

## Vector Manifestation of Chiral Symmetry

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We propose “vector manifestation” (VM) of the Wigner realization of chiral symmetry in which the symmetry is restored at the critical point by the massless degenerate pion (and its flavor partners) and the  $\rho$  meson (and its flavor partners) as the chiral partner, in sharp contrast to the traditional manifestation à la the linear sigma model where the symmetry is restored by the degenerate pion and the scalar meson. The application to the chiral phase transition of large  $N_f$  QCD is performed using the hidden local symmetry Lagrangian. Combined with the Wilsonian matching proposed recently, VM determines the critical number of massless flavors  $N_f \approx 5$  without much ambiguity.

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Chiral phase transition in QCD is discussed in various contexts such as the large  $N_f$  QCD, QCD at finite temperature and/or density, etc. In many situations the traditional linear sigma model-like Wigner realization for the chiral restoration is assumed. However, the Wigner realization does not necessarily require the massless degenerate pion and scalar meson at the critical point. The linear sigma model is merely consistent with the Wigner realization. It, therefore, is natural for us to ask the following question: Is there a manifestation of the Wigner realization other than that of the linear sigma model? The answer is yes, which we demonstrate in this paper.

In this paper we propose “vector manifestation” (VM) of the chiral symmetry as a novel manifestation of the Wigner realization in which the vector meson denoted by  $\rho$  ( $\rho$  meson and its flavor partner) becomes massless at the chiral phase transition point. Accordingly, the (longitudinal)  $\rho$  becomes the chiral partner of the Nambu-Goldstone (NG) boson denoted by  $\pi$  (pion and its flavor partners).

The essence of VM stems from the new matching of the effective field theory (EFT) with QCD (“Wilsonian matching”) recently proposed by Ref. [1] in which bare parameters of the EFT are determined by matching the current correlators in the EFT with those obtained by the operator product expansion (OPE) in QCD, based on the renormalization-group equation (RGE) in the Wilsonian sense including the quadratic divergence [2]. The quadratic divergence was identified with the presence of a pole of ultraviolet origin at  $n = 2$  in the dimensional regularization. Several physical quantities for  $\pi$  and  $\rho$  were predicted by the Wilsonian matching in the framework of the hidden local symmetry (HLS) [3,4] as the EFT, in excellent agreement with the experiments for  $N_f = 3$ , where  $N_f$  is the number of massless flavors [1]. This encourages us to perform the analysis for larger  $N_f$  up to near the critical point based on the Wilsonian matching.

Actually, the chiral symmetry restoration in Wigner realization should be characterized by the equality of the vector and axialvector current correlators. When we approach to the critical point from the broken phase (NG phase), the axialvector current correlator is still dominated by the

massless  $\pi$  as the NG boson, while the vector current correlator is by the massive  $\rho$ . The crucial ingredient of the Wilsonian matching is the quadratic divergence which yields the quadratic running of (square of) the decay constant  $F_\pi^2(\mu)$  [2], where  $\mu$  is the renormalization point. It was actually shown [2] that the order parameter  $F_\pi(0)$  can become zero for larger  $N_f$  even when  $F_\pi(\Lambda) \neq 0$ , where  $F_\pi(\Lambda)$  is not the order parameter but just a parameter of the bare Lagrangian defined at the cutoff  $\Lambda$  where the matching with QCD is made. Then the  $\pi$  contribution to the axialvector current correlator at  $\mu \neq 0$  persists,  $F_\pi(\mu) \neq 0$ , even at the critical point where  $F_\pi(0) = 0$ . Thus the only possibility for this equality to hold at any  $\mu \neq 0$  is that the  $\rho$  contribution to the vector current correlator also persists at the critical point in such a way that  $\rho$  yields a massless pole with the current coupling equal to that of  $\pi$ . Then this restoration, VM, is accompanied by the degenerate massless  $\pi$  and (longitudinal)  $\rho$  (transverse  $\rho$  is decoupled from the current correlator at the critical point, see later discussions).

