Direct Link between Coulomb Blockade and Shot Noise in a Quantum-Coherent Structure

A. Levy Yeyati,[1] A. Martin-Rodero,[1] D. Esteve,[2] and C. Urbina[2]

[1]*Departamento de Física Teórica de la Materia Condensada C-V, Facultad de Ciencias, Universidad Autónoma de Madrid, E-28049 Madrid, Spain*

[2]*Service de Physique de l'Etat Condensé, Commissariat à l'Energie Atomique, Saclay, F-91191 Gif-sur-Yvette Cedex, France*

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We analyze the current-voltage characteristic of a quantum conduction channel coupled to an electromagnetic environment with arbitrary frequency-dependent impedance. In the weak blockade regime the correction to the Ohmic behavior is directly related to the channel current fluctuations, vanishing at perfect transmission in the same way as shot noise. This relation can be generalized to describe the environmental Coulomb blockade in a generic mesoscopic conductor coupled to an external impedance, as the response of the latter to the current fluctuations in the former.

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The way in which quantum mechanics affects the laws ruling electrical circuits is presently understood only for some elementary situations. For instance, in the case of a quantum coherent nanostructure connecting two independent electron reservoirs with a voltage difference V, quantum mechanics results in current fluctuations, the so-called shot noise, which at low frequency and zero temperature have a spectrum of the form $S = 2eVG_0 \sum \tau_i(1 - \tau_i)$, where the $\{\tau_i\}$ are the transmissions of the conduction channels, and $G_0 = 2e^2/h$ is the conductance quantum [1]. The reduction of noise with increasing transmission as $(1 - \tau_i)$ is a consequence of the Fermi statistics in the reservoirs, a prediction which has been tested quantitatively in different types of nanostructures [2]. Another important consequence of quantum mechanics is that simple laws for the addition of two elements in series do not apply because each element is not simply voltage biased. The phase across each element $\phi = \frac{e}{\hbar}\int v(t)\,dt$, where $v(t)$ is the voltage drop, develops quantum fluctuations, and the electrical properties of the series connection cannot be inferred from the conductance of the separate elements. More generally, the phase differences add in series, and parallel connected branches share the same phase, but the general rules to predict the properties of the whole circuit from those of the constitutive elements are not known, except for macroscopic electromagnetic impedances. When a low transmissive nanostructure, i.e., one with negligible noise reduction, is connected in series with a macroscopic impedance $Z(\omega)$, the conductance of the series circuit is suppressed at sufficiently low voltage and temperature [3]. This phenomenon, called environmental Coulomb blockade, has been thoroughly investigated in small metallic tunnel junctions [4]. How is this phenomenon modified in the case of a coherent structure whose transmissions τ_i approach unity? One might speculate that a noiseless structure cannot be "felt" by the series impedance $Z(\omega)$, and, reciprocally, should not be affected by its presence. We show here that this naive reasoning which predicts the restoration of the linear behavior at large transmission is correct, and more precisely that Coulomb blockade is modified in the same way as shot noise.

In this Letter we address the simple case of a single channel quantum point contact with transmission τ connected in series with a macroscopic impedance $Z(\omega)$. We develop a theory of the environmental Coulomb blockade which permits one to analyze the current-voltage characteristic at arbitrary transmission for a generic frequency-dependent impedance. We show that in the limit of low impedance $Z \ll 1/G_0$ the blockade is intimately connected with the fluctuations in the current through the channel.

A single channel contact of arbitrary transmission between two electrodes can be modeled with a simple Hamiltonian resembling the usual tunneling Hamiltonian for a tunnel junction [5]. The coupling to the environment can be then introduced in the usual way [3], which leads to a model Hamiltonian $\hat{H} = \hat{H}_L + \hat{H}_R + \hat{H}_T + \hat{H}_{\mathrm{env}}$, where $\hat{H}_{L,R}$ describe the uncoupled left and right leads, characterized by flat densities of states $\rho_{L,R} \simeq 1/\pi W$, and

$$\hat{H}_T = \sum_\sigma T_0(\hat{c}^\dagger_{L\sigma}\hat{c}_{R\sigma}\hat{\Lambda}_e + \mathrm{H.c.}) \qquad (1)$$

describes the transfer of an electron between the leads in terms of a hopping element T_0. The translation operator $\hat{\Lambda}_e = e^{i\hat{\phi}}$, where $\hat{\phi}$ is the phase operator satisfying the commutation relation $[\hat{Q}, \hat{\phi}] = ie$, takes into account the change in the charge $\hat{Q}$ of the environment associated with the transfer process. Finally, $\hat{H}_{\mathrm{env}}$ describes the electromagnetic modes in the environment as a set of harmonic oscillators [6]. If the coupling to the environment is neglected the normal transmission of this model is given by $\tau = 4\beta/(1 + \beta)^2$, where $\beta = (T_0/W)^2$ [5].

