Empirical Realization of a Critical Point Description in Atomic Nuclei

R. F. Casten[1] and N. V. Zamfir[1,2,3]

[1]*Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520-8124*
[2]*Clark University, Worcester, Massachusetts 01610*
[3]*National Institute for Physics and Nuclear Engineering, Bucharest-Magurele, Romania*

(Received 12 December 2000; published 12 July 2001)

It is shown that ^{152}Sm and other $N = 90$ isotones are the first empirical manifestation of the newly predicted analytic description of nuclei at the critical point of a vibrator to axial rotor phase transition.

DOI: 10.1103/PhysRevLett.87.052503 PACS numbers: 21.60.Fw, 21.10.Re, 27.70.+q

Dynamical symmetries provide elegant and analytic paradigms for the behavior of a wide variety of physical systems, ranging from molecules to nuclei and elementary particles [1]. The concept and properties of such benchmarks can link the physical interpretations of diverse systems. In atomic nuclei, the interpretation of collective structure has benefited from such standards—the vibrator and the rotor. Most nuclei, however, do not actually satisfy these stable limits of structure but rather are in regions of changing character as a function of nucleon number. Indeed, these transition regions can be quite abrupt and, despite the finite nature of nuclear systems, can exhibit phase transitional and critical point behavior that is similar to that found in a wide variety of many-body systems.

Nuclear transition regions have historically been the most difficult to interpret since they exhibit a complicated interplay of competing degrees of freedom and, consequently, are usually treated in terms of complex numerical calculations involving parametrized Hamiltonians. Particularly challenging are nuclei at the critical point of a phase transition where structure is changing most dramatically. Yet such nuclei are, in many respects, the most important, as their structure defines the nature of the transition region itself. The development of an exactly solvable *analytic* model for critical point systems would, therefore, be a significant advance for nuclear structure and would, moreover, have wide applications to other many-body systems as well.

Recently, the concept of such critical point solutions has, in fact, been introduced [2,3] with the development of a new class of symmetries which are based on the solutions to differential equations. The first case worked out [2] is a dynamical symmetry called E(5), and, in the nuclear context, applies to a shape transition region from spherical vibrator to deformed γ-soft nuclei where the phase transition is in 1 degree of freedom only, the deformation β. A much more commonly encountered and important, but also more complex, kind of phase transition, from vibrator to axial rotor, is also far more challenging since it involves 2 degrees of freedom, β and γ, in which there is a phase transition in β for a potential which is γ dependent as well. A description of such a situation, denoted X(5) (which, as discussed in Ref. [3], is a dynamical symmetry, albeit of unusual nature), has now also been developed [3].

The basic idea of this new paradigm is sketched in Fig. 1 where the observable $R_{4/2} \equiv \mathrm{E}(4_1^+)/\mathrm{E}(2_1^+)$ is shown for nuclei which span a vibrational to axial rotor region. Included in the figure are sketches of the potential at different stages along the structural evolution. There are two competing minima, spherical and deformed, in the potential. At the critical point, these two minima cross and the shape changes from spherical to axially deformed (as discontinuously with nucleon number as the finite nature of nuclei permits). This behavior is reflected in the sharp rise in $R_{4/2}$. A simple approximation to the critical point potential, which neglects the small barrier between the two minima, gives rise to an analytically solvable model [3].

It is the main purpose of this Letter to discuss the realization of this concept in actual physical systems, specifically as it is exemplified in atomic nuclei, showing that ^{152}Sm is an excellent empirical manifestation of this critical point structure.

A derivation of the energy eigenvalues and E2 transitions at the critical point in a spherical → axially deformed shape/phase transition is given in Ref. [3]. Here we briefly outline the basic ansatz and summarize the results. The starting point is the Bohr Hamiltonian [4] where

