# Superconducting Vortices in ac Fields: Does the Kohn Theorem Work?

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(Received 13 February 2001; published 13 June 2001)

Electrodynamics of clean pinning-free type II superconductors in the mixed state is derived using the Boltzmann kinetic equations for excitations. The condition of the vortex cyclotron resonance is found. The reason why this resonance does not comply with the Kohn theorem is discussed.

DOI: 10.1103/PhysRevLett.87.017003 PACS numbers: 74.25.Nf, 74.60.Ge, 76.40.+b

Microwave and far-infrared measurements on type II superconductors offer an efficient experimental tool for studying vortex dynamics and kinetics of superconductors (see, for example, [1] and references therein). Yet theoretical aspects of electrodynamics of type II superconductors remain a matter of controversies. The full description of the electrodynamics of the mixed state has to include vortex motion, where several unresolved issues including the value of the vortex mass and the accurate account of forces acting on vortices remained lately unresolved. Recently, substantial progress has been achieved in understanding underlying microscopic mechanisms of vortex dynamics [2–7]. This opens a new route for a full description of the mixed-state electrodynamics which was treated so far within phenomenological models only [1,8–10]. In this Letter, we develop a rigorous microscopic approach to the mixed state ac response of a clean type II superconductor using the Boltzmann kinetic equation for core excitations and calculate the ac conductivity. We discuss cyclotron resonance and resonant friction effects. We consider a pinning-free sample with a thickness much smaller than the London penetration length such that bending of vortex lines can be neglected [11]. In this case, the magnetic field is nearly uniform. We find, however, that, because of coupling of normal electrons to the heat bath, the vortex cyclotron resonance even in ideal superconductors *does not* generally follow the Kohn theorem [12] formulated for a system of interacting electrons in a uniform magnetic field. The violation of the Kohn theorem can be tested in far-infrared experiments on superclean materials with the quasiparticle mean-free time $\tau > E_F/T_c^2$ (we use units with $\hbar = 1$).

*Electric fields and forces.*—The ac conductivity is determined by a general relation $\mathbf{j} = \hat{\sigma}(\omega)\mathbf{E}$. An ac electric field in a type II superconductor appears not only because of the motion of vortices: an additional field is needed to support an acceleration of Cooper pairs. Averaging the local electric field over the space and taking into account that the vortex phase is not single valued, we find

$$\overline{\mathbf{E}} = \mathbf{E}_{\rm ac} + c^{-1}[\mathbf{B} \times \mathbf{v}_L], \qquad \mathbf{E}_{\rm ac} = (m/e)\,\overline{(\partial \mathbf{v}_s/\partial t)}\,. \tag{1}$$

Here, $\mathbf{v}_L$ is the vortex velocity. The superconducting velocity $\mathbf{v}_s = (\nabla\chi - 2e\mathbf{A}/c)/2m$ is produced by a static vortex array and by nonstationary fields. The part due to static vortices averages out. We assume a uniform in space $\mathbf{E}_{\rm ac}$ as it is induced by a microwave irradiation.

The electric field $\mathbf{E}_{\rm ac}$ and the vortex velocity $\mathbf{v}_L$ act as two independent perturbations producing deviation of electrons from equilibrium. The additional electric field can exist even when vortices are totally pinned $\mathbf{v}_L = 0$, while $\mathbf{E}_{\rm ac}$ vanishes for a steady motion of vortices. As we shall see, the responses of the system to these two perturbations are different: in contrast to the additional field that simply accelerates electrons, the vortex velocity causes changes in quasiparticle distribution by affecting the entire energy spectrum of excitations. As a result, the total force exerted on vortices by the environment is

$$\mathbf{F}_{\rm env} = -\pi N d_{\rm O}(\omega)\mathbf{v}_L + \pi N d_{\rm H}(\omega)[\hat{\mathbf{z}} \times \mathbf{v}_L] + \{\beta_{\rm O}(\omega)[\hat{\mathbf{z}} \times \mathbf{E}_{\rm ac}] + \beta_{\rm H}(\omega)\mathbf{E}_{\rm ac}\}\Phi_0\,\mathrm{sgn}(e)/c\,. \tag{2}$$

We take the $z$ axis along the vortex circulation, $\hat{\mathbf{z}} = \hat{\mathbf{h}}\,\mathrm{sgn}(e)$, where $\hat{\mathbf{h}}$ is the unit vector along the magnetic field; $\Phi_0 = \pi c/|e|$ is the magnetic flux quantum. $d_{\rm O,H}$ describe the vortex friction and the Hall effect, respectively [2], while $\beta_{\rm O,H}$ parametrize the force created by the field $\mathbf{E}_{\rm ac}$. In general, $\beta_{\rm O,H} \neq (N|e|c/B)d_{\rm O,H}$. The equality would have taken place if the responses were identical.