We are interested in calculating the current through the channel under a constant bias voltage V in the presence of the environment. The current operator within this model is given by

$$\hat{I} = \frac{ie}{\hbar}\sum_\sigma T_0(\hat{c}^\dagger_{L\sigma}\hat{c}_{R\sigma}\hat{\Lambda}_e - \mathrm{H.c.})\,. \qquad (2)$$

In order to evaluate the mean current we use the Keldysh formalism [7] which is suitable for calculating averages in a nonequilibrium state. The mean current is formally given by

$$\langle \hat{I}(t) \rangle = \langle \hat{T}_c [\hat{I}_I(t) \hat{S}_c(\infty, -\infty)] \rangle, \tag{3}$$

where $\hat{T}_c$ is the chronological ordering operator along the Keldysh contour, $\hat{I}_I$ is the current operator in the interaction representation, and $\hat{S}_c(\infty, -\infty)$ is the corresponding evolution operator along the closed time contour. Introducing the series expansion of $\hat{S}_c$ in terms of $\hat{H}_T$ and applying the Wick theorem, we obtain a perturbative expansion for the evaluation of the current which can be expressed in terms of Keldysh Green functions. The lowest order diagrams within this theory are depicted in Fig. 1a. In these diagrams we associate a solid line with an arrow to the electron propagators for the uncoupled leads (denoted below by $g_{L,R}^{\alpha,\beta}$), crosses indicate hopping events, and wavy lines correspond to the environment correlators given by [8]

$$P^{\alpha,\beta}(t,t') = e^{J^{\alpha,\beta}(t,t')}, \tag{4}$$

where $\alpha, \beta \equiv +,-$ indicate the branch on the Keldysh contour for the two time arguments, and $J^{\alpha,\beta}$ are the phase correlation functions,

$$J^{\alpha,\beta}(t,t') = \langle \hat{T}_c [\hat{\phi}(t_\alpha) \hat{\phi}(t'_\beta)] \rangle - \langle \hat{\phi}^2 \rangle.$$

As usual, we assume that the modes in the environment are populated according to the equilibrium distribution at a given temperature. In this case $J^{\alpha,\beta}(t,t') = J^{\alpha,\beta}(t - t')$.

The evaluation of the complete perturbative series is a formidable task. One can, however, obtain useful results in the limit of weak impedance $Z \ll 1/G_0$. In this limit $P^{\alpha,\beta}$ can be approximated by $1 + J^{\alpha,\beta}$. To the first order in $J^{\alpha,\beta}$ one obtains the family of diagrams depicted in Fig. 1b, where the phase correlation functions are represented by a dashed line. These diagrams correspond to single mode excitation processes and can be associated into four groups depending on the types of hopping event (left to right or right to left) connected by the wavy line. Notice that the complete family of diagrams should be added (up to infinite order in T_0) to get the correct results for arbitrary transmission. This is done by introducing renormalized electron propagators and renormalized hopping amplitudes giving rise to a dressed diagram such as the one depicted in Fig. 1c. The remaining part of the calculation relies on obtaining the expression of these diagrams in terms of Keldysh Green functions.

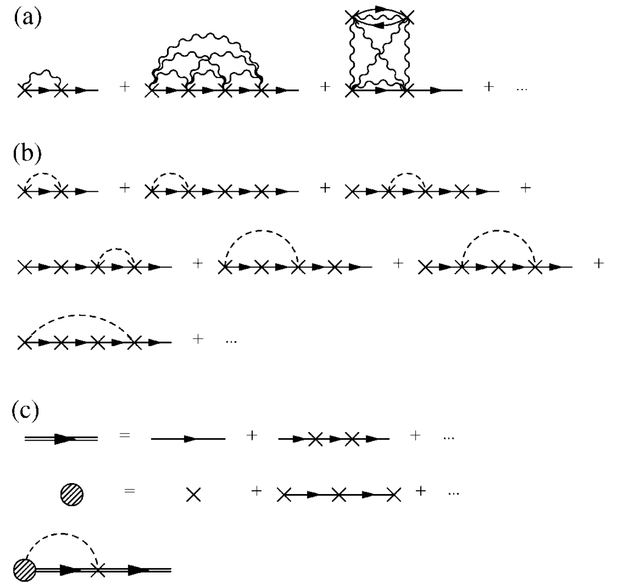

FIG. 1. Diagrammatic expansion of the current in the Keldysh formalism: (a) Unlabeled lowest order diagrams in $\hat{H}_T$. Solid lines with an arrow indicate electron propagators, crosses indicate hopping events, and wavy lines correspond to environment correlators. (b) Single mode excitation processes up to second order in $\hat{H}_T$. Dashed lines indicate phase correlation functions. (c) Renormalized diagrams arising from the addition of single mode excitation processes up to infinite order in the hopping. Double solid lines with an arrow indicate dressed electron propagators and shaded circles indicate dressed hopping amplitudes.

