# Giant Enhancement of the Thermal Hall Conductivity $\kappa_{xy}$ in the Superconductor $YBa_2Cu_3O_7$

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In high-purity $YBa_2Cu_3O_7$, the (weak-field) thermal Hall conductivity $\kappa_{xy}$ is observed to increase a thousand-fold between 90 and 30 K. The inferred quasiparticle lifetime $\tau$ increases a hundred-fold starting below 90 K, in disagreement with a recent photoemission experiment. We show that $\kappa_{xy}$ exhibits a specific scaling behavior below ~30 *K*. This scaling may bear on the issue of whether Landau quantization of the quasiparticle states occurs.

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The problem of excitations of the superconducting condensate in the cuprates at low temperatures is of strong current interest. In a *d*-wave superconductor, the energy-momentum dispersion of quasiparticles near a node is Dirac-like. The effect of an intense magnetic field on the quasiparticle (qp) states is an interesting open question [1–6]. Landau quantization of the qp states, first proposed by Gor'kov and Schrieffer [1], has been recently rederived using different arguments [2–4]. However, the case against Landau-level formation has also been argued [5,6].

A second problem is the temperature dependence of the qp mean-free path $\ell$ (in zero field) close to $T_c$. Transport evidence from thermal conductivity [7], microwave and teraHertz experiments [8–10], and thermal Hall conductivity [11–13] point to a sharp increase in the qp lifetime just below $T_c$. Recent high-resolution angle-resolved photoemission (ARPES) experiments [14,15] have started to address the lifetime issue as well, but with conflicting results (see below).

These issues reflect the strong interest in the low-lying excitations of the *d*-wave superconductor. While microwave absorption and ARPES experiments provide valuable information on the quasiparticles, they are less effective in a field. For in-field experiments, teraHertz techniques [10] and the thermal Hall effect [11–13], in particular, have emerged as powerful probes of qp transport. In a field, the qp heat current develops a transverse component that is observed as a thermal Hall conductivity $\kappa_{xy}$ (by contrast, phonons do not display a Hall effect since they are charge neutral). Hence, $\kappa_{xy}$ *selectively* senses the qp current alone [11]. To fully exploit this technique at low temperatures, however, samples with a very long $\ell$ are needed.

A recent innovation is the growth, using $BaZrO_3$ (BZO) crucibles, of crystals of $YBa_2Cu_3O_y$ (YBCO) with nearly perfect crystalline order (from x-ray rocking curves [16]) and very low impurity concentration. The stepwise improvement in crystal quality results in strong enhancements of the qp lifetime $\tau$. The weak field $\kappa_{xy}$ undergoes a remarkable thousand-fold increase between $T_c$ and 30 K. Below 30 K, the curves of $\kappa_{xy}$ vs $H$ provide new, specific information on scaling behavior at low $T$ [17]. Both features are directly relevant to the two issues mentioned above.

In BZO-grown YBCO, the anomaly in the longitudinal thermal conductivity $\kappa_{xx}$ $(-\nabla T \parallel \mathbf{a})$ is enhanced by ~80% over that in typical, non-BZO detwinned crystals (Fig. 1 inset). To isolate the qp current, we turn to $\kappa_{xy}$. The main panel of Fig. 1 displays traces of $\kappa_{xy}$ vs field $B$ from 85 to 40 K [18]. As in earlier studies [11,13], the initial slope $\kappa_{xy}^0/B \equiv \lim_{B\to 0} \kappa_{xy}/B$ increases very rapidly as the temperature $T$ falls below $T_c$. Further, the curves are strongly nonlinear in $H$. Both features reflect a $\tau$ that increases rapidly with decreasing $T$. Compared to earlier