The parameters $d$ and $\beta$ are functions of the external frequency $\omega$. In particular, $d_{\rm O,H}(\omega)$ contain the inertial effects. Indeed, in case of a small $\omega$, the first-order terms in the $\omega$ expansion of $d_{\rm O,H}(\omega)$ describe the force proportional to the vortex acceleration and thus determine the vortex mass tensor [6]. Without pinning, the environment is translationally invariant; the force $\mathbf{F}_{\rm env}$ is thus balanced by the Lorentz force $\mathbf{F}_{\rm L} = (\Phi_0/c)[\,\mathbf{j} \times \hat{\mathbf{h}}]$ from the transport current generated in the superconductor, $\mathbf{F}_{\rm env} + \mathbf{F}_{\rm L} = 0$. For $H \ll H_{c2}$, the average transport current $\mathbf{j}$ is equal to the current far from the vortex; it consists of a supercurrent and a current carried by delocalized quasiparticles: $\mathbf{j} = N_s e\overline{\mathbf{v}}_s + \mathbf{j}^{(\rm qp)}$, where $N_s$ is the superconducting density. The quasiparticle current,

  

$$\mathbf{j}^{(\mathrm{qp})} = Ne\alpha_{\mathrm{H}}(\omega)\mathbf{v}_L + Ne\alpha_{\mathrm{O}}(\omega)[\hat{\mathbf{z}} \times \mathbf{v}_L] + \sigma_{\mathrm{O}}^{(\mathrm{qp})}(\omega)\mathbf{E}_{\mathrm{ac}} + \sigma_{\mathrm{H}}^{(\mathrm{qp})}(\omega)[\mathbf{E}_{\mathrm{ac}} \times \hat{\mathbf{z}}], \quad (3)$$

is driven by $\mathbf{E}_{\mathrm{ac}}$ and $\mathbf{v}_L$. Again, $\alpha_{\mathrm{O,H}} \neq (B/N|e|c)\sigma_{\mathrm{O,H}}^{(\mathrm{qp})}$. We omit the argument $\omega$ for brevity in what follows.

*Kinetic equation.*—We consider $s$-wave superconductors in magnetic fields $H \ll H_{c2}$, where the factors $d$, $\alpha$, $\beta$, and $\sigma^{(\mathrm{qp})}$ can be explicitly calculated. Assuming that the particle wavelength is much shorter than the coherence length, $p_F\xi \gg 1$ (which holds in almost all superconductors), we use a semiclassical scheme.

The quasiparticle distribution $f$ for the states localized in vortex cores obeys the Boltzmann equation [7],

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \phi}\frac{\partial \epsilon_n}{\partial \mu} - \frac{\partial \epsilon_n}{\partial \phi}\frac{\partial f}{\partial \mu} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}, \quad (4)$$

where the quasiclassical spectrum $\epsilon_n(\mu)$ of a particle plays the role of its effective Hamiltonian, $\mu$ is the angular momentum, and $\phi$ is the azimuthal angle in the plane perpendicular to the vortex axis. The spectrum $\epsilon_n(\mu)$ also depends on the radial quantum number $n$. The spectrum has an anomalous branch [13] $n = 0$ that crosses zero of energy as a function of $\mu$ and runs from $\Delta_\infty$ to $-\Delta_\infty$ as $\mu$ varies from $-\infty$ to $+\infty$. The other branches $n \neq 0$ are concentrated [14] near the gap edges $\pm\Delta_\infty$; they do not cross zero of energy but return to the same $+\Delta_\infty$ or $-\Delta_\infty$ for $\mu \to \pm\infty$. We denote $\omega_n = -\partial\epsilon_n/\partial\mu$.