A further simplification of the calculation is obtained by considering the wide band limit, i.e., to assume that the electron bandwidth is much larger than all other relevant energy scales involved in the problem. Within this approximation the propagators for the isolated leads are given by

$$\hat{g}_{L,R}(\omega) = \begin{pmatrix} g_{L,R}^{++}(\omega) & g_{L,R}^{+-}(\omega) \\ g_{L,R}^{-+}(\omega) & g_{L,R}^{--}(\omega) \end{pmatrix} = \frac{i}{W} \begin{pmatrix} 2f_{L,R}(\omega) - 1 & 2f_{L,R}(\omega) \\ 2(f_{L,R}(\omega) - 1) & 2f_{L,R}(\omega) - 1 \end{pmatrix}, \tag{5}$$

where $f_{L,R}(\omega)$ are the Fermi distribution functions on the left and right lead, respectively.

Another basic ingredient in the calculation is the convolution of electron propagators with phase correlations $\delta g_{L,R}^{\alpha,\beta}(\omega) = \int d\omega' J^{\alpha,\beta}(\omega') g_{L,R}^{\alpha,\beta}(\omega + \omega')$. In the wide band limit, these can be approximated as

$$\delta \hat{g}_{L,R}(\omega) = \frac{i}{W} \begin{pmatrix} f_{L,R}^{+}(\omega) + f_{L,R}^{-}(\omega) & 2f_{L,R}^{+}(\omega) \\ 2f_{L,R}^{-} & f_{L,R}^{+}(\omega) + f_{L,R}^{-}(\omega) \end{pmatrix}, \tag{6}$$

where $f_{L,R}^{\pm}(\omega) = \int d\omega' J(\omega') f_{L,R}(\omega \pm \omega')$, $J(\omega) = J^{+-}(\omega)$ being the Fourier transform of the phase correlation function.

For an energy independent transmission coefficient, one then obtains the following expression for the correction to the current induced by the environment:

$$\delta I(V) = \frac{e}{h} \tau(1 - \tau) \int d\omega \{ f_R(\omega) [f_L^-(\omega) - f_L^+(\omega)] - f_L(\omega) [f_R^-(\omega) - f_R^+(\omega)] \} + \frac{e}{h} \tau^2 \int d\omega \{ f_L(\omega) [f_L^-(\omega) - f_L^+(\omega)] - f_R(\omega) [f_R^-(\omega) - f_R^+(\omega)] \}. \tag{7}$$

By analyzing the Fermi factors, this expression can be decomposed as $\delta I = \delta I^{\rightarrow} - \delta I^{\leftarrow}$, where $\delta I^{\rightarrow}$ and $\delta I^{\leftarrow}$ correspond to currents flowing in the two opposite directions. Both terms can be interpreted as arising from the coupling between the contact current fluctuations and the phase fluctuations due to the finite impedance of the environment. In fact, as described below, this expression can be directly related to the current fluctuations. In the absence of environment, the noise spectrum in a single channel contact is given by [9]

$$S(V,\Omega) = \frac{2e^2}{h}\tau(1-\tau)\int d\omega\{f_R(\omega)[1 - f_L(\omega+\Omega)] + f_L(\omega)[1 - f_R(\omega+\Omega)]\} + \frac{2e^2}{h}\tau^2\int d\omega\{f_L(\omega)[1 - f_L(\omega+\Omega)] + f_R(\omega)[1 - f_R(\omega+\Omega)]\} + [\Omega \rightarrow -\Omega]. \quad (8)$$

On the other hand, $S(V,\Omega) = \int dt\, e^{i\Omega t}[K(t) + K(-t)]$, where $K(t) = \langle \hat{I}(t)\hat{I}(0)\rangle - \langle \hat{I}^2\rangle$ is the current correlation function. By comparing Eqs. (7) and (8), we arrive to the simple relation

$$e(\delta I^{\rightarrow} + \delta I^{\leftarrow}) = \int dt\, J(t)[K(t) - K(-t)]. \quad (9)$$

This expression can be considered as a generalization of the fluctuation-dissipation theorem to the present nonequilibrium situation. As in the low impedance regime, the coupling between the contact and its environment is of the form $\hat{I}\hat{\phi}$ we expect this result to be valid for a generic situation with the same type of system-environment interaction. In particular, the result (9) would apply for any mesoscopic conductor that can be modeled as a collection of channels.