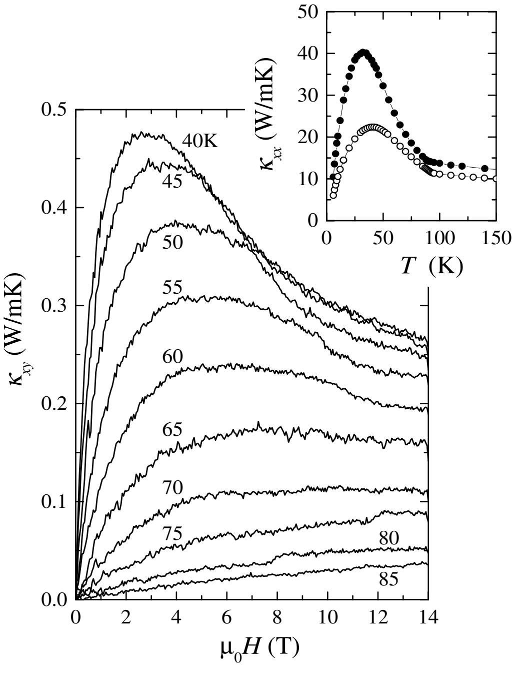

FIG. 1. (main panel) The thermal Hall conductivity $\kappa_{xy}$ vs $H$ in BZO-grown $YBa_2Cu_3O_{6.99}$ ($T_c = 89$ K) at temperature from 85 to 40 K. As $T$ decreases below $T_c$, the initial slope $\kappa_{xy}^0/B$ increases sharply. The prominent peak in $\kappa_{xy}$ below 55 K is a new feature in BZO-grown YBCO. The inset compares the zero-field $\kappa_{xx} \equiv \kappa_a$ in the BZO-grown crystal (solid circles) with a detwinned non-BZO grown crystal (open circles).

crystals, the hysteresis in $\kappa_{xy}$ is greatly reduced [18]. As $T$ falls below 40 K (see Fig. 2), the peak continues to narrow. For later reference, we note that, over a broad range of temperatures ($10 < T < 70$ K), $H_{\max}$ varies as $T^2$. Moreover, at low temperatures ($T < 28$ K), the peak magnitude $\kappa_{xy}^{\max}$ also scales as $T^2$.

The initial slope $\kappa_{xy}^0/B$, plotted as solid circles in Fig. 3, undergoes a thousand-fold increase between $T_c$ and 30 K (the $T$-linear variation of $\kappa_{xy}$ above $T_c$ is displayed as open circles [19]). We now show that this giant enhancement is driven by a hundred-fold increase in the qp lifetime.

To extract the zero-field mean-free path (mfp) $\ell$ from $\kappa_{xy}^0/B$, we apply the Boltzmann-equation approach [20], which should be valid in the *weak*-field regime $\omega_c\tau \ll 1$ ($\omega_c$ is the cyclotron frequency). In terms of the "qp heat capacity" $c_e = T^{-1}\sum_{\mathbf{k}}(-\partial f/\partial E_{\mathbf{k}})E_{\mathbf{k}}^2$, where $E_{\mathbf{k}}$ is the qp energy, the zero-$H$ thermal conductivity may be written as $\kappa_e = c_e\langle v\ell\rangle/2$, with the group velocity $\mathbf{v_k} = \nabla E_{\mathbf{k}}/\hbar$. (Close to a node $\mathbf{k}^*$, the qp energy may be approximated as $E_{\mathbf{q}} = \hbar\sqrt{(v_f q_1)^2 + (v_\Delta q_2)^2}$, where $v_f$ and $v_\Delta$ are velocity parameters normal and parallel to the Fermi surface (FS), and $\mathbf{q} = \mathbf{k} - \mathbf{k}^*$.)

