Neutrino Mass, Muon Anomalous Magnetic Moment, and Lepton Flavor Nonconservation

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(Received 26 February 2001; published 15 June 2001)

If the generating mechanism for neutrino mass is to account for both the newly observed muon anomalous magnetic moment as well as the present experimental bounds on lepton flavor nonconservation, then the neutrino mass matrix should be almost degenerate and the underlying physics should be observable at future colliders. We illustrate this assertion with two specific examples, and show that $\Gamma(\mu \to e\gamma)/m_\mu^5$, $\Gamma(\tau \to e\gamma)/m_\tau^5$, and $\Gamma(\tau \to \mu\gamma)/m_\tau^5$ are in the ratio $(\Delta m^2)^2_{\rm sol}/2$, $(\Delta m^2)^2_{\rm sol}/2$, and $(\Delta m^2)^2_{\rm atm}$, respectively, where the Δm^2 parameters are those of solar and atmospheric neutrino oscillations and bimaximal mixing has been assumed.

DOI: 10.1103/PhysRevLett.87.011802 PACS numbers: 13.40.Em, 14.60.Ef, 14.60.Pq

Any mechanism for generating a mass matrix for the three neutrinos ν_e, ν_μ, and ν_τ will have side effects [1], among which are lepton flavor violating processes such as $\mu \to e\gamma$, $\tau \to \mu\gamma$, and μ-e conversion in nuclei, as well as an extra contribution to the muon anomalous magnetic moment [2]. If the scale of this new physics is very high, as in the simplest models of neutrino mass [3,4], then these side effects are suppressed by the high scale and are totally negligible phenomenologically. However, if this scale is of order 1 TeV or less, as in two recent proposals [5,6], then the exciting possibility exists for all of these effects to be visible in present and future laboratory experiments.

In view of the newly announced measurement [2] of the muon anomalous magnetic moment,

$$a_\mu^{\rm exp} = \frac{g_\mu - 2}{2} = 116592020(160) \times 10^{-11}, \tag{1}$$

which differs from the standard-model (SM) prediction [7] by 2.6σ,

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (426 \pm 165) \times 10^{-11}, \tag{2}$$

a relatively large positive new contribution to a_μ is needed, hinting thus at possible new physics just above the electroweak scale. One may be tempted to believe that it is due to some new physics which has not appeared anywhere else before. On the other hand, a much better established hint of new physics already exists, i.e., neutrino mass from neutrino oscillations, so it is important to ask the question: *Are they related?*

In this paper we assume that the generating mechanism for neutrino mass is responsible for at least a significant part of the deviation shown in Eq. (2). We show that unless the neutrino mass matrix is almost degenerate, i.e., with three nearly equal mass eigenvalues, the a_μ measurement is in conflict with the $\tau \to \mu\gamma$ rate. This is because of the nearly maximal $\nu_\mu - \nu_\tau$ mixing for atmospheric neutrino oscillations [8], as explained below. We study two examples, one of which will be shown to be completely consistent with all other flavor-nonconserving processes as well. We predict the relative decay rates of $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ in terms of neutrino oscillation data, and show that these processes constrain the common neutrino mass scale and the solar neutrino oscillation solution in a very interesting range. In addition, the underlying new physics should be observable at future collider experiments.

Consider the following mass eigenstates of the three active neutrinos:

$$\nu_1 = \cos\theta\,\nu_e - \frac{\sin\theta}{\sqrt{2}}(\nu_\mu - \nu_\tau), \tag{3}$$

$$\nu_2 = \sin\theta\,\nu_e + \frac{\cos\theta}{\sqrt{2}}(\nu_\mu - \nu_\tau), \tag{4}$$

$$\nu_3 = \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau), \tag{5}$$

with masses $m_1 \le m_2 \le m_3$. This choice is dictated by the present knowledge of neutrino data regarding atmospheric [8] and solar [9] neutrino oscillations. Specifically, $\nu_\mu - \nu_\tau$ mixing is assumed to be maximal to explain the atmospheric data (we comment on the effect of small allowed deviations from this assumption later), and ν_e mixes with the other two neutrinos with angle θ to account for the solar data. The 3×3 Majorana neutrino mass matrix in the $(\nu_e, \nu_\mu, \nu_\tau)$ basis is then given by

