{j}_d^{nc} + \vec{j}_p^{nc})\cdot\nabla\psi\rangle = e\Gamma_i \equiv e\langle\int d^3v(\vec{v}_d\cdot\nabla\psi)f_i\rangle$. The radial guiding center drift is $\vec{v}_d\cdot\nabla\psi = Iv_\parallel\hat{b}\cdot\nabla(v_\parallel/\Omega_i)$, where $I = RB_\zeta$ (with $R$ being the major radius and $B_\zeta$ being the toroidal component of magnetic field $\vec{B}$, $\hat{b} = \vec{B}/B$), $v_\parallel$ is the particle parallel velocity, and $\Omega_i = eB/m_ic$ is the ion gyrofrequency.

The ion guiding center distribution function $f_i$ obeys the drift kinetic equation,

$$\frac{\partial f_i}{\partial t} + \frac{e}{m_i}\frac{\partial\Phi}{\partial t}\frac{\partial f_i}{\partial\varepsilon} + (\vec{v}_\parallel + \vec{v}_d)\cdot\nabla f_i = C[f_i, f_i], \quad (2)$$

where the independent velocity variables are the magnetic moment and energy $\varepsilon = v^2/2 + e\Phi/m_i$ with $v$ the particle speed, and $C$ is the ion-ion collision operator.

In the limit of the small orbit width, partial transport is intrinsically ambipolar to second order in ion poloidal gyroradius $\rho_{i\theta}$ over plasma gradient length $L$, independent of the strength of the radial electric field. In order to determine the electric field, one needs to calculate the self-collision–driven ion flux or the toroidal angular momentum flux, $\Pi_i \equiv \langle \int d^3v\,(Iv_\parallel/\Omega_i)\,(\vec{v}_d \cdot \nabla\psi) f\rangle$, which gives fourth order transport rates, where the drift kinetic equation must be solved to second order. An analytical solution was obtained in the banana regime [5]; in the case of a zero temperature gradient, the ion flux is

$$\Gamma_i = -\left\langle \frac{I^2}{\Omega_i^2} \right\rangle \frac{n_i e}{m_i} \frac{\partial^2 \Phi}{\partial\psi \partial t} + \frac{1}{\mathcal{V}'} \frac{\partial}{\partial\psi} (\mathcal{V}' \Pi_i), \tag{3}$$

where $\mathcal{V}$ is the volume enclosed by a magnetic surface $\psi =$ const, and $\mathcal{V}' = d\mathcal{V}/d\psi$; the first term gives the neoclassical polarization current and the second term gives the diffusion current. The toroidal angular momentum flux $\Pi_i$ is proportional to $(\partial^2/\partial\psi^2)(\ln n_i + e\Phi/T_i)$. The steady-state condition $\partial\Phi/\partial t = 0$ requires $\Pi_i = 0$. Thus we obtain a Boltzmann-like relation for the equilibrium potential:

$$\ln\frac{n_i}{n_a} + \frac{e\Phi}{T_i} = b\psi, \tag{4}$$

where $b$ is a constant related to the edge toroidal rotation and $n_a$ is a constant.

In order to study the problem in general and realistic toroidal plasmas, we employ $\delta f$ particle simulation. The $\delta f$ simulation solves the drift kinetic equation based on the decomposition $f_i = f_M + \delta f$, where $f_M = n_i e^{e\Phi/T_i}(m_i/2\pi T_i)^{3/2} e^{-m_i\varepsilon/T_i}$ with the density and temperature defined as $n_i \equiv \langle \int f_i\, d\vec{v}\rangle$ and $T_i \equiv \langle \int (m_i v^2/3) f_i\, d\vec{v}\rangle/n_i$. The drift kinetic equation for $\delta f$ is

$$\frac{\partial \delta f}{\partial t} + \frac{e}{m_i}\frac{\partial\Phi}{\partial t}\frac{\partial \delta f}{\partial\varepsilon} + (\vec{v}_\parallel + \vec{v}_d)\cdot\nabla\delta f - C[\delta f, f_M] = -\left(\frac{\partial f_M}{\partial t} + \frac{e}{m_i}\frac{\partial\Phi}{\partial t}\frac{\partial f_M}{\partial\varepsilon} + \vec{v}_d\cdot\nabla f_M\right) + C[f_M, \delta f], \tag{5}$$

which is solved as an initial value problem. The self-consistent radial electric field is calculated from Eq. (1) with the ion flux $\Gamma_i$ being evaluated as a moment of $\delta f$. Note that keeping the drift term in the left-hand side of Eq. (5) allows us to study the finite orbit effects while the plasma gradient is large. Our $\delta f$ simulation uses a two-weight algorithm [6] incorporating a noise reduction algorithm [7]. The two-weight algorithm, rigorously reproducing the drift kinetic equation, ensures consistent simulations in solving the equation and the noise reduction algorithm achieves its goal by introducing damping terms in the weight equations. The ion-ion Coulomb collisions are modeled by a linear Monte Carlo collision operator which was demonstrated to conserve momentum, energy, and particle number nearly perfectly [6].