The thermal Hall conductivity is related to $\kappa_e$ by $\kappa_{xy} = \kappa_e \tan\theta$. We assume that, in the weak-field limit, the thermal Hall angle $\tan\theta$ is proportional to $\omega_c\tau$, viz.

$$\tan\theta = \eta\omega_c\tau = \eta\ell/k_F\ell_B^2, \qquad (B \to 0) \tag{1}$$

where the magnetic length $\ell_B = \sqrt{\hbar/eB}$. The parameter $\eta$ is less than 1 if $\ell$ is anisotropic around the FS. To obtain $\tan\theta$ [13], we first fit the profile of $\kappa_{xx}$ vs $H$ to the empirical expression $\kappa_{xx}(B,T) = \kappa_e^0(T)/[1 + p|B|^\mu] + \kappa_{bg}(T)$, where the background term $\kappa_{bg}(T)$ is $H$ independent and identified with the phonon contribution. The initial Hall angle is then obtained as [13] $\tan\theta = \lim_{B\to 0}\kappa_{xy}(B)/[\kappa_{xx}(B) - \kappa_{bg}]$. This procedure allows us to extract $\tan\theta$ [hence, $\ell$ using Eq. (1)].

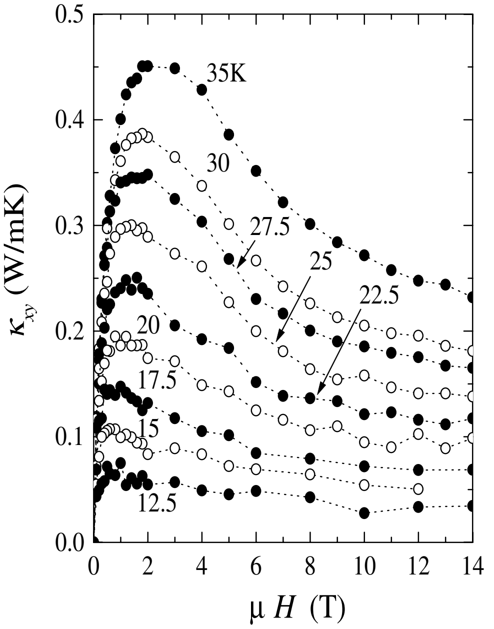

FIG. 2. The thermal Hall conductivity $\kappa_{xy}$ vs $H$ in BZO-grown $YBa_2Cu_3O_{6.99}$ between 35 and 12.5 K. Below 28 K, the peak value varies as $T^2$ (see text).

As a consistency check, we adopt a second way to obtain $\ell$ from $\kappa_{xy}^0$ that relies on measurements of the electronic heat capacity $c_e$. Using Eq. (1), we may write

$$\kappa_{xy}^0 = \frac{c_e v_f \ell^2 \eta}{4k_f\ell_B^2}. \tag{2}$$

In a $d$-wave superconductor, $c_e = \alpha_c T^2$ for $T < T_c$. Using the measured value $\alpha_c \simeq 0.064$ mJK$^{-3}$ mol$^{-1}$ [21], we may invert Eq. (2) to find $\ell$. We find that the values of $\ell$ obtained from the two methods share the *same* $T$ dependence, but differ by a fixed factor of 1.5 if $\eta = 1$. By adjusting $\eta$ to 0.6, we obtain numerical agreement between the two methods.

Figure 4 shows the $T$ dependence of $\ell$ derived from the two methods. The agreement between the two sets of data is evidence that our assumption Eq. (1) is physically reasonable. Remarkably, between $T_c$ and 20 K, the mfp increases by a factor of ~120 from 80 A to 1 $\mu$m. In the expanded scale, we show that this increase is abrupt, starting slightly below $T_c$. [For comparison, $\tan\theta$