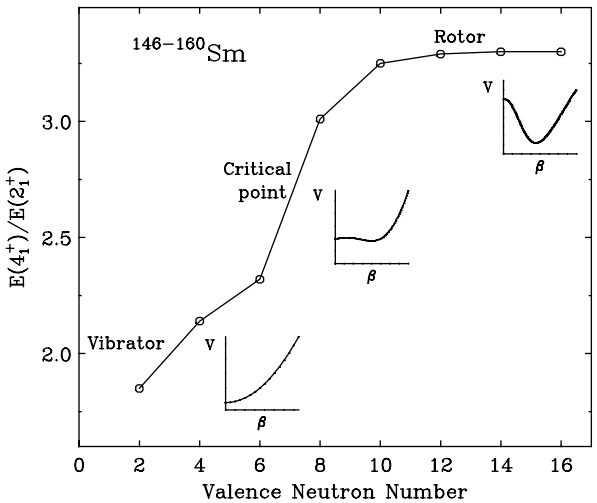

FIG. 1. Example of a vibrator to rotor transition region. $R_{4/2} \equiv \mathrm{E}(4_1^+)/\mathrm{E}(2_1^+)$ data for the Sm isotopes. The vibrator and rotor limits, and the critical point, are indicated with sketches of the potential $V(\beta)$ for each situation.

it is assumed that the potential $V(\beta, \gamma)$ can be written $V(\beta, \gamma) = V(\beta) + V(\gamma)$. If the potential $V(\beta)$ at the critical point is taken to be a square well of width β_w, then the eigenvalues are given by

$$\varepsilon_{\beta;s,L} = \frac{(x_{s,L})^2}{\beta_W^2}, \tag{1}$$

where $x_{s,L}$ is the sth zero of the Bessel function $J_\nu(z)$ ($z = \beta\sqrt{2mE_\beta}/\hbar$), and where the order of the Bessel function is

$$\nu = \left(\frac{L(L+1)}{3} + \frac{9}{4}\right)^{1/2}. \tag{2}$$

As will be discussed in more detail below, the levels belong to distinct sequences labeled by the quantum number s.

The wave functions are given by

$$\xi_{s,L}(\beta) = c_{s,L}\beta^{-3/2}J_\nu(k_{s,L}\beta); \qquad k_{s,L} = \frac{x_{s,L}}{\beta_W}, \tag{3}$$

with the normalization constants $c_{s,L}$ obtained from the condition $\int_0^\infty \beta^4 \xi_{s,L}^2(\beta)\, d\beta = 1$. Note the essential point that the order of these Bessel functions $J_\nu(z)$ is an *irrational* number in contrast to E(5) where the order is half integer.

Transition rates have also been calculated [3] by taking matrix elements of the quadrupole operator

$$T^{(\mathrm{E2})} = t\beta\left[\mathcal{D}^{(2)}_{\mu,0}\cos\gamma + \frac{1}{\sqrt{2}}(\mathcal{D}^{(2)}_{\mu,2} + \mathcal{D}^{(2)}_{\mu,-2})\sin\gamma\right], \tag{4}$$

where t is a scale factor.

Some of the predicted energy levels and B(E2) values are shown in Fig. 2 in units of $\mathrm{E}(2_1^+) - \mathrm{E}(0_1^+)$ and $\mathrm{B}(\mathrm{E2}: 2_1^+ \rightarrow 0_1^+) = 100$. More extensive results for the B(E2) values are given in Ref. [3] and will be shown in Fig. 3 in comparison with the data. We stress that all the predictions are parameter independent except for overall scale normalizations and that the levels are assigned quantum numbers, s, which determine their energies and transition rates. This situation, therefore, differs qualitatively from a numerical fit with a parametrized Hamiltonian.

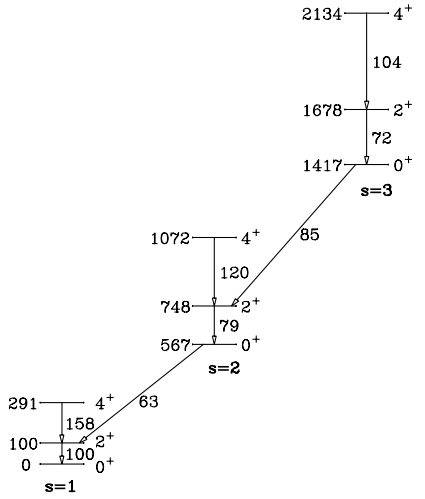

FIG. 2. Low lying levels predicted by X(5) [3] for the $s = 1, 2, 3$ sequences. Intrasequence B(E2) values and the $\mathrm{B}(\mathrm{E2}: 0_s^+ \rightarrow 2_{s-1}^+)$ values are given in units of 100 for the $\mathrm{B}(\mathrm{E2}: 2_1^+ \rightarrow 0_1^+)$ value. A more extensive set of B(E2) values is given in Fig. 3.

