PHYSICAL REVIEW LETTERS

VOLUME 86 29 JANUARY 2001 NUMBER 5

A New Look at the Accelerating Universe

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(Received 18 September 2000)

We show that the cosmological model having *zero* cosmological constant, but containing the vacuum energy of a simple quantized free scalar field of low mass (VCDM model), agrees with the cosmic microwave background radiation (CMBR) and supernova (SNe-Ia) data at least as well as the classical cosmological model with a small nonzero cosmological constant. We also show that in the VCDM model the ratio of vacuum pressure to vacuum energy density is less than -1. We display the VCDM results for a set of parameters that give a very good fit to the CMBR power spectrum, and show that the same parameters also give a good fit to the SNe-Ia data.

DOI: 10.1103/PhysRevLett.86.749 PACS numbers: 98.80.Cq, 04.62.+v, 98.80.Es

I. Introduction.—Einstein is well known to have preferred a value of zero for the cosmological constant Λ that enters into his gravitational field equations. Until recently, it was thought that observational cosmology was consistent with Λ being zero. Even early inflation coming from the vacuum energy and pressure of a self-interacting scalar field (the inflation) does not require Λ to be nonzero. However, the evidence from the cosmic microwave background radiation (CMBR) power spectrum that the universe is spatially flat, together with the relatively low average density of matter, including cold dark matter (CDM), implies that there is a significant nonmaterial component of energy and pressure in the universe.

One possible explanation for the missing energy is a nonzero present value of the classical cosmological constant, which would also account for the observed acceleration of the universe implied by the SNe-Ia data. This model, having CDM and a nonzero cosmological constant, is the ΛCDM model. Another proposed class of models involves a *classical* scalar field having a small mass and a self-interaction involving several parameters. Such a classical scalar field is known as quintessence. Although quintessence models do not require a nonzero Λ, the ones which fit the data evidently require more parameters than the ΛCDM model and involve nonrenormalizable self-interaction potentials. Although they may perhaps arise as a limit of a renormalizable theory, one cannot directly quantize them. In addition, for all quintessence models, the ratio $w_{\mathrm{quint}} \equiv p_{\mathrm{quint}}/\rho_{\mathrm{quint}}$ satisfies $w_{\mathrm{quint}} \geq -1$, where p_{quint} and ρ_{quint} are the pressure and energy density, respectively, of the quintessence field. In the ΛCDM model, the corresponding ratio is $w_\Lambda \equiv p_\Lambda/\rho_\Lambda$, where p_Λ and ρ_Λ are the pressure and energy density that the cosmological constant contributes to the Einstein equations. It satisfies $w_\Lambda = -1$.

In view of Einstein's preference for zero cosmological constant, it is important to ask if the recent cosmological data can be fit with $\Lambda = 0$, without introducing more adjustable parameters than there are in the ΛCDM model. Here we answer that question in the affirmative. We assume, as in the quintessence models, that there is a scalar field of small mass. However, our field is free in that it interacts only with gravitation. It can therefore be quantized and renormalized so that its vacuum energy and pressure can be calculated under the assumption that $\Lambda = 0$ [1–3]. The matter content is essentially the same as the CDM of the ΛCDM model. However, the nonzero value of Λ is replaced by the vacuum energy and pressure of the *quantized* free scalar field. This is the VCDM model (where V signifies the vacuum energy of the quantized field). The number of adjustable parameters in the VCDM model is the same as in the ΛCDM model. However, the VCDM model is demonstrably different from the ΛCDM model and from any quintessence model. This is because the ratio, $w_{\mathrm{V}} \equiv p_{\mathrm{V}}/\rho_{\mathrm{V}}$, where p_{V} and ρ_{V} are the pressure and energy density of the quantized free scalar field in its *vacuum* state, satisfies $w_V < -1$, and approaches the value -1 in the limit of late times.

The vacuum energy ρ_V and vacuum pressure p_V of the VCDM model is negligible until a relatively recent transition time t_j (when the universe is about one-half its current age), after which the vacuum pressure becomes negative (while the energy density remains positive) in such a way to cause the scalar curvature R to subsequently remain constant. The key to the VCDM model is that the scalar curvature is held constant by a resonance that occurs in the vacuum expectation value of the scalar field energy-momentum tensor. This resonance is a nonperturbative effect [1,4,5] that exists only in spacetimes of nonzero curvature.