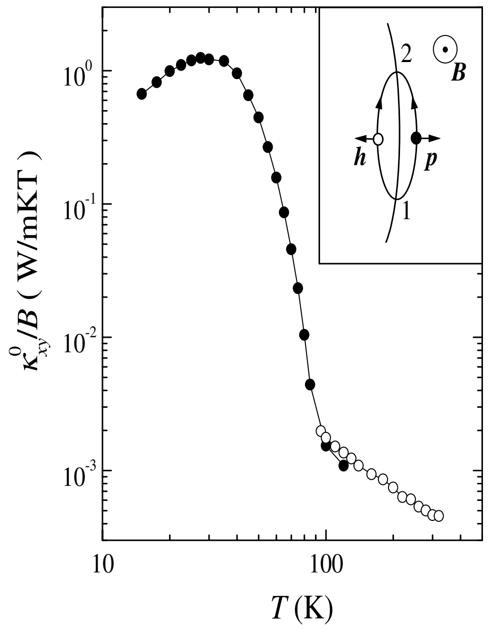

FIG. 3. The $T$ dependence of the initial Hall slope $\kappa_{xy}^0/B$ in BZO-grown YBCO (solid circles). Between $T_c$ and 30 K, $\kappa_{xy}^0/B$ increases by $10^3$. The $1/T$ dependence of $\kappa_{xy}^0/B$ above $T_c$ (measured in a non-BZO grown YCBO) is shown as open circles. The inset shows a qp energy contour on the Dirac cone. Group velocities on the particlelike ($p$) and holelike ($h$) branches are indicated.

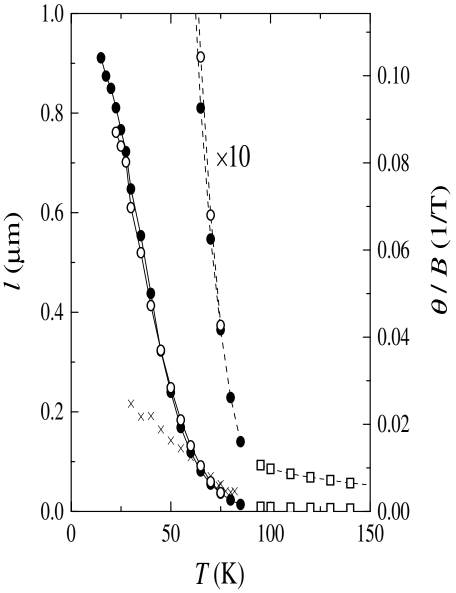

FIG. 4. The zero-field mean-free path $\ell$ extracted from the weak-field Hall angle tan$\theta$ (open circles), and from Eq. (2) (closed circles). The *equivalent* values of $\theta/B$ are shown on the right scale. The symbols (×) represent tan$\theta$ measured in a non-BZO detwinned YBCO crystal (Krishana *et al.* [13]). The expanded scale (dashed lines) highlights the steep increase below $T_c$. To extract $\ell$, we used the values $\eta = 0.60$, $v_f = 1.78 \times 10^7$ cm/s, and $k_f = 0.8A^{-1}$.

measured previously in a non-BZO crystal [13] is shown as ×. Based on the higher sensitivity and broader range in $T$ in the present experiment, we now conclude that tan$\theta$ does *not* lie on the extrapolated curve for the electrical Hall angle tan$\theta_e$.]

Beyond the weak-field regime, we need a microscopic calculation of the qp thermal Hall current to properly analyze $\kappa_{xy}$ vs $H$. As the theoretical situation is unsettled, we adopt instead scaling arguments [17]. This approach reveals some rather striking features in the data.

For states close to the node $\mathbf{k}^*$, the linear energy dispersion $E = \hbar \bar{v} q$ ($\bar{v}$ is an average velocity) implies a general relation between $k_B T$ and the magnetic length $\ell_B$ at a characteristic field scale $B_s(T)$, viz.

$$k_B T = \hbar \bar{v} \sqrt{\frac{e B_s(T)}{\hbar}}\,. \tag{3}$$

In addition to this general relation, Simon and Lee [17] have proposed that, at low $T$ (<30 K for YBCO), the magnitude of $\kappa_{xy}$ should scale as

$$\kappa_{xy}(H,T) \sim T^2 F_{xy}(\sqrt{H}/\alpha T)\,, \tag{4}$$

where $\alpha \equiv k_B/\bar{v}\sqrt{e\hbar}$, and $F_{xy}(u)$ is a scaling function of the dimensionless parameter $u = \sqrt{H}/\alpha T$. Hence, plots of $\kappa_{xy}/T^2$ versus $\sqrt{H}/T$ should collapse to the universal curve $F_{xy}(u)$.

