Coexistence of Superconductivity and Ferromagnetism in Ferromagnetic Metals

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We address the question of coexistence of superconductivity and ferromagnetism. Using a field theoretical approach we study a one-fermion effective model of a ferromagnetic superconductor in which the quasiparticles responsible for the ferromagnetism form the Cooper pairs as well. For the first time we solve self-consistently the mean-field equations for the superconducting gap and the spontaneous magnetization. We discuss the physical features which are different in this model and the standard BCS model and consider their experimental consequences.

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Recently an itinerant ferromagnet undergoing a superconducting transition was discovered in the heavy fermion compound UGe_2 [1,2] and the experimental studies have revealed that the ferromagnetic state exists even below the superconducting transition. This prompts the interesting question of the possible many-body itinerant systems supporting both types of broken symmetry.

The search for ferromagnetic superconductors goes back to the 1960s when superconducting materials with magnetic impurities were studied [3]. The research in this direction has led to the works of Larkin and Ovchinnikov [4] and Fulde and Ferrell [5], who studied a simple model of effective field theory of superconducting fermions coupled to magnetic impurities and they described the phase diagram of such a system.

In the recently discovered superconducting ferromagnet UGe_2, the electrons responsible for the ferrromagnetic order are the same as those which participate in the Cooper pair formation. Motivated by this we study a single spin-$\frac{1}{2}$ fermion model. In this model the long range ferromagnetic order is a consequence of a spontaneously broken spin rotation symmetry, as opposed to the case of a metal with ferromagnetic impurities. For the first time we solve the self-consistent equations for the superconducting gap and the magnetization simultaneously in the mean-field approximation. We find that the state in which ferromagnetism and superconductivity coexist has (i) a linear temperature dependence of the specific heat and (ii) the inverse static susceptibility vanishes at finite magnetization as opposed to the case of normal ferromagnetic metals.

Our model Hamiltonian is

$$\begin{aligned} H - \mu N = &\int d^3r\, c_\sigma^\dagger(\vec r)\Big(-\frac{1}{2m^*}\vec\nabla^2 - \mu\Big)c_\sigma(\vec r) \\ &- \frac{J}{2}\int d^3r\, \vec S(\vec r)\cdot\vec S(\vec r) \\ &- g\int d^3r\, c_\uparrow^\dagger(\vec r)c_\downarrow^\dagger(\vec r)c_\downarrow(\vec r)c_\uparrow(\vec r), \end{aligned} \qquad (1)$$

where $c_\sigma(\vec r)$ are the spin σ fermion fields, $\vec S = \frac{1}{2}c_\sigma^\dagger\vec\tau_{\sigma\sigma'}c_{\sigma'}$ is the spin field, τ_i are the Pauli matrices, and μ is the chemical potential. The exchange interaction is ferromagnetic and the four-fermion interaction is attractive.

The partition function of the model can be written as a functional integral over the Grassmann fields $c(\tau,\vec r)$ and $\bar c(\tau,\vec r)$ [6]. We introduce a real vector field $\vec M(\tau,\vec r)$ using the Hubbard-Stratonovich transformation of the exchange term and a complex scalar field $f(\tau,\vec r)$ using a Hubbard-Stratonovich transformation of the second term in Eq. (1). The vector field describes the fluctuations of the magnetization, while the complex scalar field describes the superconducting fluctuations. Performing the Gaussian integral over the fermionic fields we obtain the partition function of the model as an integral over $\vec M$, f, and $\bar f$, which we calculate using the steepest descent around the mean-field solutions $\vec M = (0,0,M)$ and $\Delta = g\langle f\rangle$. Here $M = -\langle S^z\rangle$ defines the magnetization of the system. The mean-field equations are

$$JM + \frac{\delta F_{\text{eff}}}{\delta M} = 0, \qquad \frac{2|\Delta|}{g} + \frac{\delta F_{\text{eff}}}{\delta|\Delta|} = 0, \qquad (2)$$

where F_{eff} is the free energy of a theory with the effective Hamiltonian

$$H_{\text{eff}} = \sum_{\vec p}[\epsilon_p^\uparrow c_{\vec p\uparrow}^\dagger c_{\vec p\uparrow} + \epsilon_p^\downarrow c_{\vec p\downarrow}^\dagger c_{\vec p\downarrow} + \bar\Delta c_{-\vec p\downarrow}c_{\vec p\uparrow} + \text{H.c.}], \qquad (3)$$

$$\epsilon_p^\uparrow = \frac{p^2}{2m^*} - \mu + \frac{JM}{2}, \qquad \epsilon_p^\downarrow = \frac{p^2}{2m^*} - \mu - \frac{JM}{2}. \qquad (4)$$