This is sharply contrasted with the traditional manifestation of the linear sigma model where the equality of the current correlators is trivially satisfied, since the axialvector correlator goes to zero due to  $F_\pi(\mu) \equiv 0$  independently of  $\mu$  (in the absence of the quadratic divergence), while the vector correlator has no contribution from the scalar meson and hence is simply zero. Thus the Wilsonian matching [which leads to  $F_\pi(\Lambda) \neq 0$ ] excludes the linear sigma model manifestation in QCD.

In VM we have degenerate massless  $\pi$  and (longitudinal)  $\rho$  at the phase transition point, which are the chiral partners in the representation of  $(N_f^2 - 1, 1) \oplus (1, N_f^2 - 1)$  of the chiral  $SU(N_f)_L \times SU(N_f)_R$ . This representation corresponds to  $(8, 1) \oplus (1, 8)$  for  $N_f = 3$ . This is contrasted with the linear sigma model-like manifestation in which  $\pi$  is in the representation of pure  $(3, 3^*) \oplus (3^*, 3)$  together with the scalar meson. This can be understood in the good-old-days saturation scheme of the Adler-Weisberger sum rule for the zero helicity mesons [5]:  $\pi$  and the (longitudinal) axialvector mesons denoted by  $A_1$  ( $a_1$  meson and its flavor partners) are an admixture of

$(8, 1) \oplus (1, 8)$  and  $(3, 3^*) \oplus (3^*, 3)$ , since the symmetry is spontaneously broken:

$$\begin{aligned} |\pi\rangle &= |(3, 3^*) \oplus (3^*, 3)\rangle \sin\psi + |(8, 1) \oplus (1, 8)\rangle \cos\psi, \\ |A_1\rangle &= |(3, 3^*) \oplus (3^*, 3)\rangle \cos\psi - |(8, 1) \oplus (1, 8)\rangle \sin\psi, \end{aligned} \quad (1)$$

where the experimental value of the mixing angle  $\psi$  is given by approximately  $\psi = \pi/4$  [5]. On the other hand,

$$\begin{aligned} \Pi_A^{(\text{QCD})}(Q^2) &= \frac{1}{8\pi^2} \left[ -\left(1 + \frac{\alpha_s}{\pi}\right) \ln \frac{Q^2}{\mu^2} + \frac{\pi^2}{3} \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle}{Q^4} + \frac{\pi^3}{3} \frac{1408}{27} \frac{\alpha_s \langle \bar{q}q \rangle^2}{Q^6} \right], \\ \Pi_V^{(\text{QCD})}(Q^2) &= \frac{1}{8\pi^2} \left[ -\left(1 + \frac{\alpha_s}{\pi}\right) \ln \frac{Q^2}{\mu^2} + \frac{\pi^2}{3} \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle}{Q^4} - \frac{\pi^3}{3} \frac{896}{27} \frac{\alpha_s \langle \bar{q}q \rangle^2}{Q^6} \right], \end{aligned} \quad (2)$$

where  $\mu$  is the renormalization scale of QCD,  $Q$  the Euclidean momentum carried by the current, and we neglected  $\mathcal{O}(1/Q^8)$  terms. These expressions are valid in high energy where the QCD coupling  $\alpha_s$  is small.