We proceed to plot our results in this way in Fig. 5. While the curves above 28 K are spread out, the ones below collapse onto a common curve for $H < H_{\max}$. The data taken at 25 K (and below) collectively determine the form of $F_{xy}(u)$. Its most notable feature is the nominally straight segment that extends from $u \simeq 0$ to just below $u_0 \equiv \sqrt{H_{\max}}/\alpha T$, i.e., $F_{xy}(u) \sim u$ for $0 < u < u_0$.

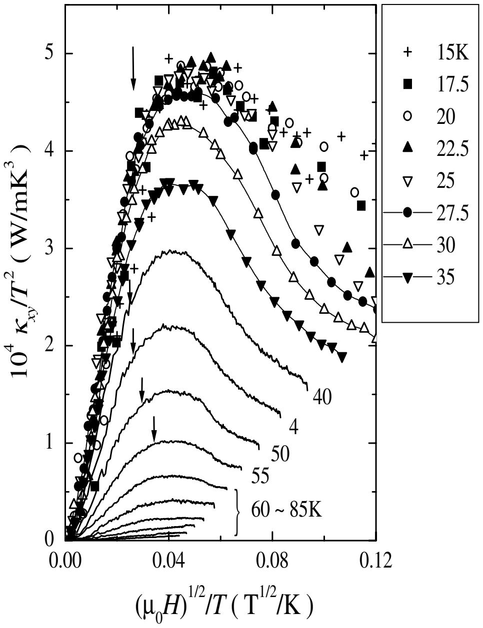

FIG. 5. Simon-Lee scaling plot of $\kappa_{xy}/T^2$ versus $\sqrt{H}/T$ [Eq. (4)]. Below 28 K, the curves collapse onto a "universal" curve $F_{xy}(u)$. Above 28 K, scaling is violated. However, the peaks still occur at the same $x$ coordinate ($\sqrt{H_{\max}}/T = 0.042$). The arrows indicate the field scale $H_{\rm arc}$.

This simple form for $F_{xy}$ implies that, below 25 K and for $H < H_{\max}$, $\kappa_{xy}$ reduces to the form

$$\kappa_{xy}(H,T) = C_0 T \sqrt{H}\,, \tag{5}$$

where the constant $C_0 = 1.51 \times 10^{-2}$ in SI units. Remarkably, when Eq. (5) applies, the magnitude of $\kappa_{xy}$ is just proportional to $T\sqrt{H}$ and is insensitive to all transport quantities such as $\ell$ and $\theta$. This interesting result has not been anticipated theoretically.

At larger values of $u$, $F_{xy}$ attains a maximum value $F_{xy}^0$ before falling slowly. The $T^2$ dependence of the peak value $\kappa_{xy}^{\max}$ noted earlier in Fig. 2 is now seen to be a simple consequence of scaling behavior (i.e., $\kappa_{xy}^{\max} \sim T^2 F_{xy}^0$).

Above 28 K, Simon-Lee scaling no longer holds. Three field regimes are now apparent. In weak fields ($0 < H < H_x$), $\kappa_{xy}$ is strictly linear in $H$. Above $H_x$, we enter a regime reminiscent of the $\sqrt{H}$ behavior at low $T$ (the $H$-linear regime is too small to resolve below 28 K). This intermediate regime appears as straight-line segments in Fig. 5. Finally, closer to $H_{\max}$, $\kappa_{xy}$ deviates from $\sqrt{H}$ behavior and goes through a broad maximum. Surprisingly, as noted earlier, the weaker scaling relation in Eq. (3) continues to hold: Between 15 and 70 K, the maximum in $\kappa_{xy}$ occurs at the *same* $x$ coordinate in Fig. 5, i.e.,

$\sqrt{H_{\max}} = 0.042T$. Substituting $H_{\max}$ for $B_s$ in Eq. (3), we find that $\bar{v} \sim 8.0 \times 10^6$ cm/s, which is close to the geometric-mean velocity $\sqrt{v_f v_\Delta} \sim 6.8 \times 10^6$ cm/s (with $v_f = 1.78 \times 10^7$ cm/s [14] and $v_f/v_\Delta \sim 7$).