Here the fermionic effective Hamiltonian H_{eff} is obtained after the Hubbard-Stratonovich transformations and setting the fields at their mean-field values.

Next we diagonalize the effective Hamiltonian using a Bogoliubov transformation. After the transformation the new dispersion relations are

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$$E_p^\alpha = \frac{JM}{2} + \sqrt{\epsilon_p^2 + |\Delta|^2}, \quad E_p^\beta = \frac{JM}{2} - \sqrt{\epsilon_p^2 + |\Delta|^2}, \tag{5}$$

where $\epsilon_p = \frac{p^2}{2m^*} - \mu$. Then the mean-field equations take the form

$$M = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}(1 - n_p^\alpha - n_p^\beta), \tag{6}$$

$$|\Delta| = \frac{|\Delta|g}{2}\int \frac{d^3p}{(2\pi)^3}\frac{n_p^\beta - n_p^\alpha}{\sqrt{\epsilon_p^2 + |\Delta|^2}}, \tag{7}$$

where n_p^α and n_p^β are the momentum distribution function of the Bogoliubov fermions.

From Eq. (5) one sees that for $M \geq 0$ (the convention that we use here) $E_p^\alpha > 0$ for all momenta p and therefore for $T = 0$, $n_p^\alpha = 0$. For E_p^β there are two possibilities. When $JM < 2|\Delta|$, $E_p^\beta < 0$ for all p and therefore $n_p^\beta = 1$ for all p. Substitution of this in Eq. (5) leads to $M = 0$. Therefore the only solution of the mean-field equations which allows for the coexistence of ferromagnetism and superconductivity is in the case when $JM > 2|\Delta|$ which we will assume. Then the equation $E_p^\beta = 0$ has two solutions:

$$p_F^\pm = \sqrt{2m^*\mu \pm m^*\sqrt{(JM)^2 - 4|\Delta|^2}}. \tag{8}$$

The dispersion of the β fermion is positive when $p_F^- < p < p_F^+$ and is negative in the complementary interval. With this in mind Eqs. (5) and (6) at $T = 0$ have the form

$$M = \frac{1}{12\pi^2}[(p_F^+)^3 - (p_F^-)^3], \tag{9}$$

$$|\Delta| = \frac{g|\Delta|}{(2\pi)^2}\left(\int_0^\infty dp\,\frac{p^2}{\sqrt{\epsilon_p^2 + |\Delta|^2}} - \int_{p_F^-}^{p_F^+} dp\,\frac{p^2}{\sqrt{\epsilon_p^2 + |\Delta|^2}}\right). \tag{10}$$

It is difficult to solve analytically these equations; however, when JM is greater, but close to 2Δ, p_F^+ is approximately equal to p_F^- and therefore M is small as follows from Eq. (8). In this case one can expand the right-hand side (rhs) of Eq. (7) in the small parameter $\sqrt{(JM)^2 - 4|\Delta|^2}$ obtaining

$$p_F^\pm = p_F \pm \frac{m^*}{2p_F}\sqrt{(JM)^2 - 4|\Delta|^2}, \tag{11}$$

where $p_F = \sqrt{2\mu m^*}$. Substitution of these expressions in Eq. (8) shows that in this approximation the magnetization is linear in $|\Delta|$, namely,

$$M = \frac{2}{J}\frac{r}{\sqrt{r^2 - 1}}|\Delta|, \tag{12}$$

where $r = Jm^*p_F/4\pi^2$ and this expression is valid for large r (i.e., $MJ - 2|\Delta| \to 0^+$).

