Dipole Blockade and Quantum Information Processing in Mesoscopic Atomic Ensembles

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(Received 7 November 2000; published 26 June 2001)

We describe a technique for manipulating quantum information stored in collective states of mesoscopic ensembles. Quantum processing is accomplished by optical excitation into states with strong dipole-dipole interactions. The resulting "dipole blockade" can be used to inhibit transitions into all but singly excited collective states. This can be employed for a controlled generation of collective atomic spin states as well as nonclassical photonic states and for scalable quantum logic gates. An example involving a cold Rydberg gas is analyzed.

DOI: 10.1103/PhysRevLett.87.037901 PACS numbers: 03.67.Lx, 03.75.Fi, 42.50.Gy, 73.23.–b

Recent advances in quantum information science have opened a door for a number of fascinating potential applications ranging from the factorization of large numbers and secure communication to spectroscopic techniques with enhanced sensitivity. But the practical implementation of quantum processing protocols such as quantum computation requires coherent manipulation of a large number of coupled quantum systems which is an extremely difficult task [1]. Challenges ranging from a long-time storage of quantum information to scalable quantum logic gates are by now well known. It is generally believed that precise manipulation of *microscopic* quantum objects is essential to implement quantum protocols. For example, in most of the potentially viable candidates for quantum computers an exceptional degree of control over submicron systems is essential for performing single-bit operations and the two-bit coupling is accomplished by interactions between nearest neighbors [2]. Related techniques are also being explored that involve photons to connect qubits [3], and to construct potentially scalable quantum networks [4]. However, since the single-atom absorption cross section is very small, reliable coupling to light requires high-finesse microcavities [5].

In the present Letter we describe a technique for the coherent manipulation of quantum information stored in collective excitations of mesoscopic many-atom ensembles. This is accomplished by optically exciting the ensemble into states with a strong atom-atom interaction. Specifically, we consider the case involving dipole-dipole interactions in an ensemble of cold atoms excited into Rydberg states. Under certain conditions the level shifts associated with these interactions can be used to block the transitions into states with more than a single excitation. The resulting "dipole blockade" phenomenon closely resembles similar mesoscopic effects in nanoscale solid-state devices [6]. In the present context it can take place in an ensemble with a size that can exceed many optical wavelengths. Combined with the exceptional degree of control that is typical for quantum optical systems and long coherence times, this allows one to considerably alleviate many stringent requirements for the experimental implementation of various quantum processing protocols. In particular, we show that this technique can be used to (i) generate superpositions of collective spin states (or Dicke states [7]) in an ensemble; (ii) coherently convert these states into corresponding states of photon wave packets of prescribed direction, duration, and pulse shapes and vice versa using the collectively enhanced coupling to light [8]; and (iii) perform quantum gate operations between distant qubits. Corresponding applications including (i) subshot noise spectroscopy and atom interferometry [9], (ii) secure cryptography protocols [10], and (iii) scalable quantum logic devices can be foreseen. In general, no strongly coupling microcavities and no single particle control are required to implement computation and communication protocols. We further anticipate that the approach can be applied to a variety of interacting many-body systems ranging from trapped ions to specifically designed semiconductor structures.

The basic element of the present scheme is an ensemble of N identical multistate atoms (Fig. 1) contained in a volume V. Using well-developed techniques all atoms can be trapped and prepared in a specific sublevel (g_i, $i = 1, \ldots, N$) of the ground state manifold. Relevant states of each atom include a pair of metastable sublevels of the ground state manifold q_i that are used for long-time storage of qubits (storage states) and long-lived Rydberg states r_i, p_i', p_i''. Additional Rydberg sublevels as well as lower electronic excited states can be used for specific applications. We assume modest atomic densities, such that interactions between atoms can safely be neglected whenever they are in the sublevels of the ground state. This also implies long coherence lifetimes — up to a few

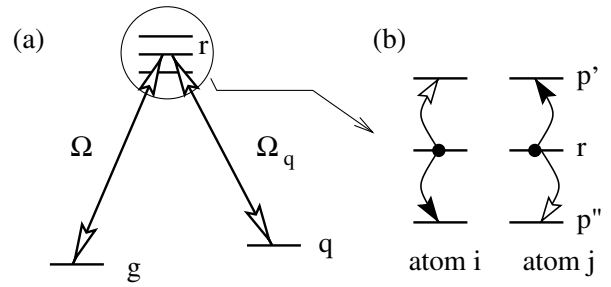

FIG. 1. (a) Relevant single-atom levels for quantum information storage (q) and processing (r, p', p'') can be accessed by optical fields. Ω, Ω_q are Rabi frequencies. (b) Resonant atom-atom interaction causes excitation hopping between atom pairs and results in collective level splitting. Arrows indicate atomic transitions in the hopping process.

seconds — associated with the storage sublevels. However when excited into the Rydberg states the atoms interact strongly with each other due to the presence of resonant long-wavelength dipole-dipole interactions [11].