In place of the cosmological constant Λ in the VCDM model is the parameter $\overline{m}$, where $\overline{m} \equiv m/\sqrt{\frac{1}{6} - \xi}$. Here m is the inverse Compton wavelength associated with the mass of the scalar field, and ξ is a constant that couples the field to the scalar curvature R in the free field equation. The field is called free because it interacts with no field except, of course, gravity, and in the limit of zero curvature it reduces to the free scalar field. When $\xi = 0$, the only interaction with gravity is through the usual covariant derivatives in the field equation. We assume that $\xi < 1/6$. For comparison with the observed cosmological data, only the parameter $\overline{m}$ is relevant. Thus, the number of parameters that we may adjust when trying to fit the data is the same in the VCDM model as in the ΛCDM model.

In our comparison with data below, we pay particular attention to the recent BOOMERANG and MAXIMA data [6,7] on fluctuations of the CMBR, and to the SNe-Ia data that imply an accelerating universe [8,9]. For comparison with the CMBR fluctuation data we have extended our VCDM model to include the presence of background radiation, and we have modified the CMBFAST [10] computer code to calculate the predicted CMBR fluctuation spectrum for our model. As measured by the χ^2 test, the fit to the CMBR fluctuation spectrum of our quantized field model is as good as the fit to the data by the ΛCDM model [11].

Comparison of our VCDM model with the SNe-Ia data shows that our predicted curve is significantly different from the curves predicted by the ΛCDM model and by quintessence models. Nevertheless, it fits the SNe-Ia data at least as well as the others. The main observational difference between our model and others is that our model gives a higher maximum and a more rapid falloff of the magnitude-redshift curve [$\Delta(m - M)(z)$ vs z] at high redshifts than the other models. Therefore, further SNe-Ia data, especially at redshifts larger than 1, may be effective in differentiating between our model and other models.

We also calculate the ratio w_V that clearly distinguishes our quantized free field model at the theoretical level from all other models that have been proposed. It has been noted by Caldwell [12] that the observed data may favor a ratio w that is less than -1, and that no quintessence model can give such a ratio of pressure to energy density. Thus, the fact that in our model $w_V < -1$ may be in its favor over quintessence and ΛCDM models. To our knowledge, only the VCDM model has $w_V < -1$ and a *positive* energy density, and has a simple microphysical basis with $\Lambda = 0$. (Caldwell [12] considers an admittedly *ad hoc* model having a negative kinetic energy term and refers to it as a phantom energy model.)

In our equations, we use units with $c = 1$.

II. CMBR Fluctuation Spectrum.—In order to modify the CMBFAST code to determine the power spectrum of the fluctuations of the CMBR temperature, we have generalized our solution of the Einstein equations to include the presence of radiation, in addition to the matter and quantized scalar field in its vacuum state that we had before [1–3]. This more general spatially flat solution corresponds to a mixed matter and radiation solution for $t < t_j$, joined with continuous first and second derivatives to a constant scalar curvature solution for $t > t_j$ containing vacuum energy, nonrelativistic matter, and radiation. The implicit solution for the cosmological scale factor $a(t)$ for $t < t_j$, in terms of the constants ρ_{mj} (the density of pressureless matter at time t_j) and ρ_{rj} (the radiation energy density at time t_j), is found to be

$$\frac{2}{3\rho_{mj}^2}\left(\rho_{mj}\frac{a(t)}{a(t_j)} + \rho_{rj}\right)^{1/2} \times \left(\rho_{mj}\frac{a(t)}{a(t_j)} - 2\rho_{rj}\right) = \sqrt{\frac{8\pi G}{3}}\, t - \frac{4}{3}\frac{\rho_{rj}^{3/2}}{\rho_{mj}^2}, \qquad t < t_j. \tag{1}$$

The time t_j, when quantum vacuum terms in the Einstein equations first become significant, is defined by the condition [1–3]

$$\rho_{mj} = \frac{\overline{m}^2}{8\pi G}. \tag{2}$$

For the range $t > t_j$, the scalar curvature R is essentially constant having the value $\overline{m}^2$, and the solution is

$$\frac{a(t)}{a(t_j)} = \left[\frac{\sqrt{12}\,H(t_j)}{\overline{m}}\sinh\left(\frac{\overline{m}}{\sqrt{3}}(t - t_j)\right) + \cosh\left(\frac{\overline{m}}{\sqrt{3}}(t - t_j)\right)\right]^{1/2}, \qquad t > t_j. \tag{3}$$

Here $H(t_j)$ is $\dot{a}/a$ evaluated at t_j. Equations (1) through (3) compose the basic set of equations defining our VCDM cosmological model when matter, radiation, and vacuum energy are present.

The above equations and their time derivatives may be used to express t_j, z_j (the redshift at time t_j), t_0 (the present age of the universe), and the mass scale $\overline{m}$ in terms of the *measurable* quantities H_0 (the present value of the Hubble constant), Ω_{r0} [the present ratio of radiation energy density to the critical density $\rho_c \equiv 3H_0^2/(8\pi G)$], and Ω_{m0} (the present ratio of nonrelativistic matter density to critical density).

