Transport Processes in Reversed-Field-Pinch Plasmas: Inconsistency with the Self-Organized-Criticality Paradigm

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A statistical analysis of the anomalous particle flux in the edge region of the RFX experiment has revealed that laminar times between bursts, which account for more than 50% of the losses, have a power law distribution and that flux fluctuations are not self-similar. These properties are found in contrast with a wide class of self-organized-criticality models so that it is concluded that there is no experimental evidence of avalanchelike process occurrence in the plasma of RFX.

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Transport in magnetically confined plasmas for controlled thermonuclear research is generally believed to be driven by plasma turbulence [1], so that the investigation of the mechanisms underlying these processes constitutes a key topic in fusion research. Understanding the origin of the turbulence is expected to contribute to improve energy and particle confinement in experimental devices by taming instabilities or mitigating the associated losses. Progress in this field is also expected to contribute to a better understanding of turbulence origin in other magnetized plasmas including astrophysical plasmas.

It is commonly observed that plasma parameters like density and temperature, which are directly related to the energy content of the plasma, exhibit large fluctuations with "bursts." Conditional spectra of these fluctuations in all experimental devices show broadband wave numbers and frequencies, characteristic of fully developed turbulence.

Among the models proposed to describe the transport processes in magnetically confined plasmas, the self-organized-criticality (SOC) model [2] appeared to be particularly suitable in thermonuclear fusion research owing to some properties of the plasma [3], like density and temperature profile resilience and power law decay in fluctuation spectra. Among the different SOC models [2,4–6] discussed in literature, the running sandpile model developed by Hwa and Kardar [4] appears particularly suitable to better mimic a magnetically confined plasma continuously powered and refueled with particles. An important feature of all SOC models is that transport occurs through avalanches covering all space scales allowed by the system size. Searching for avalanchelike processes is a challenging task demanding accurate space and time measurements. Bursty behavior and long time correlations observed in many experiments [7] have been found consistent with some SOC predictions, although some authors [8] reported long range correlations also for non-SOC systems. Recently experimental evidence of avalanchelike processes, as predicted by the SOC model, has been claimed by some authors. In particular, phenomena reminiscent of avalanche propagation in the core plasma were measured by electron cyclotron emission diagnostic in tokamak DIII-D [9]. When such accurate measurements are not available, besides the space and time spectral analysis, different statistical analysis techniques can be adopted to derive the underlying transport process from the properties of these fluctuations. In a magnetized plasma, the quantity closest to an avalanche process in a sandpile is the cross field particle flux. The anomalous particle flux is obtained by the cross correlation of density and radial $\mathbf{E} \times \mathbf{B}$ drift velocity. Identification of bursts with avalanches was conjectured in the stellarator W7-AS device, where self-similarity in the particle flux probability density function (PDF) was obtained in a range of time scales relevant for transport [10]. It was observed that bursts of particle flux could account for up to 50% of the total transport despite their smaller population (10% of the events). In the same reference the authors conjectured two types of transport associated to eddies and avalanches.

A different statistical approach, borrowed from astrophysics [11], was applied in the experiment RFX [12] in order to get insights into the physics of the bursts. The method is based on the analysis of the PDF of the laminar times, i.e., the time between two consecutive bursts. The PDF of the laminar times for density fluctuations, measured in the edge region of the RFX experiment by a Langmuir probe and a reflectometer, has been found to exhibit a power law decay. Since this decay was in contrast with predictions of the standard SOC model [2], the Bak-Tang-Wiesenfeld (BTW) model, which predicts exponential decay instead, the result was claimed to be a warning that the SOC model could not be applicable to magnetically confined plasmas.

However, as mentioned in [12], some authors [13] have conjectured that other SOC models, including the running sandpile, could allow for a power law decay for the PDF of laminar times consistent with those observed in solar corona for solar flares [11] and in laboratory plasmas.