The manipulation of atoms is done by light fields of different frequencies and polarizations, which illuminate the entire ensemble and excite all atoms with equal probability. Throughout the paper we will assume that hyperfine sublevels of the ground state and Rydberg states are excited in a Doppler free configuration, which can be done using two-photon processes. The Hamiltonian describing these interactions can be written in terms of collective operators $\hat{\Sigma}_{\mu\nu} = \sum_i |\mu_i\rangle\langle\nu_i|/\sqrt{N}$, where $|\mu_i\rangle, |\nu_i\rangle$ denote single-atom eigenstates. As a consequence only *symmetric collective states* are involved in the process. These include the ground state $|\mathbf{g}\rangle \equiv |\mathbf{g}^N\rangle = |g_1\rangle \cdots |g_N\rangle$, the n-times excited storage states $|\mathbf{q}^n\rangle \equiv |\mathbf{g}^{N-n}, \mathbf{q}^n\rangle = \sqrt{(N-n)!\,N^n/(N!\,n!)}\,(\hat{\Sigma}^+_{gq})^n|\mathbf{g}\rangle$, and the Rydberg states $|\mathbf{r}^n\rangle \equiv |\mathbf{g}^{N-n}, \mathbf{r}^n\rangle = \sqrt{(N-n)!\,N^n/(N!\,n!)}\,(\hat{\Sigma}^+_{gr})^n|\mathbf{g}\rangle$.

We now discuss the interaction between collective atomic excitations. To be specific we here consider excitation "hopping" via resonant dipole-dipole interactions between Rydberg atoms (Föster process [12]) as illustrated in Fig. 1b [13]. The energy separation between the optically excited Rydberg state $|r_i\rangle$ and the pair of sublevels of different parity $|p_i'\rangle, |p_i''\rangle$ is adjusted (using, e.g., electric fields) such that $E_r - E_{p'} = E_{p''} - E_r$. In this case, any pair of atoms excited in the $|r_i\rangle|r_j\rangle$ states would undergo a hopping transition into the states $|p_i'\rangle|p_j''\rangle$ and $|p_i''\rangle|p_j'\rangle$, etc., resulting in a splitting of the excited Rydberg components. For a system with three effective Rydberg states (Fig. 1b) the relevant process is described by the Hamiltonian:

$$\hat{V}_d = \hbar \sum_{i>j} \kappa_{ij} |r_i\rangle|r_j\rangle(\langle p_i'|\langle p_j''| + \langle p_j'|\langle p_i''|) + \text{H.c.}, \tag{1}$$

where $\hbar\kappa_{ij} \sim \wp_{rp'}\wp_{rp''}/r_{ij}^3$, $\wp_{kl}$ being the dipole matrix elements for the corresponding transitions and r_{ij} is the distance between the two atoms [14]. In general this interaction does not affect the singly excited collective states (e.g., $\hat{V}_d|\mathbf{r}^1\rangle = 0$) but leads to a splitting of the levels when two or more atoms are excited. In particular, the collective eigenstate with atoms i and j being excited and all other atoms in the ground or storage states

$$|\pm^{\mathbf{2}}_{\mathbf{ij}}\rangle = \frac{1}{\sqrt{2}}[|g_1 \cdots r_i \cdots r_j \cdots g_N\rangle \pm (|g_1 \cdots p_i' \cdots p_j'' \cdots g_N\rangle + |g_1 \cdots p_j' \cdots p_i'' \cdots g_N\rangle)/\sqrt{2}] \tag{2}$$

are split by $\hbar\kappa_{ij}$. It follows that for an ensemble contained in a finite volume V the manifold of doubly excited states has an energy gap of order $\hbar\bar{\kappa} = \wp_{rp'}\wp_{rp''}/V$ (Fig. 2). Thus if $\bar{\kappa}$ is much larger than the linewidth γ_r of the Rydberg state, resonant excitations from the singly to the doubly excited state are strongly suppressed. This is the essence of the dipole blockade.