$$\mathcal{M}_\nu = \begin{bmatrix} {\rm c}^2 m_1 + {\rm s}^2 m_2 & {\rm sc}(m_2 - m_1)/\sqrt{2} & {\rm sc}(m_1 - m_2)/\sqrt{2} \\ {\rm sc}(m_2 - m_1)/\sqrt{2} & ({\rm s}^2 m_1 + {\rm c}^2 m_2 + m_3)/2 & (-{\rm s}^2 m_1 - {\rm c}^2 m_2 + m_3)/2 \\ {\rm sc}(m_1 - m_2)/\sqrt{2} & (-{\rm s}^2 m_1 - {\rm c}^2 m_2 + m_3)/2 & ({\rm s}^2 m_1 + {\rm c}^2 m_2 + m_3)/2 \end{bmatrix}, \tag{6}$$

where ${\rm s} \equiv \sin\theta$ and ${\rm c} \equiv \cos\theta$. For $\theta = \pi/4$, it is known as bimaximal mixing.

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In the Higgs triplet model [5] with $\xi \sim (3,1)$ under the standard $SU(2)_L \times U(1)_Y$ gauge group, we have the interaction

$$f_{ij}[\xi^0 \nu_i \nu_j + \xi^+(\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j] + \text{H.c.} \tag{7}$$

which gives $(\mathcal{M}_\nu)_{ij} = 2 f_{ij}\langle \xi^0 \rangle$, and establishes a one-to-one correspondence between the neutrino mass matrix and the interaction terms. The smallness of $\mathcal{M}_\nu$ follows from the smallness of $\langle \xi^0 \rangle$ [5], while the couplings f_{ij} can be large and the triplet mass m_ξ can be the order of the electroweak scale. Therefore, it follows from Eq. (7) that the muon $g - 2$ contribution is proportional to $f_{\mu e}^2 + f_{\mu\mu}^2 + f_{\mu\tau}^2$, whereas the $\tau \to \mu\gamma$ amplitude is proportional to $f_{\tau e} f_{e\mu} + f_{\tau\mu} f_{\mu\mu} + f_{\tau\tau} f_{\tau\mu}$. The former is proportional to $(m_3^2 + \mathrm{c}^2 m_2^2 + \mathrm{s}^2 m_1^2)/2$ and the latter to $(m_3^2 - \mathrm{c}^2 m_2^2 - \mathrm{s}^2 m_1^2)/2$. This means that a suppression of the $\tau \to \mu\gamma$ rate (relative to the muon $g - 2$) is possible only if $m_1 \simeq m_2 \simeq m_3$, i.e., a nearly degenerate neutrino mass matrix.

In the leptonic Higgs doublet model [6], $\mathcal{M}_\nu$ comes from the terms

$$\tfrac{1}{2} M_i N_{iR}^2 + h_{ij} \bar{N}_{iR}(\nu_j \eta^0 - l_{jL} \eta^+) + \text{H.c.}, \tag{8}$$

where $\eta \sim (2, 1/2)$ and carries the lepton number $L = -1$, while the singlet fermions N_R have $L = 0$. We assume now that all the heavy N_R's are equal in mass. Hence Eqs. (3)–(6) imply

$$h_{ij} = \begin{bmatrix} 2\mathrm{c}h_1 & -\sqrt{2}\,\mathrm{s}h_1 & \sqrt{2}\,\mathrm{s}h_1 \\ 2\mathrm{s}h_2 & \sqrt{2}\,\mathrm{c}h_2 & -\sqrt{2}\,\mathrm{c}h_2 \\ 0 & \sqrt{2}\,h_3 & \sqrt{2}\,h_3 \end{bmatrix}, \tag{9}$$

with $m_i = 4h_i^2 \langle \eta^0 \rangle^2 / M$. Again, m_i is small because $\langle \eta^0 \rangle$ is small [6], thus allowing h_i to be large and M to be the order of the electroweak scale. In this case, the muon $g - 2$ contribution is proportional to $(m_3 + \mathrm{c}^2 m_2 + \mathrm{s}^2 m_1)/2$ and the $\tau \to \mu\gamma$ amplitude to $(m_3 - \mathrm{c}^2 m_2 - \mathrm{s}^2 m_1)/2$, again suppressing the latter relative to the former in the limit of degenerate neutrino masses.