The purpose of the present Letter is to establish a firm point on this subject, by comparing the statistical properties of the particle fluxes driven by electrostatic turbulence with those predicted by a wider class of SOC models.

The data were collected in the reversed field pinch (RFP) experiment RFX operated at low plasma current, in the same experimental conditions in which electrostatic turbulence and related transport have been extensively studied in the past [14]. In this configuration the magnetic field at the edge is mainly poloidal so that the particle flux is approximated as $\Gamma = nE_\phi/B_\theta$. In order to estimate the particle flux, the same approximation of Ref. [10] has been used, equating plasma potential with floating potential and electron density with ion saturation current. The electrostatic quantities have been measured by an array of Langmuir probes described elsewhere [15]. The main results worth mentioning here are that [14] most of the particle transport in the edge region is driven by electrostatic turbulence. The frequency resolved particle flux is mostly concentrated in the range 30–250 kHz [15] which corresponds to time scales approximately in the range 4–30 μs.

The power spectrum of particle flux, as previously observed for the primary quantities (i.e., plasma density and potential), decays with a power law behavior in the range between 50–500 kHz covering the entire range relevant for transport processes.

The PDF of the particle flux has been computed for each shot and an example is shown in Fig. 1(a). The PDF results asymmetric with a skewness of almost 4 and with a longer positive tail which corresponds to outward directed fluxes. Following the argument reported in [10], i.e., that although avalanches propagate in both directions the particle flux points only outwards, the analysis has been focused on the positive bursts. To discriminate between bursts and fluctuation background a threshold in the particle flux has been obtained by a recursive method described in [11].

The bursts beyond this threshold could correspond to possible avalanches in the plasma. In order to estimate the losses due to these events, two quantities, the fraction of events $\varepsilon(\Gamma)$ and the fraction of particle flux $\eta(\Gamma)$ beyond a given threshold value of particle flux Γ, have been defined as

$$\eta(\Gamma) = \frac{\int_\Gamma^\infty \Gamma' p(\Gamma')\, d\Gamma'}{\int_0^\infty \Gamma' p(\Gamma')\, d\Gamma'}, \qquad (1)$$

$$\varepsilon(\Gamma) = \frac{\int_\Gamma^\infty p(\Gamma')\, d\Gamma'}{\int_0^\infty p(\Gamma')\, d\Gamma'}, \qquad (2)$$

where $p(\Gamma)$ is the probability density function of particle flux. In Fig. 1(b) the behavior of η and ε are shown as a function of positive flux.

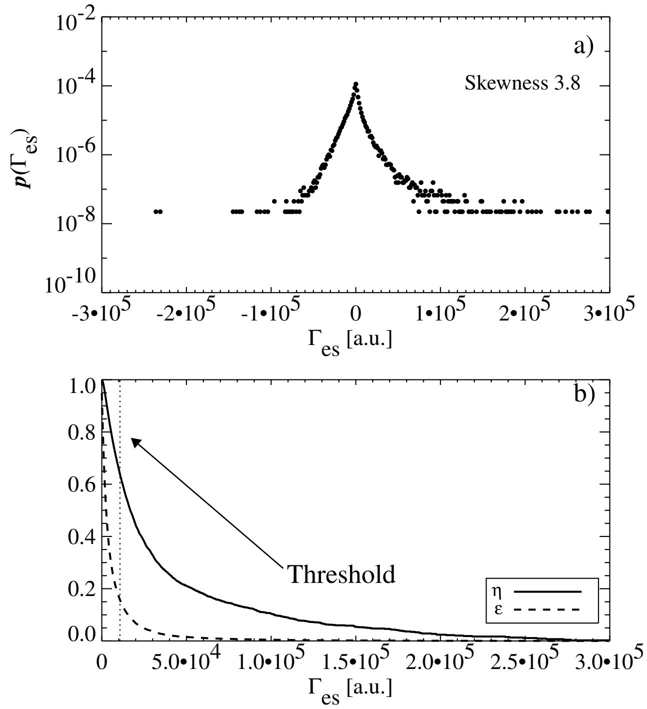

FIG. 1. (a) Probability density function of particle flux. (b) η and ε defined in Eqs. (1) and (2), respectively, as a function of positive flux.