Turning to the interaction of such many-atom systems with light we first discuss how to create single collective qubits. A weak light field tuned to the single-atom resonance frequency can excite the transition between the ground state and the first collective state $|\mathbf{r}^1\rangle$. But when the splitting of the states $|\pm^2_{\mathbf{ij}}\rangle$ is large successive transitions into these and higher states are strongly inhibited. Hence if the atomic system is initially in its ground state $|\mathbf{g}\rangle$ the evolution is given by the two-level dynamics

$$\begin{bmatrix} |\mathbf{g}(t)\rangle \\ |\mathbf{r}^1(t)\rangle \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & -i\sin\theta(t) \\ i\sin\theta(t) & \cos\theta(t) \end{bmatrix} \begin{bmatrix} |\mathbf{g}(0)\rangle \\ |\mathbf{r}^1(0)\rangle \end{bmatrix}, \tag{3}$$

where $\theta(t) = \sqrt{N}\int_0^t \Omega(\tau)/2\,d\tau$. The ensemble displays Rabi oscillations only between ground and singly excited Rydberg states with a collective Rabi rate $\sqrt{N}\,\Omega$. There are obviously several conditions that restrict the validity of Eq. (3). First of all, the Hamiltonian evolution takes place for times t much shorter than the dephasing time $\gamma_{\mathbf{r}}^{-1}$. Secondly, there is a finite probability to populate the doubly excited states $|\pm^2_{\mathbf{ij}}\rangle$ that scales as $\sim\bar{\kappa}^{-2}$. For example, at a time $t = T$ where a single collective excitation is generated [i.e., where $\theta(T) = \pi$] the probabilities of errors due to populating the doubly excited states and dephasing

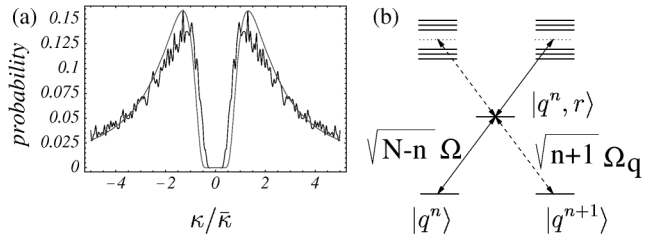

FIG. 2. (a) Probability distribution for frequency shifts of doubly excited Rydberg states. Shown is a Monte Carlo simulation for 3×10^4 positions in a rectangular box and analytical approximation for a random gas $p(x) = \sqrt{2}\,\pi \exp[-\pi^3/18x^2]/(6x^2)$, $x = \kappa/\bar{\kappa}$. (b) Successive generation of Fock states $|\mathbf{q}^n\rangle$ using dipole blockade.

scale, respectively, as

$$p_{\mathrm{doub}} \sim \frac{1}{N^2}\sum_{i,j}\frac{1}{\kappa_{ij}^2 T^2} \approx \frac{1}{4\pi(\bar{\kappa}T)^2}, \qquad p_{\mathrm{deph}} \sim \gamma_{\mathbf{r}} T. \tag{4}$$

As long as $p_{\mathrm{doub}}, p_{\mathrm{deph}} \ll 1$ the system can be driven into superpositions of collective states $\alpha_0|\mathbf{g}\rangle + \alpha_1|\mathbf{r}^1\rangle$. The single-quantum excitation can now be stimulated into a storage sublevel (e.g., $|\mathbf{q}^1\rangle$) by a π pulse [$\int \Omega_q(\tau)\,d\tau = \pi$] and we can associate a qubit with the state $\alpha_0|\mathbf{g}\rangle + \alpha_1|\mathbf{q}^1\rangle$. As indicated in Fig. 2, this procedure can be generalized to the generation of higher order collective states or their superpositions using a sequence of properly timed pulses. For example, the sequence of two pulses $\sqrt{N-n}\int\Omega(\tau)\,d\tau = \pi$ and $\sqrt{n+1}\int\Omega_q(\tau)\,d\tau = \pi$ results in $|\mathbf{q}^n\rangle \rightarrow |\mathbf{q}^{n+1}\rangle$. Note that under the conditions of dipole blockade the transitions into collective Rydberg states other than $|\mathbf{r}^1\mathbf{q}^m\rangle$ are always inhibited. The scaling of the error probability in each step is given by Eq. (4). To prove that a synthesis of arbitrary superpositions $|\Psi_n\rangle = \sum_{m=0}^{n}\alpha_m|\mathbf{q}^m\rangle$ ($n \le N$) is possible we note that the inverse procedure, i.e., the evolution from a known state $|\Psi_n\rangle$ into the ground state $|\mathbf{g}\rangle$ can easily be constructed. In particular, $2n$ pulses of proper duration can be used to sequentially empty the states $|\mathbf{q}^n\rangle, |\mathbf{r}^1\mathbf{q}^{n-1}\rangle, |\mathbf{q}^{n-1}\rangle$, etc., using a procedure analogous to that described in Ref. [15], and the system would ultimately evolve into its ground state. Since this evolution is unitary a reverse sequence results in $|\mathbf{g}\rangle \rightarrow |\Psi_n\rangle$.