We put $f = f^{(0)} + f_1$. The equilibrium part is $f^{(0)} = 1 - 2n_\epsilon = \tanh(\epsilon/2T)$, where $n_\epsilon$ is the Fermi function. The driving term in Eq. (4) is $\partial f/\partial t = (\partial f^{(0)}/\partial\epsilon)(\partial\epsilon_n/\partial t)$. The energy contains a time dependence through $\mu(t) = [(\mathbf{r} - \mathbf{v}_L t) \times \mathbf{p}] \cdot \hat{\mathbf{z}}$ and due to the work produced by $\mathbf{E}_{\mathrm{ac}}$:

$$\frac{\partial \epsilon_n}{\partial t} = \frac{\partial \epsilon_n}{\partial \mu}([\mathbf{p}_F \times \mathbf{v}_L] \cdot \hat{\mathbf{z}}) + e(\mathbf{v}_F \cdot \mathbf{E}_{\mathrm{ac}})\zeta_n .$$

Here, $\zeta_n \mathbf{v}_F$ is the "group velocity" in the state $\epsilon_n$. The factor $\zeta_n \sim 1$; we will not need an explicit expression for $\zeta_n$ in what follows. We use the relaxation-time approximation for the collision integral $(\partial f/\partial t)_{\mathrm{coll}} = -f_1/\tau_n$, where $\tau_n \sim \tau$. With this approximation, the mean-free time can be of any origin. We assume that the most effective relaxation is brought about by impurities, as is the case in almost all practical superconducting compounds.

Delocalized particles with $|\epsilon| > \Delta_\infty$ move mostly far from the vortex cores, where $\Delta$ is constant and the Doppler energy $\mathbf{p}_F\mathbf{v}_s$ is small. It can be shown that the kinetic equation for delocalized excitations is

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{p}} \cdot \mathbf{f} + \mathbf{v}_g \cdot \frac{\partial f}{\partial \mathbf{r}} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}, \quad (5)$$

similar to that for a particle in a magnetic field with a semiclassical spectrum $\epsilon_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta_\infty^2} + \mathbf{p}_F\mathbf{v}_s$, where $\xi_{\mathbf{p}} = \mathbf{p}^2/2m - E_F$. The derivation of Eq. (5) will be given elsewhere (see also Ref. [15]). The elementary Lorentz force and the energy derivative in Eq. (5) are

$$\mathbf{f} = \frac{e}{c}\mathbf{v}_g \times \mathbf{B} = \frac{\omega_c}{g_0}[\mathbf{p}_F \times \hat{\mathbf{z}}],$$

$$\frac{\partial \epsilon}{\partial t} = \mathbf{v}_L \cdot \mathbf{f} + e\mathbf{v}_F \cdot \mathbf{E}_{\mathrm{ac}} .$$

Here, $\omega_c = (e)B/mc$ is the cyclotron frequency, $\omega_c \sim (H/H_{c2})\omega_0 \ll \omega_0$ for $H \ll H_{c2}$; $\mathbf{v}_g = \partial\epsilon_{\mathbf{p}}/\partial\mathbf{p} = \mathbf{v}_F/g_0$ is the group velocity, and $g_0 = \epsilon/\sqrt{\epsilon^2 - \Delta_\infty^2}$. The second term in $\partial\epsilon/\partial t$ comes from the Doppler energy $\mathbf{p}\mathbf{v}_s$. The spatial derivative of $f_1$ drops out: For an extended state, $f_1$ should be independent of coordinates since a particle trajectory goes through many vortex unit cells at various distances from vortices. For the spectrum $\epsilon_{\mathbf{p}}$ as above, the collision integral is [16] $(\partial f/\partial t)_{\mathrm{coll}} = -f_1/g_0\tau$.

We put $f_1 = f_v + f_{\mathrm{ac}}$, where

$$f_v = -\frac{\partial f^{(0)}}{\partial \epsilon}\{([\mathbf{v}_L \times \mathbf{p}_F) \cdot \hat{\mathbf{z}}]\gamma_{\mathrm{O}} + (\mathbf{v}_L \cdot \mathbf{p}_F)\gamma_{\mathrm{H}}\}, \quad (6)$$

$$f_{\mathrm{ac}} = -\frac{e}{m}\frac{\partial f^{(0)}}{\partial \epsilon}\{(\mathbf{p}_F \cdot \mathbf{E}_{\mathrm{ac}})\gamma'_{\mathrm{O}} + ([\mathbf{p}_F \times \mathbf{E}_{\mathrm{ac}}] \cdot \mathbf{z})\gamma'_{\mathrm{H}}\}. \quad (7)$$