The calculated threshold value, shown in Fig. 1(b), gives that almost 60% of the outward flux is carried by less than 20% of the events, in agreement with the estimate given in [10].

The PDF of the laminar times between successive bursts of Γ has been examined. A power law distribution $P(\tau_L) \sim \tau_L^{-\alpha}$, with $\alpha = (1.79 \pm 0.02)$, has been identified between $3 \leq \tau \leq 80$ μs. Therefore the particle flux exhibits power law decay in the laminar time PDF which is inconsistent with predictions of BTW SOC models in agreement with conclusions reported in Ref. [12]. It is worth noting that a similar power law decay has been obtained by applying the same analysis to all bursts, including the negative ones.

In order to compare with SOC models applied to plasmas in fusion experiments and to verify the warning given in [13], we have applied the same analysis to the running sandpile model. This model, developed by Hwa and Kardar [4], is characterized by a continuous finite drive, and tuning the input rate J_{in} allows avalanches to interact among themselves. In particular, the system behaves as the BTW model in the limit of vanishing J_{in} [4], which corresponds to the case of nonoverlapping avalanches. As is usually done [2,4–6] to describe the dynamics of these systems, the dissipated energy, which is defined as the total number of overturning (i.e., unstable) sites [4] per step, i.e., per unit time, has been considered. With the threshold identified as described above, the analysis of the PDF of the laminar times has shown that at low J_{in} the PDF has an

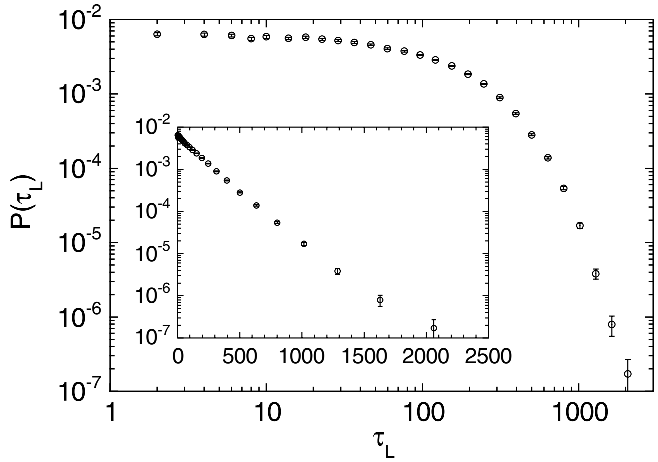

FIG. 2. PDF of laminar times $P(\tau_L)$ between subsequent bursts in energy dissipated signal for the Hwa-Kardar model at low input rate ($J_{\rm in} = 0.2$). Inset: the same graph in linear-log scale.

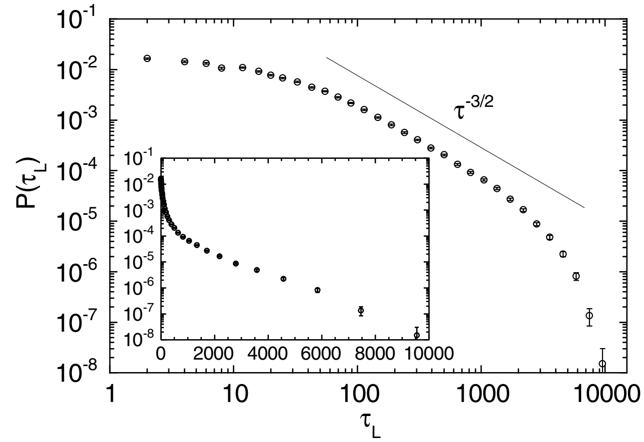

FIG. 3. PDF of laminar times $P(\tau_L)$ between subsequent bursts in energy dissipated signal for the Hwa-Kardar model at high input rate ($J_{\rm in} = 4$). Inset: the same graph in linear-log scale.