Before proceeding we note several important properties of collective spin states generated by means of the dipole blockade. First of all, the states $|\Psi_N\rangle$ with $\alpha_0 = \alpha_N = 1/\sqrt{2}$, $\alpha_i = 0, i = 1,\ldots,N-1$ maximize spectroscopic sensitivity [9] beyond the standard quantum limit. Second, by using standard techniques of atom optics it is possible to couple all atoms in a specific internal state (e.g., q) out of the trap. When these techniques are applied to the atoms in collective state $|\mathbf{q}^n\rangle$, the pulsed beam of atoms with any prescribed atom number $1 < n < N$ can be coupled out. (Note that the atoms are indistinguishable, hence we will have a superposition with all possible permutations in such an outcoupled state.) Techniques of this kind can be used to enhance the sensitivity of atom interferometers [16,17]. Finally, the spin states $|\mathbf{q}^n\rangle$ exhibit a collectively enhanced coupling to light, when excited in a proper two-photon configuration [8]. Consequently, the state $|\Psi_n\rangle$ can be transferred from the spin degrees of freedom to the optical field. This allows one to generate wave packets on demand with arbitrary quantum states, pulse shapes, and duration without the use of high-Q cavities as necessary in single-atom cavity QED [15]. The quantum state of these wave packets can be transferred to distant ensembles thereby making quantum networking possible.

The dipole blockade mechanism also facilitates quantum logic operations. For instance, in one approach qubits stored in few-μm-spaced atomic clouds can be entangled if the transitions in one ensemble are inhibited when a collective Rydberg state is excited in a second ensemble. In essence, this scheme utilizes the same principle as that of Ref. [18] with the single-particle excitations replaced by collective qubits stored, e.g., in $\{|\mathbf{g}\rangle, |\mathbf{q}^1\rangle\}$. In what follows we discuss another approach that allows for such operations between distant ensembles, which is instrumental for scalability. In this approach the qubit states of two different ensembles $\{|\mathbf{g}\rangle_1, |\mathbf{q}^1\rangle_1\}$ and $\{|\mathbf{g}\rangle_2, |\mathbf{q}^1\rangle_2\}$ are first transferred to one ensemble using light. Specifically, the qubit from ensemble 1 is transferred to ensemble 3, followed by a transfer of the qubit from ensemble 2 to ensemble 3. Different internal states of the atoms in the third ensemble are used (see Fig. 3a) to realize the following mapping: $|\mathbf{g}\rangle_1|\mathbf{g}\rangle_2 \rightarrow |\mathbf{g}\rangle_3$, $|\mathbf{g}\rangle_1|\mathbf{q}^1\rangle_2 \rightarrow |\mathbf{q}^1_-\rangle_3$, $|\mathbf{q}^1\rangle_1|\mathbf{g}\rangle_2 \rightarrow |\mathbf{q}^1_+\rangle_3$, and $|\mathbf{q}^1\rangle_1|\mathbf{q}^1\rangle_2 \rightarrow |\mathbf{q}^1_-\mathbf{q}^1_+\rangle_3$. The state $|\mathbf{q}^1_-\mathbf{q}^1_+\rangle_3$ refers here to a collective excitation of the ensemble 3 with one atom being in the internal state q_- *and* one atom being in state q_+. A universal logic gate is then performed in three steps. First, all atoms in the storage states q_- are excited into a Rydberg state r_- by a π pulse, $\int_0^T \Omega_-(\tau)\,d\tau = \pi$, which leads to the transitions $|\mathbf{q}^1_-\rangle_3 \rightarrow i|\mathbf{r}^1_-\rangle_3$ and $|\mathbf{q}^1_-\mathbf{q}^1_+\rangle_3 \rightarrow i|\mathbf{r}^1_-\mathbf{q}^1_+\rangle_3$. Second, the atoms in the storage state q_+ are excited into the state r_+ and back by a 2π pulse, $\int_0^T \Omega_+(\tau)\,d\tau = 2\pi$, which leads to the transitions $|\mathbf{q}^1_+\rangle_3 \rightarrow -|\mathbf{q}^1_+\rangle_3$. Note, however, that transitions into states with two Rydberg atoms are inhibited due to the dipole blockade. As a result, the atoms in the collective state $|\mathbf{r}^1_-\mathbf{q}^1_+\rangle_3$ do not acquire any phase rotation. In a final step, the atoms in state r_- are stimulated back into the storage sublevels with a π pulse. The entire pulse sequence results in a total phase shift of π for the states $|\mathbf{q}^1_\pm\rangle_3$ and $|\mathbf{q}^1_+, \mathbf{q}^1_-\rangle_3$. Since the state $|\mathbf{g}\rangle_3$ is obviously unaffected by pulses, a conditional phase shift is obtained. The gate is completed by sending the qubits back to the original registers.