As in the standard BCS theory of superconductivity, the pairing of the quasiparticles occurs in the vicinity of p_F, which must include the interval between p_F^- and p_F^+. Then the integration in the first integral on the rhs of Eq. (9) is limited to a shell of width 2Λ, i.e.,

$$\frac{1}{g} = \frac{1}{(2\pi)^2}\left(\int_{p_F-\Lambda}^{p_F+\Lambda} dp\,\frac{p^2}{\sqrt{\epsilon_p^2 + |\Delta|^2}} - \int_{p_F^-}^{p_F^+} dp\,\frac{p^2}{\sqrt{\epsilon_p^2 + |\Delta|^2}}\right). \tag{13}$$

Here we have assumed that $|\Delta| \neq 0$, $p_F + \Lambda > p_F^+$, and $p_F - \Lambda < p_F^-$.

Substitution of the approximate expressions for $p_F^\pm$ from Eq. (11) in the second term on the rhs of Eq. (13) leads to

$$\frac{1}{g} = \frac{m^*p_F}{2\pi^2}\int_{\sqrt{(\frac{JM}{2})^2 - |\Delta|^2}}^{\Lambda}\frac{d\epsilon}{\sqrt{\epsilon^2 + |\Delta|^2}}. \tag{14}$$

Performing the integration in the above expression we obtain

$$\frac{1}{g} = \frac{m^*p_F}{2\pi^2}\ln\frac{2\Lambda}{\frac{MJ}{2} + \sqrt{(\frac{MJ}{2})^2 - |\Delta|^2}}. \tag{15}$$

Substitution of M from Eq. (12) leads to the expression for the gap:

$$\frac{1}{g} = \frac{m^*p_F}{2\pi^2}\ln\frac{2\Lambda}{|\Delta|}\sqrt{\frac{r-1}{r+1}}. \tag{16}$$

In that approximation the solution is

$$|\Delta| = \sqrt{\frac{r-1}{r+1}}\,\Lambda e^{-\frac{2\pi^2}{gm^*p_F}}, \tag{17}$$

$$M = \frac{2}{J}\frac{r}{r+1}\Lambda e^{-\frac{2\pi^2}{gm^*p_F}}. \tag{18}$$

Along with the above nonzero solution, describing the coexistence of ferromagnetism and superconductivity, there is a solution with a vanishing gap describing a normal ferromagnetic state. For the transition from the normal ferromagnetic state to the superconducting ferromagnetic state to take place the energy of the former state must be lower than the energy of the latter state. One can calculate the difference between the two energies using the standard integral representation [7], namely,

$$\Omega_{sf} - \Omega_{nf} = \int_0^g d\bar{g}\,\bar{g}^{-1}\langle H_{\rm BCS}\rangle, \tag{19}$$

where $H_{\rm BCS}$ is the BCS part of the Hamiltonian in Eq. (1). Using Eq. (16) we transform the integral from an integral over $\bar{g}$ to an integral over the gap and performing the integration we obtain

$$\Omega_{sf} - \Omega_{nf} = -\frac{m^*p_F}{4\pi^2}|\Delta|^2. \tag{20}$$

This shows that the superconducting ferromagnetic state has lower energy than the normal ferromagnetic state and therefore will be realized at low enough temperature.

In the case of magnetic impurities interacting with conduction electrons (RKKY interaction) one considers the magnetization as an external parameter, independent of the superconducting gap. In that case [8] the normal ferromagnetic state has lower energy than the superconducting ferromagnetic state and ferromagnetism and superconductivity do not exist. In the case of ferromagnetism which results from a spontaneously broken spin rotation symmetry the magnetization and the gap are related through the system of Eqs. (7) and (8) [see also Eq. (13)] and this leads to ouř new result; i.e., the superconducting ferromagnetic state will appear at low temperature.

When the magnetization increases the domain of integration in the second integral on the rhs of Eq. (12) can exceed the size of the domain around p_F in which the pairing occurs and which is the integration domain in the first integral of the same equation. In that case the second integral dominates and this leads to the absence of solutions with a finite gap. Taking the limiting case when the two integration domains are equal, i.e., $p_F + \Lambda = p_F^+$ and $p_F - \Lambda = p_F^-$, where $p_F^{\pm}$ are the values of the momenta from Eq. (10) with $\Delta = 0$, we obtain the critical value of the magnetization

$$M_c = \frac{\Lambda}{m^* J}(2p_F + \Lambda), \tag{21}$$

above which the superconductivity disappears even for an attractive four-fermion interaction.