exponential behavior, as shown in Fig. 2, as expected for a BTW SOC system, whereas at high $J_{\rm in}$, the PDF exhibits a power law decay, as shown in Fig. 3. The same results have been found by considering systems of different size. The data herein refer to a system with a grid of 512 points. In particular, in this case, to detect a power law decay in more than an order of magnitude in τ_L, $J_{\rm in}$ larger than 4 is required. Therefore we have proved that the conjecture in Ref. [13] correctly applies so that power law behavior in the PDF of laminar time between bursts cannot rule out SOC dynamics since it could be consistent with the properties of a class of SOC systems wider than the original BTW. Thus other statistical properties should be investigated to settle upon the consistency with SOC dynamics. It is known that an important feature of SOC dynamics is self-similarity in a wide range of scales. Therefore it is mandatory to verify whether the particle flux reflects this fundamental property.

According to the Kolmogorov theory, turbulence can be described as a random process with stationary increment [16]. Thus to measure any departure from self-similarity, the PDFs at different scales τ of the increments, defined as $\delta_\tau \Gamma(t) = \Gamma(t + \tau) - \Gamma(t)$, must be compared [16].

To carry out the analysis of the PDF of Γ fluctuations at different time scales, an efficient tool is the wavelet decomposition of the signal. It has been proved indeed that the wavelet coefficients $C(t, \tau)$ have the same scaling properties of $\delta_\tau \Gamma(t)$ with τ [17]. To compare the shape of the PDF at the different scales, for each time scale the coefficients $C(t, \tau)$ have been normalized by subtracting the average and dividing by the standard deviation $c_\tau(t) = [C(t, \tau) - \langle C(t, \tau) \rangle]/\sigma_\tau$. Therefore the probability distribution function of the coefficients at each scale $P(c_\tau)$ has been obtained.

In Figs. 4(a) and 4(b) an example of normalized PDFs for two different time scales ($\tau = 4$ and 36 μs) is shown. It appears that the PDFs are different and, in particular, at the smaller scale the PDF develops non-Gaussian tails. The dependence of the PDFs with τ can be recovered by fitting the data by a three parameter stretched exponential function $P(X) \sim A \exp(-b|X|^\zeta)$, where the scaling behavior of the PDF is derived from the scaling of the coefficient $\zeta(\tau)$ with τ. In the same figure the result of a reduced χ^2 fit is superimposed to the experimental PDF. In Fig. 4(c) $\zeta(\tau)$ as a function of τ is shown. In the range of scales relevant for transport, $\zeta(\tau)$ exhibits two different