We next discuss a number of important practical issues associated with the present technique. First of all the decoherence of collective excitations is of particular importance here: the qubit states such as $|\mathbf{q}^1_\pm\rangle, |\mathbf{r}^1\rangle$ are, in

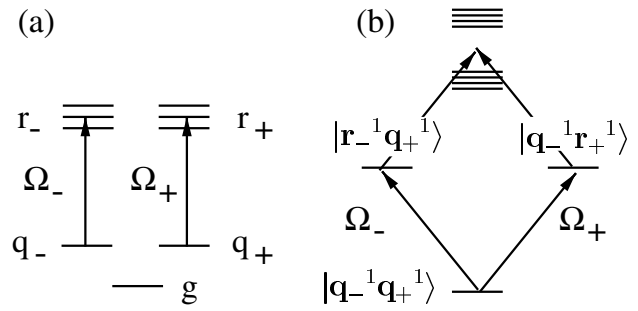

FIG. 3. (a) Relevant single-atom energy levels for generation of conditional phase shifts. (b) Coupling scheme within the manifold of doubly excited collective states.

fact, entangled superpositions of N single-particle states and these might be expected to be extremely fragile. This is not the case, since the symmetric entangled states involved are known to be very robust against, e.g., particle loss [19]. As a result, collective dephasing rates are equal to the corresponding single-particle rates, if the dephasing is dominated by single-particle rather than collective processes. The last assumption is well justified as long as the average interatomic distance is larger than the reduced optical wavelength $\lambda/(2\pi)$: $N/V \leq (2\pi/\lambda)^3$ [20]. Errors due to atom-number fluctuations (ΔN) are negligible if $\Delta N \ll N$. Finally, because the states that are involved in strong atomic interactions are, in the ideal limit, never populated, the present approach avoids mechanical interaction between atoms and leaves the qubit degrees of freedom decoupled from atomic motion [18].

Present day magnetic or optical traps allow one to contain about $N \sim 10^4$ cold alkali atoms (Li, Cs, or Rb) within $V < (10\ \mu\text{m})^3$. For the atoms excited to Rydberg states ($n \sim 50$), typical coherence lifetimes limited by radiative relaxation and black-body radiation correspond to $\gamma_{\mathbf{r}} < 10$ kHz, whereas dipole-dipole interactions correspond to values of $\bar{\kappa}$ in the range of 10–100 MHz. This implies that for excitation on a fast time scale such as $T < 100$ ns, the probability of loss $p_{\text{doub}}, p_{\text{loss}} < 1\%$. Such ensembles are opaque near the resonant optical transitions of the $D_{1,2}$ absorption lines, thereby making efficient coupling to light possible [8]. In a potentially scalable approach one could employ arrays of such traps created on nanofabricated surfaces [21], in which case the traps could also be coupled using low-Q cavities and low-loss fiber waveguides. Finally, since the atomic motion is never coupled to the qubits, the temperature is limited only by the requirement that the atomic distribution should not change significantly on the time scale of the gate operation T ("frozen" gas approximation). This is the case even for temperatures as high as a few mK.

In summary, we have shown that the dipole blockade can be used for coherent manipulation and entanglement of collective excitations in mesoscopic ensembles of cold atoms. We anticipate a number of potential applications ranging from atomic clocks and secure quantum communication to quantum computation with mesoscopic atomic samples.

We thank E. Eyler, P. Gould, T. Gallagher, and V. Kharchenko for many stimulating discussions. This work was supported by the Austrian Science Foundation, the Europe Union project EQUIP, the ESF, the European TMR network Quantum Information, the NSF through the grant to the ITAMP and the NSF ITR program. M. F. thanks the Deutsche Forschungsgemeinschaft for financial support and ITAMP for the hospitality during his stay.

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