Our fully toroidal $\delta f$ code FORTEC-E follows a toroidal plasma to steady state in a magnetic coordinate system. In each simulation the initial toroidal plasma has a local Maxwellian distribution with given density and temperature profiles. The change in density is negligible because it occurs on a time scale longer than that for $E_r$. Since the initial toroidal rotation is zero, the angular momentum relaxation associated with the electric-field development is insignificant, leading to the fast establishment of a steady state typically in about ten ion-collision times. The temperature change is also insignificant on this time scale and is neglected for simplicity. The simulations presented here are carried out for large aspect ration circular geometry with the magnetic field $\vec{B} = (B_0/h)\hat{\zeta} + r/(qR_0)(B_0/h)\hat{\theta}$, where $\zeta$ is a toroidal angle. The equilibrium parameters used are $B_0 = 3$ T, major radius $R_0 = 3$ m, and minor radius $a = 0.5$ m. The safety factor $q(r)$ is uniform but may vary for each simulation. A Gaussian or super-Gaussian profile is used for temperature and density: $T_i = T_0 \exp[-\alpha_t (r/a)^2]$ and $n_i = A_n + B_n \exp[-\alpha_n (r/a)^{\beta_n}]$. The temperature gradient length $L_T$ is chosen to be much larger than the ion banana orbit width, $\Delta_b = \rho_{i\theta}(r/R)^{1/2}$, so that the local Maxwellian equation is justified and the $\delta f$ method is applicable. The number $\Delta_b/L_n$ which measures the steepness of the density gradient or orbit size is changed by adjusting parameters $\alpha_n$, $\beta_n$, $T_0$, and $q$. It varies over the simulations from 0.04 to 0.45 ($\rho_{i\theta}/L_n \gtrsim 1$, corresponding to a steep gradient). The typical value of $\Delta_b/L_T$ is 0.04. The collisionality ranges radially from a banana regime to a plateau regime near the axis in each simulation. We used $4 \times 10^6$ particles.

The ambipolarity of neoclassical transport with a self-consistent electric field is numerically demonstrated and compared to the case without $E_r$ in Fig. 1. With no electric field a considerable self-collision–driven ion flux is found in the region within the inner half radius, violating ambipolarity. The time history of a local ion flux which evolves to a positive steady-state value is shown in Fig. 1(a). Moreover, the self-collision–driven ion flux is observed regardless of the shape of the density profile. Of course, with nonzero toroidal rotation, a steady state is possible with no electric field. To check the reliability of our result, we also performed a simulation of the zero orbit width limit (neglecting the drift term). As shown in Fig. 1(c), no self-collision ion flux is produced, which excludes the possibility that the ion flux might be created nonphysically by the collision model if used inappropriately, for instance, without momentum conservation. The nonambipolarity implies that a radial electric field is required in a toroidal plasma. Indeed, when we include the self-consistent electric field through Eq. (1) in the simulation, ambipolarity is restored with a vanishing ion flux, as shown in Fig. 1(c). The corresponding time history with the self-consistent electric field is shown in Fig. 1(b).

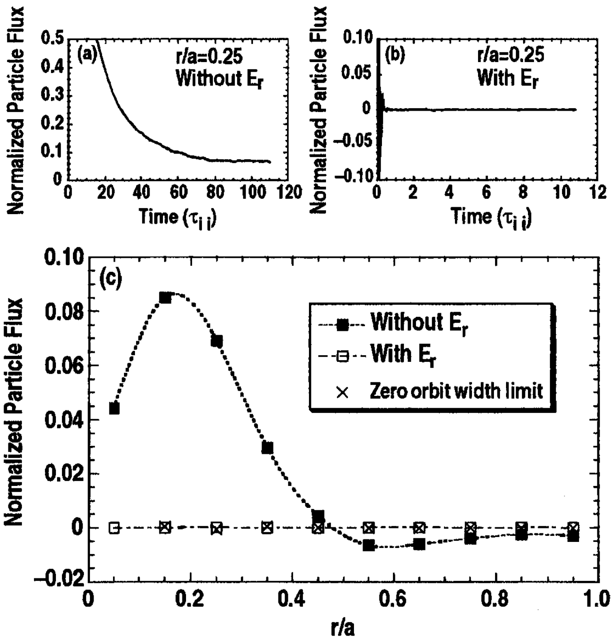


FIG. 1. Self-collision–driven ion flux: time history of normalized ion flux in simulation (a) without $E_r$ and (b) with $E_r$; (c) ion fluxes at steady state.