$f_v$ and $f_{\mathrm{ac}}$ correspond to the off-diagonal and diagonal terms, respectively, in the perturbation Hamiltonian of Ref. [17]. Equation (4) gives, for localized states,

$$\gamma_\pm = \frac{\omega_n}{\omega_n \mp \omega \mp i/\tau_n}, \qquad \gamma'_\pm = \frac{\zeta_n}{\omega_n}\gamma_\pm , \quad (8)$$

where $\gamma_\pm = \gamma_{\mathrm{H}} \pm i\gamma_{\mathrm{O}}$. Equation (5) for delocalized excitations yields

$$\gamma_\pm = \frac{\omega_c}{\omega_c \mp \omega g_0 \mp i/\tau}, \qquad \gamma'_\pm = \frac{g_0}{\omega_c}\gamma_\pm . \quad (9)$$

The responses of localized and delocalized excitations are different. They resemble, respectively, "vortex core" and "current pattern" responses introduced in Ref. [1].

*The quasiparticle current* far from the vortex core is

$$\mathbf{j}^{(\mathrm{qp})} = -\frac{e}{\pi}\int \frac{dp_z}{2\pi}\frac{d\phi}{2\pi}\frac{d\epsilon}{2}\mathbf{p}_F g_0 f_1 . \quad (10)$$

Using Eqs. (6) and (7), we recover Eq. (3) with

$$\alpha_{\mathrm{O,H}} = \int_{\epsilon>\Delta_\infty} g_0\gamma_{\mathrm{O,H}}\frac{\partial f^{(0)}}{\partial \epsilon}d\epsilon , \quad (11)$$

$$\sigma_{\mathrm{O,H}}^{(\mathrm{qp})} = \frac{Ne^2}{m\omega_c}\int_{\epsilon>\Delta_\infty} g_0^2\gamma_{\mathrm{O,H}}\frac{\partial f^{(0)}}{\partial \epsilon}d\epsilon , \quad (12)$$

where $\gamma_{\mathrm{O,H}}$ are to be taken from Eq. (9).

*The force from the environment* is [2]

$$\mathbf{F}_{\mathrm{env}} = -\frac{1}{2}\sum_n\int \frac{dp_z}{2\pi}\frac{d\phi\, d\mu}{2\pi}\omega_n[\mathbf{z} \times \mathbf{p}_F]f_1 - \int_{\mathrm{del}}\frac{d\epsilon}{2}\frac{dp_z}{2\pi}\frac{d\phi}{2\pi}[\mathbf{z} \times \mathbf{p}_F]f_1 .$$

The first term takes care of localized excitations while the second one is due to delocalized states. As a result, we get Eq. (2) where the vortex friction parameters are

$$d_{\rm O,H} = d_{\rm O,H}^{\rm (loc)} + \int_{\rm del} \frac{d\epsilon}{2} \frac{\partial f^{(0)}}{\partial \epsilon} \gamma_{\rm O,H}\,, \qquad (13)$$

while $\beta_{\rm O,H} = \beta_{\rm O,H}^{\rm (loc)} + (Ne^2/m\omega_c)\alpha_{\rm O,H}$, where

$$d_{\rm O,H}^{\rm (loc)} = \left\langle\!\!\left\langle \sum_n \int \frac{\omega_n d\mu}{2} \frac{\partial f^{(0)}}{\partial \epsilon} \gamma_{\rm O,H} \right\rangle\!\!\right\rangle, \qquad (14)$$

$$\beta_{\rm O,H}^{\rm (loc)} = \frac{Ne^2}{m} \left\langle\!\!\left\langle \sum_n \int \frac{\omega_n d\mu}{2} \frac{\partial f^{(0)}}{\partial \epsilon} \gamma'_{\rm O,H} \right\rangle\!\!\right\rangle \qquad (15)$$

come from localized excitations. We write $\langle\!\langle \ldots \rangle\!\rangle$ for an average over the Fermi surface with the weight $p_\perp^2 = p_F^2 \sin^2\theta$, where $\theta$ is the angle between $\mathbf{p}_F$ and the $z$ axis.