In both of the above models, there are large contributions to Δa_μ as well as $l_i \to l_j \gamma$ coming from the interactions of Eqs. (7) and (8), as shown in Fig. 1. In the triplet model,

$$\Delta a_\mu = \sum_l \frac{10}{3} \frac{f_{\mu l}^2}{(4\pi)^2} \frac{m_\mu^2}{m_\xi^2}. \tag{10}$$

In the limit of a degenerate neutrino mass matrix, i.e., $m_1 = m_2 = m_3 = 2f\langle \xi^0 \rangle$, this implies

$$m_\xi < 1174\sqrt{\alpha_f}\ \text{GeV}, \tag{11}$$

where $\alpha_f = f^2/4\pi$ and the 90% confidence-level limit $\Delta a_\mu > 215 \times 10^{-11}$ has been used [7]. In the doublet model,

$$\Delta a_\mu = \sum_i \frac{h_{i\mu}^2}{(4\pi)^2} \frac{m_\mu^2}{m_\eta^2} F_2(s_{N_i}), \tag{12}$$

where $s_{N_i} \equiv m_{N_i}^2 / m_\eta^2$ and

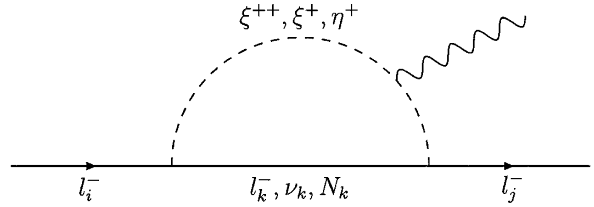

FIG. 1. Diagrams giving rise to Δa_μ and $l_i \to l_j \gamma$. The photon can be attached to any charged line.

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1 - x)^4}. \tag{13}$$

Assuming $s_{N_i} = 1$ [which gives $F_2(1) = 1/12$] and using Eq. (9) with all h's equal, we then obtain

$$m_\eta < 371\sqrt{\alpha_h}\ \text{GeV}, \tag{14}$$

where $\alpha_h = h^2/4\pi$. Comparison of Eq. (11) and Eq. (14) implies that masses below 1 TeV are expected in either model.

The $l_i \to l_j \gamma$ rate divided by the $l_i \to l_j \nu_i \bar{\nu}_j$ rate is given by

$$R(l_i \to l_j \gamma) = \frac{96\pi^3 \alpha}{G_F^2 m_{l_i}^4} (|f_{M1}|^2 + |f_{E1}|^2), \tag{15}$$

where $\alpha \simeq 1/137$ and G_F is the Fermi constant. In the doublet model, the magnetic and electric dipole moment form factors are given by

$$f_{M1} = f_{E1} = \sum_k \frac{h_{kl_i} h_{kl_j}}{4(4\pi)^2} \frac{m_{l_i}^2}{m_\eta^2} F_2(1). \tag{16}$$

For $\tau \to \mu\gamma$,

$$\sum_k h_{k\tau} h_{k\mu} = 2(h_3^2 - \mathrm{c}^2 h_2^2 - \mathrm{s}^2 h_1^2)$$

$$\simeq 2(h_3^2 - h_2^2) \simeq \frac{h^2}{m_\nu^2} (\Delta m^2)_{\text{atm}}, \tag{17}$$

where m_ν is the common mass of the three neutrinos. Hence the $\tau \to \mu\gamma$ branching fraction is given by

$$B(\tau \to \mu\gamma) = B(\tau \to \mu\nu\bar{\nu}) \frac{\pi\alpha}{192 G_F^2} \left(\frac{\alpha_h}{m_\eta^2}\right)^2 \frac{(\Delta m^2)_{\text{atm}}^2}{m_\nu^4}. \tag{18}$$

Suppose we do not have neutrino mass degeneracy, but rather a hierarchical neutrino mass matrix, then $(\Delta m^2)_{\text{atm}}/m_\nu^2$ would be equal to 1, and, by using Eq. (14), we would obtain $B(\tau \to \mu\gamma) > 8.0 \times 10^{-6}$, well above the experimental upper limit of 1.1×10^{-6}. Note that this result, while presented for a specific model, is actually very general. If $\nu_3 = \mathrm{c}\nu_\mu + \mathrm{s}\nu_\tau$, there would be a suppression factor of $\mathrm{s}^2/\mathrm{c}^2$, but this is not available because atmospheric neutrino data require nearly maximal $\nu_\mu - \nu_\tau$ mixing.