A general feature of the electric-field dynamics is the appearance of geodesic acoustic oscillations [8] in the initial phase of the $E_r$ development, which then relax to a steady state, consistent with the previous studies [9]. Further relaxation to the equilibrium defined by $\Pi_i = 0$ would be accompanied by the toroidal angular momentum transport, which would be significant when there is an initial nonuniform toroidal rotation.

In the case of zero temperature gradient, the equilibrium electric field, as shown in Fig. 2, is in excellent agreement with the Boltzmann relation. Moreover, simulation with a steep density gradient ($\Delta_b/L_n = 0.45$) shows that the Boltzmann relation is again obtained as long as the temperature is uniform.

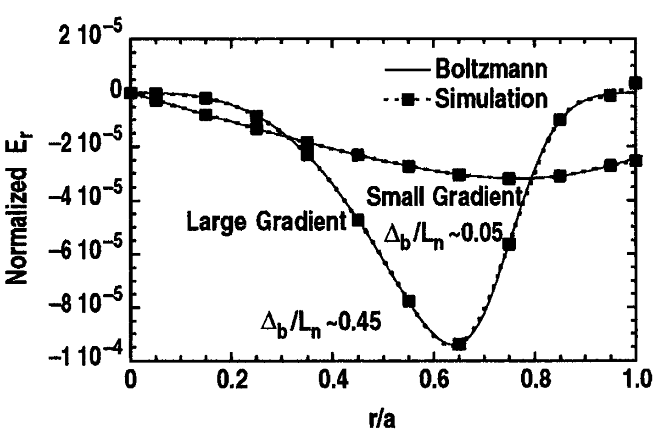


FIG. 2. Equilibrium $E_r$ [in units of $m_i a^2 \Omega_{i0}^2/(ea)$] for zero temperature gradient plasmas with small and large density gradients.

With the above rigorous benchmark, we next simulate toroidal plasmas with a nonzero temperature gradient. The equilibrium electric field is plotted in Fig. 3 for both small ($\Delta_b/L_n = 0.04$) and large ($\Delta_b/L_n = 0.4$) density gradients. It is interesting to compare our simulations with the standard neoclassical expression for the ion parallel flow (or, equivalently, the toroidal rotation):

$$u_{i\|} = \frac{I}{\Omega_i \psi'} \frac{T_i}{m_i} \left[ (k-1) \frac{\partial \ln T_i}{\partial r} - \frac{\partial \ln n_i}{\partial r} + \frac{e}{T_i} E_r \right], \tag{6}$$

where $k$ is a function of ion collisionality. Because initial toroidal rotation is zero in the simulations and because collisions conserve momentum, the parallel flow (and toroidal rotation) at the steady state is very small compared with the $E_r$ term in Eq. (6). The equilibrium $E_r$ is found to be consistent with Eq. (6) with $u_{i\|} = 0$ even when the plasma gradient is large. This result suggests that Eq. (6) can continue to be used in experiments to infer $E_r$ from rotation measurements when the density gradient is steep.

We now study the effects of the self-consistent electric field on ion thermal transport. We discuss the case of the small density gradient ($\Delta_b/L_n \sim 0.04$). The ion heat fluxes from the simulations with $E_r$, without $E_r$, and without a drift term are plotted in Fig. 4, and compared with the neoclassical formula [10]. All results for the heat flux show good agreement with each other in the region outside half minor radius. The radial electric field does not make the ion heat flux differ from the standard neoclassical prediction, except in the region near the magnetic axis, where nonstandard orbits may modify the standard neoclassical calculations. This is consistent with the fact that neoclassical transport rates are independent of $E_r$ in the small orbit limit. In comparison, the simulation with no $E_r$ gives a smaller ion heat flux in the region within the inner half radius. Moreover, because of a significant contribution from the self-collision–driven ion flux, the energy flux is inconsistent with the heat flux, while the simulation with self-consistent $E_r$ gives energy flux the same as heat flux. In the zero orbit limit the simulation recovers the standard neoclassical result, as expected.

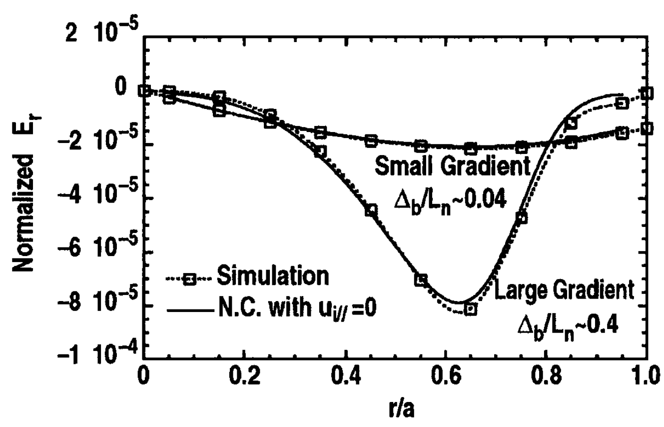


FIG. 3. Equilibrium $E_r$ for the nonzero temperature gradient case.