Next we consider the expression of the current correlators in the EFT which is valid in the low energy below the matching scale  $\Lambda$ . As an EFT to describe the VM we need a model having both  $\pi$  and  $\rho$  fields. Here we use the HLS model [3,4] which includes  $\pi$  and  $\rho$  consistently with the chiral symmetry and actually reproduces experiments nicely through the Wilsonian matching [1]. The axialvector and vector current correlators in the HLS are well described by the tree contributions with including  $\mathcal{O}(p^4)$  terms when the momentum is around the matching scale  $\Lambda$  [1]:

$$\begin{aligned} \Pi_A^{(\text{HLS})}(Q^2) &= \frac{F_\pi^2(\Lambda)}{Q^2} - 2z_2(\Lambda), \\ \Pi_V^{(\text{HLS})}(Q^2) &= \frac{F_\sigma^2(\Lambda)[1 - 2g^2(\Lambda)z_3(\Lambda)]}{M_v^2(\Lambda) + Q^2} - 2z_1(\Lambda), \end{aligned} \quad (3)$$

where  $g(\Lambda)$  is the bare HLS gauge coupling,  $F_\sigma^2(\Lambda) = a(\Lambda)F_\pi^2(\Lambda)$  is the bare decay constant of the would-be NG boson  $\sigma$  (not to be confused with the scalar meson in the linear sigma model) absorbed into the HLS gauge boson, and  $M_v^2(\Lambda) \equiv g^2(\Lambda)F_\sigma^2(\Lambda)$  is the bare HLS gauge boson mass. In Ref. [1] these correlators are matched with those in Eq. (2) up to the second derivative in terms of  $Q^2$  for  $Q^2 = \Lambda^2$ . The resultant Wilsonian matching condition relevant to the present analysis is given by [1]:

$$\begin{aligned} \frac{F_\pi^2(\Lambda)}{\Lambda^2} &= \frac{1}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle}{\Lambda^4} \right. \\ &\quad \left. + \pi^3 \frac{1408}{27} \frac{\alpha_s \langle \bar{q}q \rangle^2}{\Lambda^6} \right]. \end{aligned} \quad (4)$$

Let us now obtain constraints on the bare parameters of the HLS in the VM through the Wilsonian matching. At

the longitudinal  $\rho$  belongs to  $(8, 1) \oplus (1, 8)$  and the scalar meson to  $(3, 3^*) \oplus (3^*, 3)$ . Then the conventional linear sigma model-like manifestation corresponds to the limit  $\psi \rightarrow \pi/2$ , while the VM to the limit  $\psi \rightarrow 0$  in which case  $A_1$  goes to a pure  $(3, 3^*) \oplus (3^*, 3)$ , now degenerate with the scalar meson in the same representation  $(3, 3^*) \oplus (3^*, 3)$ , but not with  $\rho$  in  $(8, 1) \oplus (1, 8)$ .

Now we formulate the VM more explicitly. Let us write the axialvector and vector current correlators evaluated by the OPE in QCD [6]:

the critical point the quark condensate  $\langle \bar{q}q \rangle$  vanishes, while the gluonic condensate  $\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle$  is independent of the renormalization point of QCD and hence it is expected that it does not vanish. Then the right-hand side (RHS) of Eq. (4) is nonzero, implying that  $F_\pi^2(\Lambda)$  is nonzero even at the critical point.

Then how do we know by the bare parameters defined at  $\Lambda$  whether or not the chiral symmetry is restored? As we discussed before, a clue comes from the fact that  $\Pi_A^{(\text{QCD})}$  and  $\Pi_V^{(\text{QCD})}$  in Eq. (2) agree with each other for any value of  $Q^2$  when the chiral symmetry is restored with  $\langle \bar{q}q \rangle = 0$ . Thus, we require that  $\Pi_A^{(\text{HLS})}$  and  $\Pi_V^{(\text{HLS})}$  in Eq. (3) agree with each other for any value of  $Q^2$ . This agreement is satisfied only if the following conditions are met:

$$\begin{aligned} g(\Lambda) &\rightarrow 0, & a(\Lambda) &\rightarrow 1, \\ z_1(\Lambda) - z_2(\Lambda) &\rightarrow 0. \end{aligned} \quad (5)$$

This is nothing but the VM of the chiral symmetry in terms of the HLS parameters. Note that  $a(\Lambda) \simeq 1$  is satisfied in QCD already for  $N_f = 3$  in the broken phase [1]. The first two in Eq. (5) are the values in the Georgi's vector limit [7], which was simply assumed in Ref. [2] to be a consistent way to incorporate the chiral phase transition of the large  $N_f$  QCD into the HLS. Thanks to the Wilsonian matching it is now clear that Eq. (5) is the precise HLS expression of the Wigner realization in QCD.