In order to understand the effects on the I-V characteristic, it is instructive to consider first the case of an environment with just a single mode at zero temperature, for which $J(\omega) = \pi G_0 Z_0[\delta(\omega - \omega_0) - \delta(\omega)]/2$ [6]. For this model the conductance exhibits a discontinuity at $eV = \hbar\omega_0$. For $eV < \hbar\omega_0$, there is a reduction in the conductance $\delta G = -\pi G_0^2 Z_0 \tau(1-\tau)/2$ while for $eV > \hbar\omega_0$, $\delta G = 0$. This simple case shows that the blockade is proportional to shot noise and vanishes at perfect transmission.

In the general situation the environment is characterized by a complex impedance $Z(\omega)$. The phase correlation function is related to the impedance by the expression [6]

$$J(t) = G_0 \int d\omega \frac{\mathrm{Re}Z(\omega)}{\omega}\frac{e^{i\omega t} - 1}{1 - e^{-\beta\hbar\omega}}, \quad (10)$$

which at zero temperature leads to a correction in the differential conductance given by

$$\frac{\delta G}{G} = -G_0(1-\tau)\int_{eV}^{\infty} d\omega \frac{\mathrm{Re}Z(\omega)}{\omega}. \quad (11)$$

This is the same result one obtains for a tunnel junction except for the reduction factor $(1-\tau)$. In the simple but realistic case in which the impedance $Z(\omega)$ is composed by the resistance R of the leads embedding the contact in parallel with the capacitance C of the contact itself, $Z(\omega) = R/(1 + i\omega RC)$, and the integral in (11) yields

$$\frac{\delta G}{G} = -G_0 R(1-\tau)\ln\sqrt{1 + \left(\frac{\hbar\omega_R}{eV}\right)^2}, \quad (12)$$

where $\omega_R = 1/RC$. For finite temperature, δG can be evaluated numerically from Eq. (7). Figure 2 shows the evolution of the correction to the differential conductance with temperature. For an energy independent transmission, the temperature dependence is the same as for a tunnel junction except for a global factor $(1-\tau)$. For increasing temperatures, the dip in the conductance at zero bias is progressively washed out.

The multichannel extension of these results is straightforward. One can, for instance, analyze the case of a diffusive conductor by replacing the reduction factor $(1-\tau)$ by its average over many channels (which in the diffusive case is known to be $1/3$) and R by the resistance of the conductor itself [10]. This analysis leads to quantitative agreement with recent experimental results [11].

As a final remark, we would like to stress that the results of the present theory can be thoroughly tested experimentally using atomic contacts that can be produced by scanning tunneling microscope or break junction techniques [12]. These contacts accommodate a small number of channels, and the ensemble of the transmissions $\{\tau_i\}$ can be determined experimentally and varied over a wide range including the $\tau \rightarrow 1$ limit [13]. Moreover, the impedance of the environment embedding such contacts can be tuned within a desired range using nanolithography [14]. Experimental work along these lines is currently under progress.

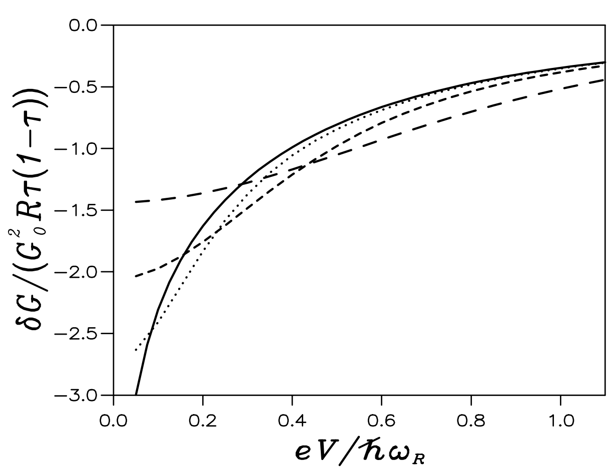

FIG. 2. Correction to the conductance of a single conduction channel with transmission τ due to an Ohmic environment for different temperatures $k_B T = 0$ (solid line), 0.005 (dotted line), 0.01 (short-dashed line), and 0.02 (long-dashed line) in units of $\hbar\omega_R$.

In conclusion, we have presented a theoretical analysis of the environmental Coulomb blockade in coherent nanostructures. We have considered the weak blockade regime and showed that the corrections in the current-voltage characteristic can be related to the structure current fluctuations. The blockade vanishes at perfect transmission as $\tau(1 - \tau)$, in the same way as shot noise. The temperature dependence of the conductance is similar to the one observed in ultrasmall tunnel junctions. The present calculation provides a first step towards the understanding of Coulomb blockade effects in coherent nanostructures. Although derived for a particular model, it has been argued that the expression (9) is more general and valid for a generic mesoscopic conductor coupled to an arbitrary external impedance. The strong blockade limit could be addressed through the analysis of multiple mode excitation processes along the lines suggested in this Letter.

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