Next we calculate the distribution functions $n_p^{\uparrow}$ and $n_p^{\downarrow}$ of the spin-up and spin-down quasiparticles. In terms of the distribution functions of the Bogoliubov fermions these momentum distribution functions are

$$\begin{aligned} n_p^{\uparrow} &= u_p^2 n_p^{\alpha} + v_p^2 n_p^{\beta}, \\ n_p^{\downarrow} &= u_p^2(1 - n_p^{\beta}) + v_p^2(1 - n_p^{\alpha}), \end{aligned} \tag{22}$$

where u_p^2 and v_p^2 are the coefficients in the Bogoliubov transformation. They have the same form as in the BCS theory.

At zero temperature n_p^{α} is zero and $n_p^{\beta} = \theta(p_F^- - p) + \theta(p - p_F^+)$. Then the spin-up and spin-down quasiparticles have the following momentum distribution functions:

$$n_p^{\uparrow} = v_p^2[\theta(p_F^- - p) + \theta(p - p_F^+)], \tag{23}$$

$$\begin{aligned} n_p^{\downarrow} = {} & \theta(p_F^+ - p) - \theta(p_F^- - p) \\ & + v_p^2[\theta(p_F^- - p) + \theta(p - p_F^+)]. \end{aligned} \tag{24}$$

The functions are depicted in Fig. 1.

The appearance of the Fermi surfaces of the Bogoliubov fermion β is unexpected in the superconducting phase, but it is a necessary condition for the existence of itinerant ferromagnetism. Therefore in the case of coexistence of superconductivity and ferromagnetism caused by the same

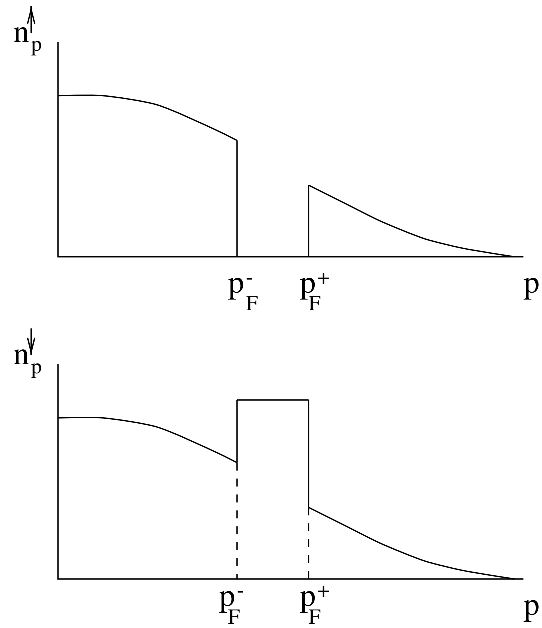

FIG. 1. The zero temperature momentum distribution functions for spin-up and spin-down fermion.

quasiparticles the existence of the two Fermi surfaces is a generic property of this state. These Fermi surfaces are reflected in the spin-up and spin-down momentum distribution functions as well as in the anomalous Green's functions. It is easy to show that the anomalous Green's function,

$$\mathcal{F}(\tau - \tau', \vec{p}) = -\langle T c_{\downarrow}(\tau, -\vec{p}) c_{\uparrow}(\tau', \vec{p}) \rangle, \tag{25}$$

in the case $\tau = \tau'$ is

$$\mathcal{F}(0, \vec{p}) = \frac{|\Delta|}{2\sqrt{\epsilon_p^2 + |\Delta|^2}} \tag{26}$$

when $0 < p < p_F^-$ and $p > p_F^+$ and is zero when the momentum p is between the two Fermi surfaces p_F^- and p_F^+.