The VM in the HLS is similar to the Georgi's "vector realization" [7], but is different in an essential way: The "vector realization" is claimed to be a different realization than either the Wigner or NG realizations in such a way that the NG boson does exist [ $F_\pi(0) \neq 0$ ] while the chiral symmetry is still unbroken. On the contrary, our VM is precisely the Wigner realization having  $F_\pi(0) = 0$ . Technically, the bare HLS Lagrangian in the VM coincides with the parameter choice of the Georgi's vector realization;  $g(\Lambda) = 0$ ,  $a(\Lambda) = 1$ , and  $F_\pi(\Lambda) \neq 0$ . However, an essential difference comes from the Wilsonian RGE's

whose quadratic divergence leads to the Wigner realization with  $F_\pi(0) = 0$  at the low-energy limit (on-shell of NG bosons). On the other hand, the vector realization lacking the quadratic divergence leads to  $F_\pi(0) = F_\pi(\Lambda) \neq 0$ . In contrast to the Georgi's vector realization, the VM in the Wigner realization is consistent with the chiral Ward-Takahashi identity [8,9].

We now examine the chiral symmetry restoration in the large  $N_f$  QCD ( $N_f < \frac{11}{2}N_c$ ) which was implied by the fact that the coupling at the infrared fixed point becomes very small [10]. Such a restoration was indeed observed by various methods like lattice simulation [11], ladder Schwinger-Dyson (SD) equation [12,13], dispersion relation [14], instanton calculus [15], etc. As pointed out in Ref. [13] the phase transition for large  $N_f$  QCD may be characterized by the "conformal phase transition." In such a case the Ginzburg-Landau effective theory (linear sigma model-like manifestation) simply breaks down. The VM may be a manifestation of the chiral symmetry restoration consistent with the conformal phase transition.

The chiral restoration in terms of HLS was obtained in Ref. [2] without VM and Wilsonian matching. What was shown in Ref. [2] is that the RGE for  $F_\pi^2$  including the quadratic divergence reduces the value of  $F_\pi^2(0)$  (the pole residue of the massless pion pole) from  $F_\pi^2(\Lambda)$  (the bare parameter of the HLS Lagrangian defined at a cutoff scale  $\Lambda$ ) in such a way that the larger  $N_f$ , the smaller the value of  $F_\pi(0)$  is. It eventually goes to zero, the chiral restoration, at a certain critical number of  $N_f$ .

Now in the VM, the bare parameters are characterized by the vector limit  $g(\Lambda) = 0$  and  $a(\Lambda) = 1$  [see Eq. (5)] which is actually the fixed point of RGE [2]. Then the VM justifies the previous derivation of the RGE for  $F_\pi$  in the vector limit which relates the order parameter with the bare parameter as [2]:

$$\frac{F_\pi^2(m_\rho = 0; N_f)}{\Lambda_f^2} = \frac{F_\pi^2(\Lambda_f; N_f)}{\Lambda_f^2} - \frac{N_f}{2(4\pi)^2}, \quad (6)$$

where we expressed the matching scale by  $\Lambda_f \equiv \Lambda(N_f)$  since it generally depends on  $N_f$ . It should be noticed that this equation holds only at the critical flavor  $N_f^{\text{cr}}$ , and the left-hand side vanishes at the critical point.