Note that the ratio $R_{4/2}(s = 1) = 2.91$, which is indeed appropriate for a nucleus at the critical point of a vibrator to rotor phase transition. Also, the yrast $\mathrm{B}(\mathrm{E2}: J + 2 \rightarrow J)$ values increase with J at a rate intermediate between that of a vibrator and a rotor. Particularly interesting is the prediction that the energy of the 0_2^+ level is fixed by the symmetry. We also note that the higher lying sequences ($s = 2, 3, \ldots$) have successively lower $R_{4/2}$ values and successively smaller *intrasequence* B(E2) values, reflecting lower expectation values of the deformation β. Finally, we note (see Fig. 3 below) that the symmetry predicts specific ($\Delta s = 1$) *inter-s*-sequence E2 transitions, which show substantial and characteristic variations: the $J \rightarrow (J + 2)$B(E2) values being the most collective. These intersequence transitions reflect strong phase mixing at the critical point which is a characteristic feature of this description. The $\Delta s \geq 2$ transitions are extremely weak.

Since nuclei contain integer numbers of nucleons, their properties change discretely with N and Z, and therefore, in any given transition region, there is no assurance that any specific nucleus will occur at the critical point. Nevertheless, the new critical point description immediately brings to mind ^{152}Sm where recent measurements [5–7], which have substantially revised previous data, have independently led to the conclusion [8,9] that ^{152}Sm is very near the critical point of a vibrator to rotor transition region and shows evidence of phase coexistence. Indeed, $R_{4/2}(^{152}\mathrm{Sm}) = 3.01$, which is in the crossover region in Fig. 1. Also, $R_{4/2}$ for the states built on the 0_2^+ state is 2.69. The difference in these two values was one of the original indicators of a coexistence of two distinct phases in ^{152}Sm—near deformed for the ground state and (anharmonic) vibrational for the 0_2^+ level. Moreover, empirically, these coexisting states have collective connecting B(E2) values suggesting a mixing of phases.

A comparison of the data [5–7] for ^{152}Sm with the X(5) predictions [3] is shown in Fig. 3. The agreement is remarkable. Recall that, after a single normalization of the energy and transition rate scales, the predictions are completely parameter free.

First, we see that the $R_{4/2}(s = 1)$ value is very close to experiment. Indeed, the predicted yrast energies agree very well with the data and reflect the nature of the critical point structure, intermediate between a vibrator and a rotor. This is shown explicitly in Fig. 4, which also includes another $N = 90$ isotone, ^{150}Nd. (Other $N = 90$ isotones, such as ^{154}Gd and ^{156}Dy, are likewise similar.) The slight deviations for both nuclei suggest that ^{152}Sm and ^{150}Nd are just past the actual critical point, towards the rotor limit,