We have modified the CMBFAST [10] package to incorporate our spatially flat solution in terms of the measurable parameters. Our modification of the CMBFAST code consists of adding to the usual matter and radiation pressure and energy density, the pressure and energy density contributions of the vacuum. This straightforward modification enables the CMBFAST code to compute the scale factor of the VCDM model, as well as the fluctuations about the background solution that it requires to compute the CMB anisotropy spectrum. No other modifications were made to CMBFAST. The vacuum pressure and energy density are taken to be homogenous in our modification to the CMBFAST code. We expect inhomogeneities in the vacuum pressure and energy density to be significant only over scales comparable to the size of the universe, which would not affect the small-angle CMBR fluctuations.

The modified package requires the input parameters H_0, Ω_b (the present ratio of the density of baryonic matter to critical density), $\Omega_{\rm CDM}$ (the present ratio of the density of nonbaryonic cold dark matter to critical density), and $T_{\rm CMB}$ (the present temperature of the cosmic microwave background), in addition to other standard early universe parameters such as the number of massless and massive neutrinos and the spectral index of primordial fluctuations. It should be kept in mind that $\Omega_{\rm CDM} + \Omega_b = \Omega_{m0}$ and that $T_{\rm CMB}$ and the number of massless neutrinos together fix the value of Ω_{r0}. The input parameters $\Omega_{\rm CDM}$, Ω_b, $T_{\rm CMB}$, and the number of massless neutrinos are used by the modified CMBFAST code to compute $\overline{m}$, which determines the time evolution of our cosmological model.

The results of the modified CMBFAST code are given in Fig. 1. The data points shown in Fig. 1 are the combined data from the BOOMERANG [6] and MAXIMA [7] measurements. The solid curve shown is the theoretical curve predicted by our spatially flat VCDM model, with $\Omega_{\rm CDM} = 0.50$, $\Omega_b = 0.06$, and $h = 0.7$, where $h = H_0/(100\ {\rm km\,s^{-1}\,Mpc^{-1}})$. These parameters give the best fit to the BOOMERANG and MAXIMA data points in our VCDM model, with a χ^2 value of $\chi^2 = 24.96$ (assuming spatial flatness and a scale-invariant spectrum of initial perturbations, as follows from early inflation). For comparison, we have also shown the predicted curve of the ΛCDM model (dashed curve) with the *same* values of $\Omega_{\rm CDM}$, Ω_b, and h as the VCDM model.

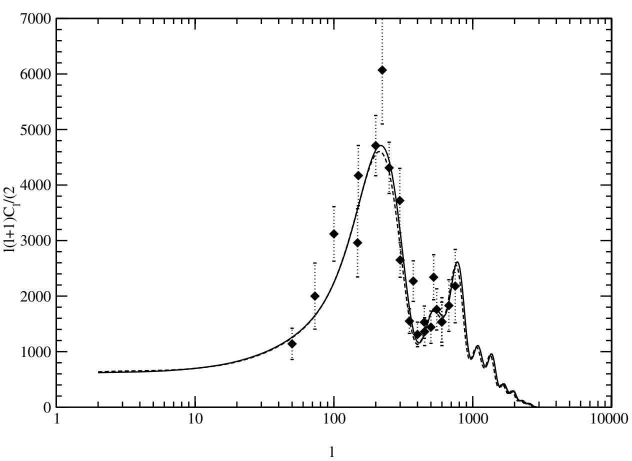

FIG. 1. A plot of the CMBR fluctuation spectrum $l(l+1)C_l/(2\pi)$ [in units of $(\mu K)^2$] versus multipole number l. The plotted data points with error bars are the combined data from the BOOMERANG and MAXIMA experiments. The solid curve refers to the spatially flat VCDM model, with $h = 0.7$, $\Omega_b = 0.06$, $\Omega_{\rm CDM} = 0.5$. The dashed curve is obtained from the spatially flat classical ΛCDM model with the same parameters.

Note that the parameter values we consider are within the observationally allowed ranges given by Tegmark and Zaldarriaga [11]. The baryon density is larger than that currently favored by big bang nucleosynthesis. A high baryon density is generally required to suppress the second acoustic peak of the CMBR fluctuation spectrum if one assumes early inflation, in our model as well as in other models. Tegmark and Zaldarriaga find that the best fit ΛCDM model, assuming early inflation, has $\Omega_\Lambda \equiv \rho_\Lambda/\rho_c = 0.43$, $h = 0.63$, $\Omega_{\rm CDM} = 0.50$, and $\Omega_b = 0.076$. Their best fit value of h is lower than the best fit value for the VCDM model primarily because they fit theoretical curves to *all* the data on CMBR fluctuations, not just the recent BOOMERANG and MAXIMA data. If we were to fit to all the CMBR data we would also obtain a smaller value of h (and a correspondingly larger age for the universe), while our fit to the SNe-Ia data would continue to be good.