The existence of the Fermi surfaces leads to different thermodynamic properties of the system, compared to the standard BCS theory. The specific heat has a linear temperature dependence at low temperatures as opposed to the exponential decrease of the specific heat in the BCS theory:

$$C = \frac{2\pi^2}{3} N(0) T. \tag{27}$$

Here

$$N(0) = \frac{m^*}{4\pi^2} \frac{p_F^+ + p_F^-}{\sqrt{1 - \frac{4|\Delta|^2}{J^2 M^2}}} = N^+(0) + N^-(0) \tag{28}$$

is the sum of the density of states on the two Fermi surfaces of the Bogoliubov fermion β. When the magnetization is small, from Eqs. (10) and (11) follow that the density of states increases with r as $N(0) \to \frac{m^*}{2\pi^2} r p_F$, as opposed to the case of ordinary weak ferromagnets, where the density of states is $N(0) = \frac{m^*}{2\pi^2} p_F$ in this limit. Hence, the specific heat is large even at very low temperatures. In the case of a superconductor in an external magnetic field [3] although there are gapless fermionic excitations the specific heat is not linear as opposed to our case. This can also be contrasted with some of the unconventional superconductors which have power law dependence of the specific heat on the temperature, depending on the nodal structure of the gap function.

Another consequence of the existence of the Fermi surfaces is the existence of paramagnons which describes the longitudinal spin fluctuations [9]. They exist in ferromagnetic normal metals and in our theory they survive even in the ferromagnetic superconducting phase. Their propagator is given by

$$D_l(\omega, p) = \frac{1}{\delta + a\frac{|\omega|}{p} + bp^2}, \tag{29}$$

where a, b, and δ are constants. The constant

$$a = \frac{J\pi}{4}\left(1 - \frac{4|\Delta|^2}{J^2M^2}\right)^{-1/2}\left(\frac{N^+(0)}{v_F^+} + \frac{N^-(0)}{v_F^-}\right) \tag{30}$$

defines the analytical properties of the paramagnon and is different from zero because of the existence of the Fermi surfaces. The constant δ is

$$\delta = 1 - \frac{J}{2}N(0) \tag{31}$$

and b is a positive constant. As we mentioned earlier, the density of states, Eq. (22), increases as the magnetization M decreases and therefore, at a small, but finite value of the magnetization, $M = M_0$, the inverse of the static susceptibility, δ, becomes zero. This is a quite different behavior from the one in weak ferromagnetic metals where δ becomes zero at zero magnetization. This observation is important, because in contrast to the spin waves and superconducting fluctuations which are a consequence of the spontaneously broken symmetry, the paramagnon is not a consequence of the broken symmetry but depends on the properties of the metal under consideration. In the case of the coexistence of the superconductivity and ferromagnetism the superconductivity prevents the magnetization from becoming arbitrarily small, because when the magnetization is smaller than the critical value M_0, δ is negative and the paramagnon fluctuations lead to an instability of that phase. In the superconducting phase, with zero magnetization (BCS-like regime) the spin fluctuations of the paramagnon type are absent.

In this paper we considered the possibility of the coexistence of ferromagnetism and superconductivity and the physical features of such a system. We arrived at a system of self-consistent equations for the magnetization and the superconducting gap and solved analytically these equations at small magnetizations. This is the first time a mean-field theory was found with coexisting ferromagnetism and superconductivity. The solutions with coexistence of superconductivity and ferromagnetism describe Bogoliubov fermions one of which has two Fermi surfaces. Therefore the spin-up and spin-down quasiparticles have two Fermi surfaces each. The thermodynamic properties of the coexistence phase are different from the standard BCS theory. The specific heat has a linear temperature dependence as in normal ferromagnetic metals, but increases anomalously at small magnetizations. These results are obtained in a mean-field approximation, but they are generic for the coexistence state and can be used as a starting point for calculations beyond mean field. In our model the quantum critical point is dressed; i.e., the superconducting state occurs at zero magnetization, because the superconducting gap is generated not by the spin fluctuations, but by some other means. This is to be contrasted with the theory of spin fluctuation mediated pairing in weak ferromagnetic metals [10] where the quantum critical point is naked and the superconducting ferromagnetic critical temperatures go to zero at the quantum critical point.

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