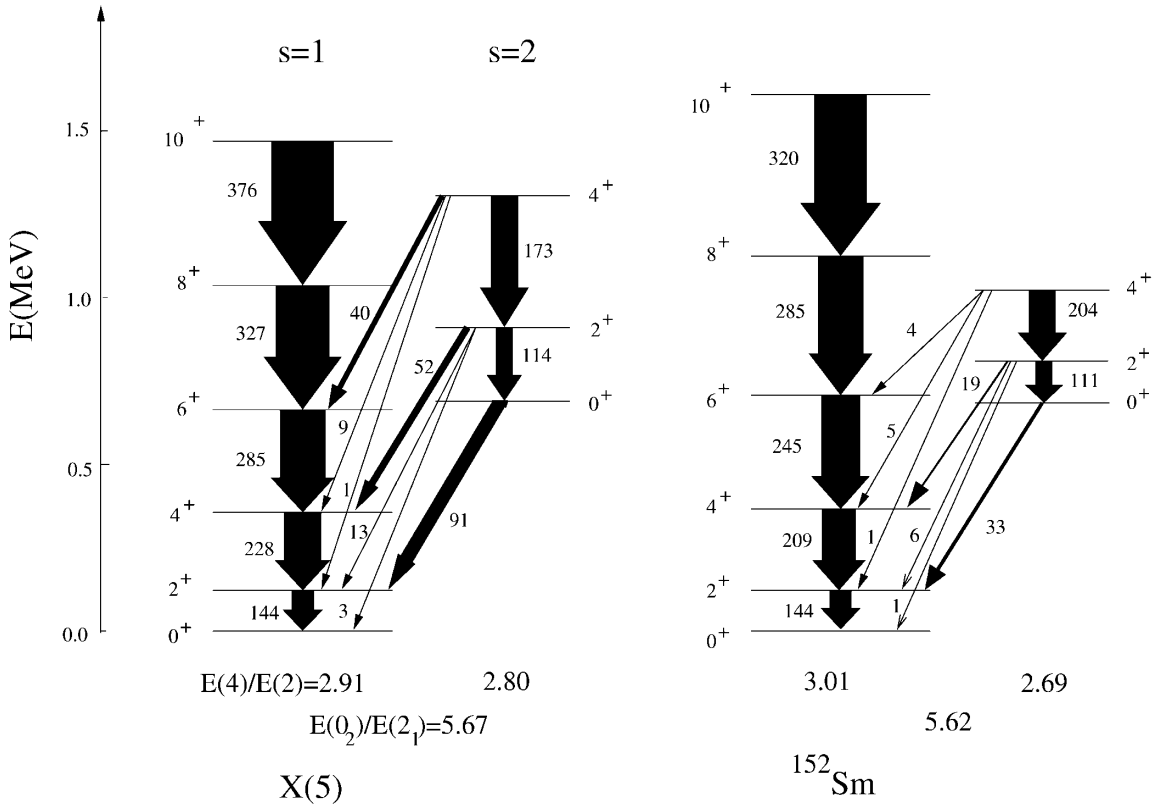

FIG. 3. Comparison of X(5) with the data [5–7] for ^{152}Sm. The $E(2_1^+)$ value (in keV) and the $B(E2: 2_1^+ \rightarrow 0_1^+)$ value (in W.u.) are normalized to the data. The $R_{4/2}$ values given refer to relative energies within an s sequence of levels.

which is also consistent with the fact that, for the $s = 1$ sequence, $R_{4/2}[\text{exp}] > R_{4/2}[\text{X(5)}]$.

We next note that $R_{4/2}(s = 2) < R_{4/2}(s = 1)$ both in X(5) and the data, consistent with a lower expectation value of β in the excited sequence, reflecting the phase coexistence that we have mentioned. This difference in the structure of the $s = 1$ and $s = 2$ sequences is also seen in the $B(E2: 2_2^+ \rightarrow 0_2^+)$ value, which is about 25% lower than the corresponding $s = 1$ value. These predictions agree almost exactly with the data. Finally, the comparison of predicted and observed "bandhead" energies of the $s = 2$ sequence is of interest: $E(0_2^+)$ is 5.67 times $E(2_1^+)$ in X(5) compared to 5.62 in the data. One significant disagreement is the expanded energy scale for the $s = 2$ states compared to the data. This also occurs in numerical fits to ^{152}Sm [6,10], and its origins need to be further studied.

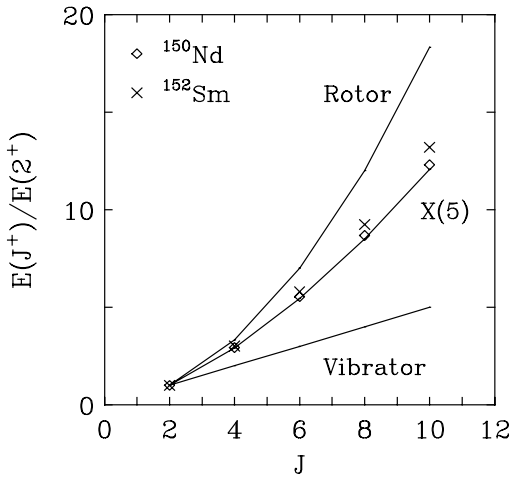

FIG. 4. Energy levels of the $s = 1$ sequence in X(5) compared to yrast energies in the harmonic vibrator and symmetric rotor limits. Data for ^{152}Sm (crosses) and ^{150}Nd (open symbols) are shown.

The $s = 1$ and $s = 2$ intrasequence B(E2) values are also in excellent agreement, displaying a J dependence that is transitional between vibrator and rotor. Finally, as noted above, the model predicts a full set of intersequence B(E2) values. The $J \rightarrow J + 2$ transitions are predicted to be quite collective; the others much less so. This is true empirically as well. However, the absolute magnitudes of the predicted intersequence B(E2) values are nearly 3 times larger than observed. Nevertheless, the relative values (with the exception of the $4_2^+ \rightarrow 6_1^+$ transition) are in striking agreement with the data, over a range of nearly 2 orders of magnitude, as shown in Fig. 5, and, once again, show a pattern that is that of neither a vibrator nor a rotor.