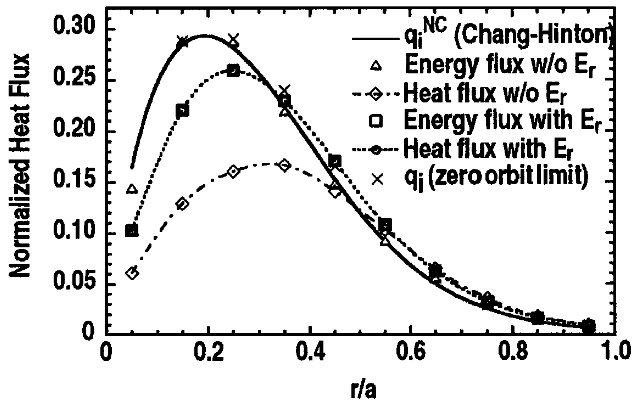


FIG. 4. Ion heat flux versus $r/a$ for the small density gradient plasma

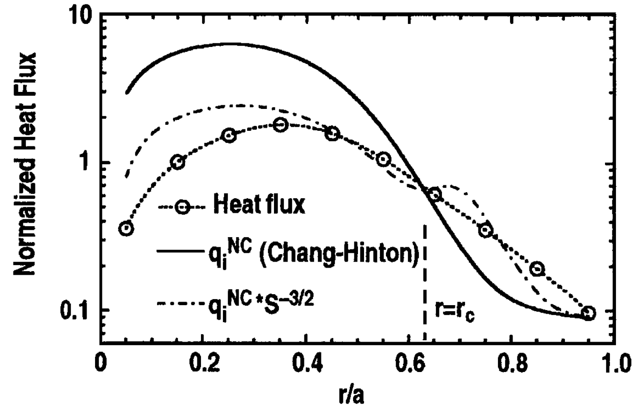


FIG. 5. Ion heat flux versus $r/a$ for the large density gradient plasma.

Self-consistent neoclassical transport in large gradient plasma is more interesting. As shown in Fig. 3, a deep radial electric-field well develops in the region of the large density gradient. The shear of $E_r$ changes direction at the well bottom $r = r_c$ (which is close to the location of the steepest density gradient). In the neighboring regions of the well bottom, ion thermal transport is significantly modified with this self-consistent radial electric field. As shown in Fig. 5, the modifications in the two sides of $r = r_c$ go in opposite directions, depending on the $E_r$ shear. Ion heat flux is largely reduced in the inner side ($r < r_c$) and increased in the outer side ($r > r_c$), relative to the standard neoclassical value. This is consistent with the orbit squeezing effects of the $E_r$. In a sheared radial electric field, the ion banana width is reduced by a factor of $\sqrt{S}$ and the trapped ion fraction is increased by the same amount, where the orbit squeezing factor $S = 1 - (e/m_i\Omega_{i\theta}^2)\partial E_r/\partial r$. As a consequence, the standard neoclassical result of ion heat flux is modified by a factor of $S^{-3/2}$ [4]. For the self-consistent electric field obtained in the simulation, $S > 1$ at $r < r_c$ and $S < 1$ at $r > r_c$. This modified neoclassical result is shown to agree reasonably well with the simulation result, except near the magnetic axis. The smoother radial variation of the ion flux from the simulation may be attributed to the nonlocal effect of the ion radial drifts near $r_c$, which span a significant portion of the minor radius. Near the magnetic axis, ion thermal transport is reduced nearly 1 order of magnitude relative to the standard neoclassical level by electric-field effects and nonstandard orbits. A previous study without the self-consistent radial electric field gave a much larger reduction of more than 2 orders [3]. Also, there is a large discrepancy between the simulation results and the $S^{-3/2}$ curve near the magnetic axis, showing that the orbit squeezing effect needs to be reformulated there. Our result is not compatible with analytic transport calculations close to the axis [11], which give ion heat flux $\langle \vec{q} \cdot \vec{r} \rangle \propto 1/r$ for the Gaussian temperature profile, or imply a profile flatter than Gaussian near the axis.

Future simulations allowing nonuniform initial toroidal rotation will give more comprehensive $E_r$ dynamics and spatial structure. The large scale neoclassical radial electric field with strong shear, like small scale turbulence-generated zonal flow, is also believed to play an important role in suppressing turbulence and associated transport through $\vec{E} \times \vec{B}$ flow shear decorrelation. This is a topic of ongoing research [12].

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