The value of  $N_f^{\text{cr}}$  is determined in terms of the parameters in the OPE by combining Eqs. (4) and (6) with taking  $\langle \bar{q}q \rangle = 0$ . The resultant expression is given by

$$N_f^{\text{cr}} = 4 \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle}{\Lambda_f^4} \right]. \quad (7)$$

Here we estimate this by using the parameters for  $N_f = 3$  [1]:  $(\Lambda_3, \Lambda_{\text{QCD}}) = (1.2, 0.35)$  GeV,  $\alpha_s = 0.56$  and  $\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle = 0.012$  GeV<sup>4</sup>. The result is given by

$$N_f^{\text{cr}} \simeq 4.9. \quad (8)$$

The precise estimation of this will be done by determining the  $N_f$  dependences of the QCD coupling  $\alpha_s$  and  $\Lambda_f$

in the forthcoming paper [9]. Here we just quote the result  $N_f^{\text{cr}} \simeq 4.8$ , which is consistent with the above estimate.

To study the critical behaviors of the parameters when approaching to the critical point, we need to know how the bare parameters  $g(\Lambda_f; N_f)$  and  $a(\Lambda_f; N_f)$  approach to the vector limit Eq. (5). Comparing the difference between vector and axialvector correlators in Eq. (2) with that in Eq. (3), we know that the critical behavior of  $g^2(\Lambda_f; N_f)$  is given as  $g^2(\Lambda_f; N_f) \sim \langle \bar{q}q \rangle^2$ . Since we do not know the scaling of  $\langle \bar{q}q \rangle$  except for the ladder SD approach [16], we here tentatively adopt the following ansatz on the behavior of the HLS gauge coupling approaching to zero:

$$g^2(\Lambda_f; N_f) = \bar{g}^2 \epsilon^q, \quad \epsilon \equiv 1/N_f - 1/N_f^{\text{cr}}, \quad (9)$$

where  $\bar{g}$  is independent of  $N_f$  [17]. Moreover, we fix  $a(\Lambda_f; N_f) = 1$  even off the critical point, since the Wilsonian matching conditions with the physical inputs  $F_\pi(0) = 88$  MeV and  $m_\rho = 770$  MeV leads to  $a(\Lambda) \simeq 1$  already for  $N_f = 3$  [1]. The RGE's for  $F_\pi^2$  and  $g^2$  are analytically solvable for  $a = 1$ . A careful analysis [9] leads to that  $q$  in Eq. (9) must satisfy  $q \geq 1$  for the consistency. The resultant critical behaviors of the order parameter and the mass of  $\rho$  are given by

$$\begin{aligned} F_\pi^2(0; N_f)/\Lambda_f^2 &\sim \epsilon \rightarrow 0, \\ m_\rho^2(N_f)/\Lambda_f^2 &\sim \epsilon^{1+q} \rightarrow 0, \end{aligned} \quad (10)$$

which shows that  $m_\rho$  approaches to zero faster than  $F_\pi$ . This is a salient feature of the VM [18].

Let us consider the critical behaviors of the physical quantities listed in Ref. [1]. The  $\rho$ - $\gamma$  mixing strength  $g_\rho$  and the  $\rho$ - $\pi$ - $\pi$  coupling constant  $g_{\rho\pi\pi}$  go to zero as

$$\begin{aligned} g_\rho(m_\rho) &= g(m_\rho) F_\pi^2(m_\rho) \sim \epsilon^{1+q/2} \rightarrow 0, \\ g_{\rho\pi\pi}(m_\rho, 0, 0) &= \frac{g(m_\rho)}{2} \frac{F_\pi^2(m_\rho)}{F_\pi^2(0)} \sim \epsilon^{q/2} \rightarrow 0, \end{aligned} \quad (11)$$