*ac response.*—Collecting all the terms in the force balance with the help of Eqs. (1)–(3), we obtain

$$(N_s/N)[\mathbf{v}_s \times \hat{\mathbf{z}}] + a_{\rm H}[\hat{\mathbf{z}} \times \mathbf{v}_L] - a_{\rm O}\mathbf{v}_L = (e/m\omega_c)(b_{\rm O}[\hat{\mathbf{z}} \times \mathbf{E}] + b_{\rm H}\mathbf{E}). \quad (16)$$

where $\mathbf{v}_s$ and $\mathbf{E}$ now stand for the space-averaged values,

$$b_{\rm O,H} = \frac{m\omega_c}{Ne^2}(\sigma_{\rm O,H}^{\rm (qp)} - \beta_{\rm O,H}^{\rm (loc)}) - \alpha_{\rm O,H}\,,$$

$$a_{\rm O,H} = (d_{\rm O,H} - \alpha_{\rm O,H}) + b_{\rm O,H}\,.$$

The term with $\beta^{\rm (loc)}$ in $a_{\rm O,H}$ is proportional to the vortex density and is thus small as compared to $d_{\rm O,H}^{\rm (loc)}$.

The solution is conveniently written for a circularly polarized wave. We put $E_\pm = E_x \pm iE_y$ and $v_{L\pm} = v_{Lx} \pm iv_{Ly}$, etc., for two polarizations $E_x = E \times \cos\omega t$, $E_y = \mp \sin\omega t$ and introduce $d_\pm = d_{\rm H} \pm id_{\rm O}$, $\alpha_\pm = \alpha_{\rm H} \pm i\alpha_{\rm O}$, and so on. The parameters $d_\pm$, $\alpha_\pm$, $\beta_\pm$, and $\sigma_\pm^{\rm (qp)}$ are expressed through the factors $\gamma_\pm$ via Eqs. (11) to (15) in the same way as the corresponding $d_{\rm O,H}$, etc. are expressed through $\gamma_{\rm O,H}$. Solving Eqs. (1) and (16), we write the current in the form $j_\pm = \sigma_\pm(\omega)E_\pm$, where

$$\sigma_\pm(\omega) = \sigma_{\rm O}^{\rm (qp)} \mp i\sigma_{\rm H}^{\rm (qp)} + \frac{ie^2N_s}{m\omega} - \frac{iNe^2}{m\omega_c}\Lambda_\pm\,,$$

$$\Lambda_\pm = \left(\frac{N_s}{N}\frac{\omega_c}{\omega} \mp \tilde{b}_\pm\right)\left(\frac{N_s}{N}\frac{\omega_c}{\omega} \mp b_\pm\right) \times \left(\frac{N_s}{N}\frac{\omega_c}{\omega} \mp a_\pm\right)^{-1}. \qquad (17)$$

Here, $\tilde{b}_\pm = (m\omega_c/Ne^2)\sigma_\pm^{\rm (qp)} - \alpha_\pm$.

In some publications (see, for example, Ref. [9]), a simple model is used that neglects $\mathbf{E}_{\rm ac}$ in the force balance:

$$c^{-1}\Phi_0[\mathbf{j} \times \hat{\mathbf{z}}] = \eta\mathbf{v}_L + \eta'[\mathbf{v}_L \times \hat{\mathbf{z}}].$$

At the same time, the transport current is assumed to be simply $\mathbf{j} = N_s e\mathbf{v}_s$. This model corresponds to $\alpha = \beta = \sigma^{\rm (qp)} = b = \tilde{b} = 0$ such that $a_{\rm O,H} = d_{\rm O,H}$ while $\eta = \pi N d_{\rm O}$, $\eta' = \pi N d_{\rm H}$. Our analysis shows that this model is justified only in the case when the delocalized quasiparticles do not participate in the response. This regime is realized when $\omega_c\tau \ll 1$ as explained below.

*Cyclotron resonance.*—In the normal state $N_s = \Lambda_\pm = 0$, while $g_0 = 1$. For $\omega_c\tau \gg 1$, the conductivity $\sigma_\pm(\omega) = \sigma_\pm^{\rm (qp)}$ in Eq. (17) has resonances at $\omega = \pm\omega_c$ [see Eq. (9)] in accordance with the Kohn theorem. In the superconducting state, however, the Kohn theorem is violated (see discussion below): each resonance transforms into an absorption band at $\omega < \omega_c$ because the poles in Eq. (9) acquire an energy dispersion through $g_0(\epsilon)$. The attenuation results from the Landau damping at delocalized states [15]. Note that the condition $\omega_c\tau \gg 1$ is not realistic for practical superconducting materials.