Similarly, the $\mu \to e\gamma$ and $\tau \to e\gamma$ branching fractions are given by

$$B(\mu \to e\gamma) = \frac{\pi\alpha}{192G_F^2}\left(\frac{\alpha_h}{m_\eta^2}\right)^2 [2s^2c^2]\frac{(\Delta m^2)^2_{\rm sol}}{m_\nu^4}, \tag{19}$$

$$B(\tau \to e\gamma) = B(\tau \to e\nu\bar{\nu})B(\mu \to e\gamma). \tag{20}$$

Hence we have the interesting relationship

$$\frac{\Gamma(\mu \to e\gamma)}{m_\mu^5}:\frac{\Gamma(\tau \to e\gamma)}{m_\tau^5}:\frac{\Gamma(\tau \to \mu\gamma)}{m_\tau^5} = 2s^2c^2(\Delta m^2)^2_{\rm sol}:2s^2c^2(\Delta m^2)^2_{\rm sol}:(\Delta m^2)^2_{\rm atm}. \tag{21}$$

The μ-e conversion ratio $R_{\mu e}$ in nuclei is given by

$$R_{\mu e} = \frac{8\alpha^5 m_\mu^5 Z_{\rm eff}^4 Z|\overline{F_p}(p_e)|^2}{\Gamma_{\rm capt} q^4}[|f_{E0} + f_{M1}|^2 + |f_{E1} + f_{M0}|^2], \tag{22}$$

where $q^2 \simeq -m_\mu^2$ and, for ^{13}Al, $Z_{\rm eff} = 11.62$, $\overline{F_p} = 0.66$, and $\Gamma_{\rm capt} = 7.1 \times 10^5\ {\rm s}^{-1}$ [10,11]. The charge-radius form factors are given by

$$f_{E0} = -f_{M0} = \sum_i \frac{h_{i\mu}h_{ie}}{2(4\pi)^2}\frac{m_\mu^2}{m_\eta^2}F_1(s_{N_i}), \tag{23}$$

where

$$F_1(x) = \frac{2 - 9x + 18x^2 - 11x^3 + 6x^3\ln x}{36(1 - x)^4}, \tag{24}$$

with $F_1(1) = 1/24$. In Fig. 2, using

$$(\Delta m^2)_{\rm atm} = 3 \times 10^{-3}\ {\rm eV}^2, \tag{25}$$

and assuming the large-angle matter-enhanced solution of solar neutrino oscillations with

$$(\Delta m^2)_{\rm sol} = 3 \times 10^{-5}\ {\rm eV}^2, \tag{26}$$

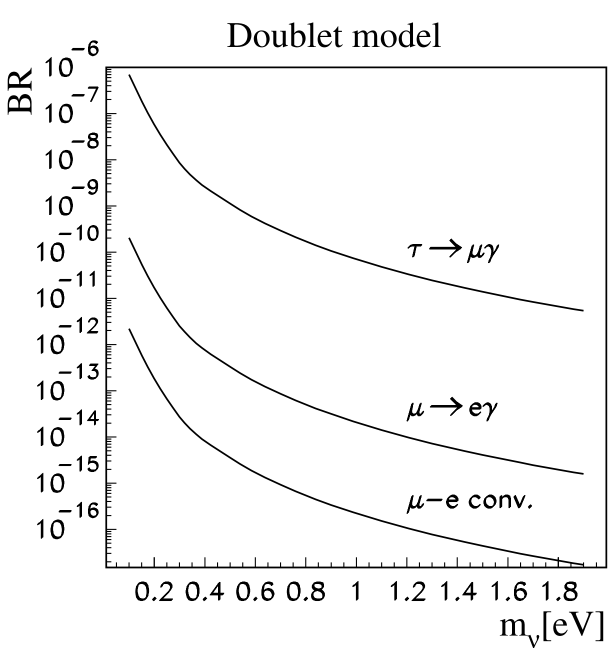

FIG. 2. Lower bounds on $B(\tau \to \mu\gamma)$, $B(\mu \to e\gamma)$, and $R_{\mu e}$ from the measurement of a_μ in the leptonic Higgs doublet model, assuming bimaximal mixing of degenerate neutrinos.

we plot $B(\tau \to \mu\gamma)$, $B(\mu \to e\gamma)$, and $R_{\mu e}$ as functions of m_ν for $s^2 = c^2 = 1/2$ and $\alpha_h/m_\eta^2 = (371\ {\rm GeV})^{-2}$. Hence these lines should be considered as *lower* bounds in the case of bimaximal mixing for neutrino oscillations.