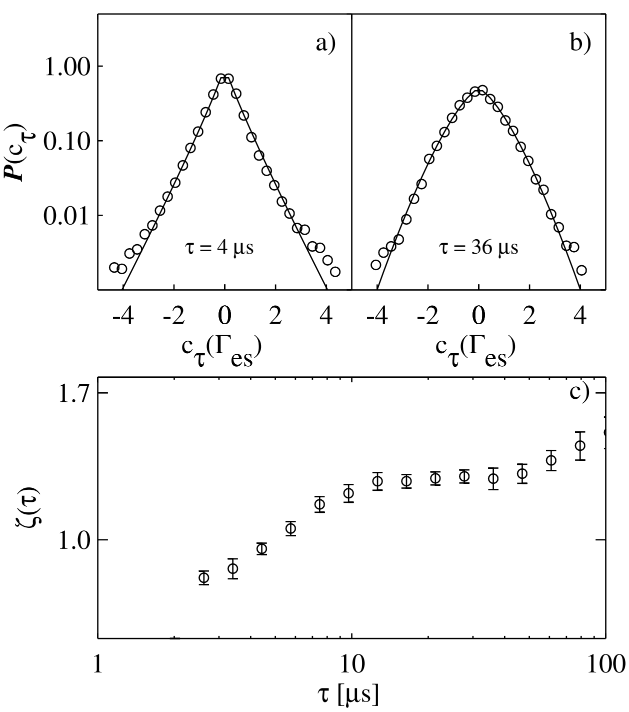

FIG. 4. (a), (b) PDF of particle flux fluctuations at two different time scales; (c) log-log plot of coefficient $\zeta(\tau)$ calculated from stretched exponential fit as a function of scale τ. The data refer to a statistical ensemble of different shots and the error bars are the standard deviation of the average.

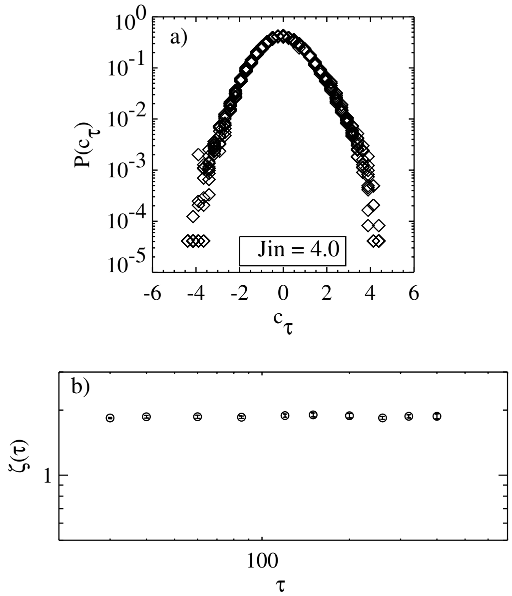

FIG. 5. (a) PDF of fluctuations of dissipated energy at ten different time scales for a Hwa-Kardar model. (b) Scaling behavior of coefficients $\zeta(\tau)$ vs τ.

behaviors: in the range $2 \leq \tau \leq 10$ μs, $\zeta(\tau)$ is not constant and therefore the particle flux fluctuations are not self-similar in character, while in the range $10 \leq \tau \leq 30$ μs, $\zeta(\tau)$ is almost constant with a value 1.24. In order to compare these two behaviors with SOC models, the same PDF analysis using the wavelet decomposition has been applied to the dissipated energy fluctuations in the running sandpile model at a high input rate, i.e., in the regime where the PDF of laminar times has a power law decay consistent with experimental data. In Fig. 5(a) the PDFs at ten different time scales in the range 30–400 time steps and with $J_{\text{in}} = 4$ are shown: the PDFs collapse to a single one close to a Gaussian distribution. Indeed, as shown in Fig. 5(b), $\zeta(\tau)$ results almost constant and almost 1.9. Therefore, even in the limit of high J_{in} where the laminar time statistics was found consistent with the experimental results, the experimental properties of the particle flux fluctuations appear inconsistent with those predicted by SOC models. Indeed fluctuations are not self-similar in character in a wide range of scales and where they are self-similar, their PDF is far from a Gaussian one.

In conclusion, a wavelet decomposition of particle flux fluctuations in the edge region of RFX has revealed that fluctuations are not self-similar in character and that the PDFs have non-Gaussian tails in a range of time scales where most of the transport is concentrated. Moreover the laminar times between bursts, which carry more than 50% of the particle transport at the edge, exhibit a power law decay. These two properties are in contrast with statistical properties of avalanches in a wide class of SOC models including the running sandpile model, which allows avalanche interaction. Therefore we conclude that the statistical analysis performed on the particle flux rules out avalanchelike processes in a RFP configuration.

As a final remark we mention that the similarities in electrostatic turbulence and transport between RFP and other magnetic configurations [14] should encourage applying the same statistical analysis to fluctuations in other experiments to check the consistency of SOC models with fusion plasmas in general.

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