In the superconducting state, $\sigma_\pm(\omega)$ has resonances at

$$\omega a_{\rm H}(\omega) = \pm(N_s/N)\omega_c\,. \qquad (18)$$

It is the condition $a_{\rm O} \lesssim a_{\rm H}$ that is required for a sharp resonance rather than $\omega_c\tau \gg 1$. The inequality $a_{\rm O} \lesssim a_{\rm H}$ is well satisfied in the limit $\omega_0\tau \gg 1$, i.e., $\tau \gg E_F/T_c^2$, provided $\omega, \omega_c \ll \tau^{-1}$, when the width of the resonance is associated mostly with the localized states: $a_{\rm O}^{\rm (loc)} \sim (\omega_0\tau)^{-1}$ and $a_{\rm O}^{\rm (del)} \sim (\omega_c\tau)e^{-\Delta_\infty/T}$ while $a_{\rm H} \sim 1$. The dissipation at delocalized states grows with increasing $\tau$ and reaches its maximum for $\omega_c \sim \tau^{-1}$. With a further increase in $\tau$, the dissipation at delocalized states goes down and the low-damping condition is again satisfied if $\omega > \omega_c$. However, if $\omega < \omega_c$, a finite attenuation comes from the Landau damping discussed above; it is generally not small: ${\rm Re}a_{\rm O} \sim 1$ even for $\omega_c\tau \gg 1$ unless $T \ll \Delta_\infty$.

The resonant frequency is of the order of, but does not coincide exactly with, $\omega_c$. This violates the Kohn theorem [12] which states that the only resonance in the system of electrons should occur at the frequency $\omega_c$ irrespective of their mutual interaction. This is not surprising though: the Kohn theorem may not work in superconductors (despite the statement of Refs. [9,10]) because there are, in fact, *two different kinds of interacting charge carriers,* i.e., superconducting and normal electrons, that do not form a Galilean invariant system. Indeed, the normal electrons are in equilibrium with the heat bath (crystal lattice, sample boundaries, etc.) in absence of perturbations and make thus a preferable frame of reference.

One has to distinguish between ac and dc drives at this point. Upon switching a dc electric field, both superconducting and normal components adjust to it after a characteristic relaxation time in which a steady-state motion is established. Consider this case in more detail. For a steady-state motion of vortices, $\omega \to 0$, the additional field $\mathbf{E}_{\rm ac} = 0$. Equation (17) gives $j_\pm = \mp i(Ne^2/m\omega_c) \times d_\pm E_\pm$, i.e.,

$$\mathbf{j} = (Ne^2/m\omega_c)(d_{\rm O}\mathbf{E} + d_{\rm H}[\mathbf{E} \times \hat{\mathbf{z}}])\,, \qquad (19)$$

as it should be according to the force balance and Eqs. (2) and (3) with $\mathbf{E}_{\rm ac} = 0$. The effective conductivity is then determined by the vortex friction parameters $d_{\rm O,H}$. This couples the vortex velocity $\mathbf{v}_L$ with the superflow $\mathbf{v}_s$ through the factors $d_{\rm O,H}$ and $\alpha_{\rm O,H}$. For example, in an ideal superconductor with $\tau \to \infty$, one has $\gamma_H = 1$, $\gamma_O \to 0$. Therefore $d_O, \alpha_O \to 0$. The function under the integral in Eq. (14) reduces to the full derivative of $f^{(0)}$. The terms with $n \neq 0$ vanish because $\epsilon_n(\mu)$ returns to the same gap energy $\Delta_\infty$ (or to $-\Delta_\infty$) as $\mu$ goes to $\pm\infty$. The term $n = 0$ does not disappear since $\epsilon_0(\mu)$ varies from $\Delta_\infty$ to $-\Delta_\infty$. One obtains $d_H^{(\mathrm{loc})} = \tanh(\Delta_\infty/2T)$ and $d_H^{(\mathrm{del})} = 1 - \tanh(\Delta_\infty/2T)$. Thus, $d_H = 1$ and $\mathbf{j} = Ne\mathbf{v}_L$ [see Eq. (19)]. At the same time $\alpha_H = N_n/N$, where $N_n$ is the density of normal electrons. Equation (3) results in $\mathbf{j}^{(\mathrm{qp})} = N_n e \mathbf{v}_L$. Since $\mathbf{j} = N_s e \mathbf{v}_s + N_n e \mathbf{v}_L$, we have $\mathbf{v}_s = \mathbf{v}_L$: in an ideal superconductor, vortices move together with the superflow in accordance with the Helmholtz theorem. In turn, the quasiparticle current $\mathbf{j}^{(\mathrm{qp})} = N_n e \mathbf{v}_L$ implies that delocalized quasiparticles with $|\epsilon| > \Delta_\infty$ also move with the velocity $\mathbf{v}_L$. The quasiparticles localized in the cores have the distribution Eq. (6) with $\gamma_H = 1$ and $\gamma_O = 0$, i.e., they also move with $\mathbf{v}_L$. Therefore, the steady-state solution for $\tau \to \infty$ corresponds to a motion of the entire system with the superflow velocity $\mathbf{v}_s$. The Galilean solution is restored.