In this Letter, we have focused on excitations involving the β degree of freedom. A full treatment of the γ degree of freedom, involving β-γ coupling terms in the potential $V(\beta, \gamma)$, is more complex and will be discussed later in

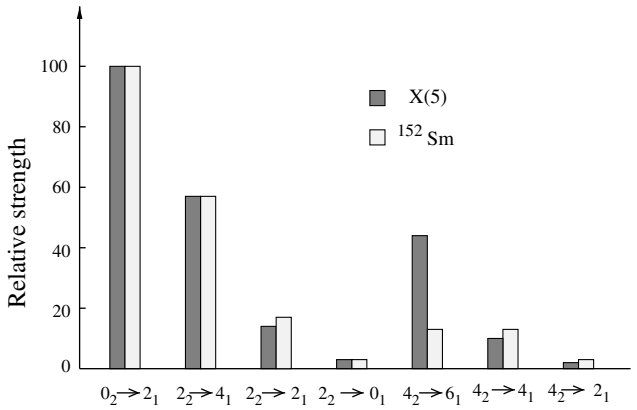

FIG. 5. Comparison of intersequence B(E2) values in X(5) with the data for ^{152}Sm. Theory and experiment are normalized for the $0_2^+ \rightarrow 2_1^+$ transition.

a longer publication. Reference [3] contains an approximate analysis. Numerical calculations of the critical point structure [8,11] that automatically include the γ degree of freedom and higher order terms in the E2 transition operator suggest that even better agreement may be obtained when the γ degree of freedom and β-γ coupling is taken into account.

In summary, we have shown evidence that the new benchmark, denoted X(5) in Ref. [3], which describes a system at the critical point of a first order phase transition in two variables (vibrator to axial rotor transition region in atomic nuclei), is closely manifested empirically in ^{152}Sm. Other $N = 90$ isotones are very similar in structure. Overall, the agreement of the data and predictions is excellent. The discrepancies that do exist may reflect the fact that ^{152}Sm is slightly to the deformed side of the critical point, or that the treatment of the γ degree of freedom is only approximately incorporated.

This is the first empirical example of this new description of nuclei at the critical point of a first order phase transition. This description provides a new structural paradigm for nuclei and should have applications in other many-body systems as well.

We are very grateful to F. Iachello for his development of this critical point model and for extensive and illuminating discussions of critical point structures in general. We are also grateful to Peter von Brentano and colleagues for collaborations on several experiments on ^{152}Sm in the last few years and for numerous discussions. This work was supported by U.S. DOE under Grants No. DE-FG02-91ER-40609 and No. DE-FG02-88ER-40417.

[1] See, for example, F. Iachello, in *Group Theoretical Methods in Physics,* Lecture Notes in Physics, Vol. 94, edited by A. Böhm *et al.* (Springer, Berlin, 1979), p. 420; in *Symmetries in Particle Physics,* edited by A. Chodos and C. Tze (Plenum Press, New York, 1984), p. 47.
[2] F. Iachello, Phys. Rev. Lett. **85**, 3580 (2000).
[3] F. Iachello, preceding Letter, Phys. Rev. Lett. **87**, 052502 (2001).
[4] A. Bohr, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. **87**, No. 14 (2001).
[5] R. F. Casten, M. Wilhelm, E. Radermacher, N. V. Zamfir, and P. von Brentano, Phys. Rev. C **87**, 052502 (2001).
[6] N. V. Zamfir, R. F. Casten, M. A. Caprio, C. W. Beausang, R. Krücken, J. R. Novak, J. R. Cooper, G. Cata-Danil, and C. J. Barton, Phys. Rev. C **60**, 054312 (1999).
[7] T. Klug, A. Dewald, V. Werner, P. von Brentano, and R. F. Casten, Phys. Lett. B **495**, 55 (2000).
[8] F. Iachello, N. V. Zamfir, and R. F. Casten, Phys. Rev. Lett. **81**, 1191 (1998).
[9] R. F. Casten, D. Kusnezov, and N. V. Zamfir, Phys. Rev. Lett. **82**, 5000 (1999).
[10] Jing-ye Zhang, M. A. Caprio, N. V. Zamfir, and R. F. Casten, Phys. Rev. C **60**, 061304(R) (1999).
[11] O. Scholten, F. Iachello, and A. Arima, Ann. Phys. (N.Y.) **115**, 325 (1978).