Semiclassically, the time for a qp to move from 1 to 2 along the arc is $\Delta t = (\hbar/eH)\int_1^2 ds_{\mathbf{k}}\,|\mathbf{v_k}|^{-1}$ (Fig. 3 inset). For this time to equal $\tau$, the field required is $H_{\rm arc} = \pi E/(e v_\Delta v_f \tau)$. Using the measured $\ell \simeq v_f\tau$ at each $T$ and setting $E = k_B T$, we indicate $H_{\rm arc}$ as arrows in Fig. 5. This rough estimate shows that the peak is related to the maximum arclength of the dominant energy contour on the Dirac cone. Hence, a detailed analysis of the Hall results should shed important light on the current debate about how vortices affect the qp spectrum. The presence of Landau levels [1–4] or absence [5,6] will presumably have a large effect on $\kappa_{xy}$. Moreover, the direct measurement of the scaling function $F_{xy}$ (Fig. 5) together with the other scaling features uncovered should stringently narrow the range of possibilities in this interesting problem.

The new results on $\kappa_{xy}$ also bear on the issue of the change in qp lifetime at $T_c$. As discussed, $\ell$ derived from transport undergoes a steep increase just below $T_c$ [7–9,11]. Recently, ARPES has attained enough resolution to probe the qp spectral peak along the nodal direction in $\mathrm{Bi_2Sr_2CaCu_2O_8}$. Valla *et al.* [14] find that the width $\Delta k$ ($\sim 1/\ell_{\rm ARPES}$) retains its $T$-linear dependence across $T_c$ (near $T_c$ $\ell_{\rm ARPES} \simeq 25$–$30$ Å). This appears to be in striking contrast with the transport results. However, Kaminski *et al.* [15] resolve a new feature of the qp peak that appears below $T_c$. They infer that well-defined qp states at the nodes exist only below $T_c$. The steep increase in $\ell$ shown in Fig. 4 is in agreement with Kaminsky *et al.* The data in Fig. 4 show that $\ell$ increases to $\simeq 1$ $\mu$m below 20 K (implying a peak 200 times narrower than the peaks resolved in the current ARPES studies). Hence, in high-purity YBCO, there are exceedingly sharp qp peaks in the spectral function that remain to be resolved and investigated. Understanding the abrupt appearance of the qp state below $T_c$, as implied by the steep increase in $\ell$ and $\kappa_{xy}^0/B$ near $T_c$, seems a key problem in the cuprates.

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[17] Steve H. Simon and Patrick A. Lee, Phys. Rev. Lett. **78**, 1548 (1997). The $\kappa_{xy}$ data shown in this reference (from K. Krishana and N. P. Ong) were taken on twinned 90-K (YSZ-grown) YBCO. Below 40 K, the signal levels are about 20 times weaker than here and must be regarded as preliminary. For example, the zero crossing at 20 K is not reproduced in crystals with longer $\ell$.

[18] With the thermal gradient $-\nabla T \parallel \mathbf{a} \parallel \mathbf{x}$ and the field $\mathbf{H} \parallel \mathbf{c} \parallel \mathbf{z}$, the thermal Hall current is $\parallel \mathbf{y}$. Above 35 K, the "Hall" gradient $-\partial_y T$ is measured continuously with a chromel-constantan thermocouple as $H$ is swept slowly (0.2 T/min). Below 35 K (Fig. 2), we adopt a high-resolution mode. Each point is taken with $H$ stabilized. The signal is sampled with $-\partial_x T$ turned on, then off. Each Hall trace is the average of two traces, from $-14$ to 14 T and from 14 to $-14$ T. Because of the relatively weak pinning in BZO-grown YBCO, the hysteresis in $\kappa_{xy}$ between the two traces is very small above 40 K. Below 40 K, it remains small below 4 T (the scaling analysis is applied only to $H - T$ values where the hysteresis in $\kappa_{xy}$ is under 10%).

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