However, the Galilean invariance holds only for a steady-state flow. In time-dependent conditions, the responses of superconducting and normal subsystems are different. In the limit $\omega\tau \to \infty$, these subsystems do not come to equilibrium with each other, and the steady-state solution is never reached. Thus, the Kohn theorem which treats a single system of electrons is not applicable.

Consider the most realistic regime $\omega_c \tau \ll 1$ in more detail. The subsystem of electrons with $|\epsilon| > \Delta_\infty$ has $\gamma_{O,H} \ll 1$ [see Eq. (9)]; it is in equilibrium with the heat bath. We have $a_H = d_H^{(\mathrm{loc})}$. If $\omega_0 \tau \gg 1$, electrons in the vortex cores are in equilibrium with the superconducting component rather than with the heat bath. In this limit, $a_O \sim (\omega_0 \tau)^{-1} \ll 1$ while $\gamma_H = 1$ for $|\epsilon| < \Delta_\infty$ and $a_H = \tanh(\Delta_\infty/2T)$. Therefore, there are two interacting subsystems: one is composed of superconducting electrons in equilibrium with localized excitations, another subsystem is represented by delocalized excitations with $|\epsilon| > \Delta_\infty$ in equilibrium with the heat bath. It is clear that the resonance of the entire system cannot be described by the Kohn theorem. The ratio $N_s/Na_H$ that stands in Eq. (18) represents the temperature-dependent relative weight of the two subsystems. If delocalized excitations with $|\epsilon| > \Delta_\infty$ are absent, the resonant frequency turns to $\omega_c$, and the Kohn theorem is restored. This takes place for $T \ll \Delta_\infty$. On the contrary, $\omega_{\mathrm{res}}/\omega_c \sim \Delta/T_c$ according to Eq. (18); the ratio decreases for $T \to T_c$.

*Resonant friction.*—The vortex response displays another interesting feature at higher frequencies $\omega \sim \omega_0$ if $\omega_0 \tau \gg 1$: The factors $\gamma_{O,H}$ have resonances at localized states when $\omega = \omega_0(p_z)$. This may happen if $\omega > \min\{\omega_0(p_z)\}$. Since $\omega_0(p_z)$ is an increasing function of $|p_z|$, the friction parameter $d_O$ has a Van Hove singularity at the edge of the absorption band $p_z = 0$ [18,19]: $d_O \to \infty$. Simultaneously, $\beta_O^{(\mathrm{loc})} \to \infty$, both $d_O$ and $\beta_O^{(\mathrm{loc})}$ being real. The quantities $\alpha_O$ and $\sigma_O^{(\mathrm{qp})}$ remain finite. The parameter $\gamma_O$ for delocalized states with $|\epsilon| > \Delta_\infty$ becomes $\gamma_O = i\omega_c/\omega g_0$. As a result, the last term in Eq. (17) gives an imaginary contribution which is much smaller than the first two terms. Equation (12) results in $\sigma_O^{(\mathrm{qp})}(\omega) = iN_n e^2/m\omega$; hence, the total conductivity becomes imaginary $\sigma_\pm(\omega) = iNe^2/m\omega$. The response displays an *antiresonance*: the vortex dissipation vanishes because vortex motion freezes when the friction becomes infinitely large as noticed in [9,17,19].

In conclusion, we have developed a microscopic theory for the ac response of the mixed state of type II superconductors. We derived the ac conductivity in a pinning-free sample and discussed cyclotron resonance and resonant friction effects.

We are grateful to G. Blatter, V. Geshkenbein, and G. Volovik for illuminating discussions. This research is supported by the U.S. DOE, Office of Science, under Contract No. W-31-109-ENG-38, and by the Russian Foundation for Basic Research Grant No. 99-02-16043.

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