We note that, at $m_\nu = 0.2$ eV, $B(\mu \to e\gamma)$ is at its present upper limit [12] of 1.2×10^{-11}. If $m_\nu > 0.2$ eV is desired, then the constraint from the nonobservation of neutrinoless double beta decay [13] requires the m_{ee} element of Eq. (6) to be less than 0.2 eV. This is easily achieved by making $m_1 < 0$ but keeping $m_{2,3} > 0$, without affecting any of our results presented so far. However, we *must* then choose the large-angle mixing solution of solar neutrino oscillations, implying the observation of $\mu \to e\gamma$ and μ-e conversion in the planned experiments with the sensitivities down to 2×10^{-14} [14] and 2×10^{-17} [15], respectively. From Fig. 2 we see that an order-of-magnitude improvement of the present $\tau \to \mu\gamma$ bound will also test this specific prediction. Thus $B(\tau \to \mu\gamma)$, neutrinoless double beta decay, $B(\mu \to e\gamma)$, and μ-e conversion are all complementary to one another in probing the connection between m_ν and Δa_μ.

However, the neutrino mixings need not be exactly bimaximal. Indeed, the mixing element $|V_{e3}|$ is constrained to be small but may still be nonzero. Obviously the rate $B(\tau \to \mu\gamma)$ is completely independent of this parameter and our conclusion that neutrinos must be degenerate in mass to explain the observed Δa_μ remains unchanged. However, $B(\mu \to e\gamma)$ and $R_{\mu e}$ receive additional contributions proportional to $|V_{e3}|^2(\Delta m^2)^2_{\rm atm}$ [16]. For example, if $|V_{e3}| \sim 0.1$, one needs $m_\nu \sim 1$ eV to satisfy the present experimental bounds. Therefore, no fine-tuning in the parameters of Fig. 2 is needed to comply with data if $|V_{e3}| \neq 0$. Nevertheless, the planned $\mu \to e\gamma$ experiments offer sensitive probes of the small mixing angle $|V_{e3}|$ in this scenario.

In the triplet model, the relevant form factors are calculated in Ref. [10]. We again have the relationship given by Eq. (21), but the corresponding $R_{\mu e}$ is not suppressed as in the doublet model. The reason is that the form factors $f_{E0,M0}$ are now functions of $m_{l_i}^2/m_\xi^2$ which are different for different charged leptons, unlike $m_{N_i}^2/m_\eta^2$ which are the same for all N's. As a result, $R_{\mu e}$ is of order 10^{-6} independent of m_ν, which is definitely ruled out by experiment. In addition, the $\mu \to eee$ branching fraction [which occurs at tree level, but is suppressed by $(\Delta m^2)^2_{\rm sol}$] also exceeds the present experimental bound for $m_\nu < 2.7$ eV if $s^2 = c^2 = 1/2$, again assuming the large-angle matter-enhanced solution of solar neutrino oscillations. Thus the triplet model cannot explain Δa_μ even if neutrino masses are degenerate. It is, of course, still perfectly viable as a model of neutrino masses [5], but it will have no significant contribution to the muon $g - 2$.

Since the $g - 2$ announcement [2], there have been many papers [17] dealing with its possible explanation. Ours is the only one relating it to another *existing* hint of new physics, i.e., neutrino mass from neutrino oscillations. A glance at Fig. 2 shows that $m_\nu = 0.2$ eV is a very interesting number. It is the present upper limit of a Majorana neutrino mass from neutrinoless double beta decay; it also corresponds to the present upper limits of $B(\mu \to e\gamma)$ and μ-e conversion in nuclei. Planned experiments on all three fronts are in progress and will test our proposed connection between m_ν and Δa_μ. They will also probe the possibly nonzero neutrino mixing angle V_{e3}. In addition, the $\tau \to \mu\gamma$ branching fraction is just an order of magnitude away, and Eq. (14) implies that the leptonic Higgs doublet (η^+, η^0) as well as the fermion singlets N_{iR} are not far away from being discovered in future colliders, as proposed in Ref. [6]. A neutrino mass of 0.2 eV is also very relevant in cosmology [18] and astrophysics [19].

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-94ER40837 and Estonian ETF Grant No. 3832.

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