where  $a(\Lambda) = a(m_\rho) = 1$  was used. As discussed in Ref. [1], the KSRF (I) relation for the low-energy quantities  $g_\rho(0) = 2g_{\rho\pi\pi}^2(0, 0, 0)F_\pi^2(0)$  holds as a low energy theorem of the HLS [4,19,20] for any  $N_f$ . The relation for on-shell quantities is violated by about 10% for  $N_f = 3$  [1]. As  $N_f$  goes to  $N_f^{\text{cr}}$ ,  $g_\rho(m_\rho)$  and  $g_{\rho\pi\pi}(m_\rho, 0, 0)$  approach to  $g_\rho(0)$  and  $g_{\rho\pi\pi}(0, 0, 0)$ , respectively, and hence the on-shell KSRF (I) relation becomes more accurate for larger  $N_f$ . On the other hand, the (on-shell) KSRF (II) relation  $m_\rho^2 = 2g_{\rho\pi\pi}^2(m_\rho, 0, 0)F_\pi^2(0)$  becomes less accurate. Near the critical flavor it reads as  $m_\rho^2 = 4g_{\rho\pi\pi}^2(m_\rho, 0, 0)F_\pi^2(0) \rightarrow 0$ .

Several comments are in order:

In the VM both the axialvector and vector current correlators in Eq. (3) take the form of  $F_\pi^2(\Lambda)/Q^2 - 2z_2(\Lambda)$ . For the axialvector current correlator, the first term  $F_\pi^2(\Lambda)/Q^2$  comes from the  $\pi$ -exchange contribution, while for the vector current correlator it can be easily understood as the  $\sigma$  (would-be NG boson absorbed into  $\rho$ )-exchange contribution in the  $R_\xi$ -like gauge. Thus only

the longitudinal  $\rho$  couples to the vector current, and the transverse  $\rho$  with the helicity  $\pm 1$ , which belongs to the representation  $(N_f, N_f^*) \oplus (N_f^*, N_f)$ , is decoupled from it. This can be also seen in the unitary gauge [9].

The parameters  $L_{10}(m_\rho)$  and  $L_9(m_\rho)$  defined in Ref. [1] diverge as  $N_f$  approaches to  $N_f^{\text{cr}}$ . However, we should note that, even for  $N_f = 3$ , both  $L_{10}(\mu)$  and  $L_9(\mu)$  have the infrared logarithmic divergences when we take  $\mu \rightarrow 0$  in the running obtained by the chiral perturbation theory [21]. Thus we need more careful treatment of these quantities for large  $N_f$ . This is beyond the scope of this paper.

The  $A_1$  in the VM is resolved and/or decoupled from the axialvector current near the critical flavor since there is no contribution in the vector current correlator to be matched with the axialvector current correlator. As to the scalar meson [22], although the mass is smaller than the matching scale adopted in Ref. [1] for  $N_f = 3$  [23], we expect that the scalar meson is also resolved and/or decoupled near the chiral phase transition point, since it is in the  $(N_f, N_f^*) \oplus (N_f^*, N_f)$  representation together with the  $A_1$  in the VM.

In this paper we applied the VM to the chiral restoration in the large  $N_f$  QCD. It may be checked by the lattice simulation: The vanishing ratio  $m_\rho/F_\pi(0)$  is a clear indication of the VM.

The VM may be applied to other chiral phase transitions such as the one at finite temperature and/or density. In such a case, the position of the  $\rho$  peak of the dilepton spectrum would move to the lower energy region in accord with the picture shown in Ref. [24], and Eqs. (10) and (11) would further imply smaller  $\rho$  width ( $\Gamma/m_\rho \sim g_\rho^2 \pi \pi \sim \epsilon^q$ ) and larger peak value [ $\Gamma_{ee} \Gamma_{\pi\pi} / \Gamma^2 \sim g_\rho^2 / (g_\rho^2 \pi \pi m_\rho^4) \sim 1/\epsilon^{2q}$ ] near the critical point. If it is really the case, these would be clear signals of VM tested in the future experiments.

The VM studied in this paper may be applied to the models for the composite  $W$  and  $Z$ . Our analysis shows that the mass of the composite vector boson approaches to zero faster than the order parameter, which is fixed to the electroweak symmetry breaking scale, near the critical point. The VM may also be applied to the technicolor with light techni- $\rho$ .

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