Comment on "Do Semiclassical Zero Temperature Black Holes Exist"

In a recent Letter [1], the claim was made that "in all physically realistic cases, macroscopic zero temperature black hole solutions do not exist." We will show this conclusion was reached on the basis of an incorrect calculation.

The Reissner-Nordström metric is parametrized as

$$ds^2 = -[1 + 2\rho(r)]\left(1 - \frac{2m(r)}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2m(r)}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2. \quad (1)$$

Defining $m(r) = M[1 + \mu(r)]$, with $\epsilon = \hbar/M^2$, the authors find the semiclassical Einstein equations,

$$\frac{d\mu}{dr} = -\frac{4\pi r^2}{M\epsilon}\langle T_t^t\rangle,$$
$$\frac{d\rho}{dr} = \frac{4\pi r}{\epsilon}\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}(\langle T_r^r\rangle - \langle T_t^t\rangle), \quad (2)$$

and try to find a perturbative solution for μ and ρ. At the unperturbed horizon $r_+ = M + \sqrt{M^2 - Q^2}$, the authors set $\mu(r_+) = C_1$ and $\rho(r_+) = C_2$.

They then go on to state that the perturbed horizon lies at $r = M_R + \sqrt{M_R^2 - Q^2}$, where $M_R = M(1 + C_1)$. The perturbed horizon is a solution of

$$r^2 - 2m(r)r + Q^2 = 0. \quad (3)$$

Here they perform a double expansion in ϵ and $r - r_+$, keeping $|r - r_+| \le \mathcal{O}(\epsilon)$. Keeping only terms of order ϵ in m gives the result the authors find. However, when one approaches the extremal limit $[M^2 - Q^2 \le \mathcal{O}(\epsilon)]$, it becomes necessary to expand m to order ϵ^2 to obtain r_h correctly to order ϵ. This follows simply from the form of the solution to the quadratic equation. When one does this the perturbed horizon does not lie at $M_R + \sqrt{M_R^2 - Q^2}$ in general.

Another way to see this is to view the equation $r = r_+(r)$ with $r_\pm(r) \equiv m(r) \pm \sqrt{m(r)^2 - Q^2}$ as determining the position of the horizon (where we define $\sqrt{A^2} = |A|$, for real A). In general, this equation need not have solutions, and the horizon appears as a solution of $r = r_-(r)$ instead. An explicit example is given by taking $m(r) = [1 + \epsilon(r - 1)]/(1 + \epsilon^2)$ and $Q^2 = 1/(1 + \epsilon^2)$. By inserting these expressions into (3), one finds a quadratic with a double zero at $r = r_h = 1/(1 - \epsilon)$. However, when $\epsilon < 0$, $r_+[1/(1 - \epsilon)] = (1 - \epsilon)/(1 + \epsilon^2)$ and the argument of [1], identifying the horizon with $r_+(r_h)$, would lead to an incorrect horizon position.

The correct near-horizon solution of the semiclassical Einstein equations at extremality (expanding in ϵ and $r - M$, keeping $|r - M| \le \mathcal{O}(\epsilon)$ and denoting $d\mu/dr$ by μ') is

$$m(r) = M - M^3\epsilon^2[\mu'(M)]^2 + M\epsilon\mu'(M)(r - M) + \dots \quad (4)$$

with the extremal charge $Q = M - \frac{1}{2}\epsilon^2 M^3[\mu'(M)]^2 + \dots$. Note $\mu'(M)$ is determined by (2). The $r - M$ independent term in (4) is an integration constant of (2) which may be chosen for convenience. The quantum correction to the extremality relation between M and Q is fixed by demanding that the discriminant of the quadratic equation (3) vanishes. The solution of (3) gives the position of the horizon $r_h = M + \mu'(M)\epsilon M^2 + \dots$, to leading order in ϵ. Inserting (4) into the metric, one finds that the surface gravity of the black hole vanishes on the horizon. This is most easily seen by noting that (3) has a double zero at $r = r_h$ (at order ϵ^2). This solution holds regardless of the sign of $\mu'(M)$, and smoothly matches onto the classical solution as $\epsilon \to 0$, contradicting the calculation performed in [1].

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[1] P. R. Anderson, W. A. Hiscock, and B. E. Taylor, Phys. Rev. Lett. **85**, 2438 (2000).