It is noteworthy that the VCDM model gives a higher first acoustic peak than the corresponding ΛCDM model with the same parameter values. This may be a general feature of models that have a vacuum equation of state such that $w_{\rm V} < -1$, as has been pointed out for phantom models by Caldwell [12].

With the above best fit of our model to the CMBR fluctuation data, we have $\Omega_{m0} = \Omega_b + \Omega_{\rm CDM} = 0.56$, from which we obtain for $\overline{m}$ a mass of 4.5×10^{-33} eV. This gives a value of $t_0 = 11.7$ Gyr for the age of the universe, and a value of $z_j = 0.76$. The age is compatible with the age of the oldest globular clusters, which lies in the range 10–17 Gyr at the 95% confidence level [13]. (The corresponding age in the ΛCDM model, with the same values of h and Ω_{m0}, is 11.2 Gyr.) With the same value of $\overline{m}$, the predicted magnitude-redshift relation gives a good fit to the SNe-Ia data, as we now show.

III. High Redshift Type-1a Supernovae.—The VCDM model has a remarkably different recent evolution than the ΛCDM model and the usual quintessence models that involve a classical rolling scalar field. The difference is clear from the form of the vacuum energy density and pressure. In an earlier work [3], we showed that the vacuum equation of state for $t > t_j$ in our model (this equation of state is not altered by the presence of radiation) is given by

$$\rho_{\rm V} = 3p_{\rm V} + \frac{\overline{m}^2}{8\pi G}\left\{1 - \left(1 + \frac{32\pi G}{\overline{m}^2}p_{\rm V}\right)^{3/4}\right\}, \quad (4)$$

where $p_{\rm V}$ and $\rho_{\rm V}$ are the vacuum pressure and energy density, respectively, each of which can be separately expressed as functions of redshift z as follows:

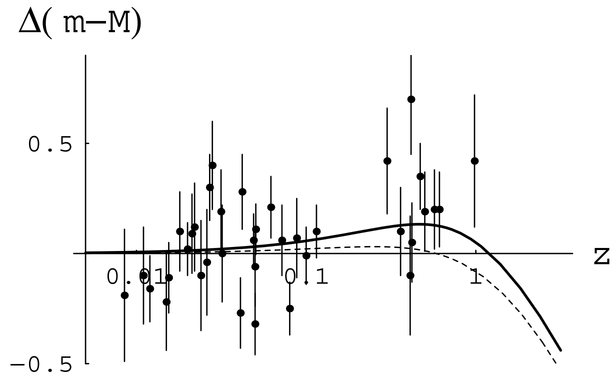

FIG. 2. A plot of the magnitude-redshift relation for high-redshift type Ia supernovae, with the data points of the Hi-Z group [7]. The y axis is the difference between apparent and absolute magnitudes, normalized to that difference in an open universe with $\Omega_{m0} = 0.2$. The solid curve corresponds to our VCDM model, with the parameters of Fig. 1. The dashed curve corresponds to the ΛCDM model with the same parameters.

$$\rho_{\rm V}(z) = \frac{\overline{m}^2}{32\pi G}(1 - 4x^3 + 3x^4), \qquad z < z_j, \tag{5}$$

$$p_{\rm V}(z) = \frac{\overline{m}^2}{32\pi G}(x^4 - 1), \qquad z < z_j, \tag{6}$$

where $x \equiv (1 + z)/(1 + z_j)$. Hence,

$$w_{\rm V}(z) \equiv \frac{p_{\rm V}(z)}{\rho_{\rm V}(z)} = \frac{x^4 - 1}{1 - 4x^3 + 3x^4}. \tag{7}$$

It is straightforward to verify that $w_{\rm V}(z) < -1$ and that $w_{\rm V}(z)$ is monotonically decreasing in the relevant interval $-1 < z < z_j$ (i.e., $0 < x < 1$), with the late time value $w_{\rm V}(-1) = -1$ and the transition time value $w_{\rm V}(z_j) = -\infty$. Although $w_{\rm V}$ is unbounded as $z \rightarrow z_j^-$, the quantity $w_{\rm tot} \equiv p_{\rm tot}/\rho_{\rm tot}$ is well behaved and has a positive value small with respect to 1 in this limit. Here $p_{\rm tot}$ and $\rho_{\rm tot}$ are the total pressure and energy density coming from matter, radiation, and the vacuum. Physically measurable quantities therefore are well behaved at all times.

A consequence of the unique feature that $w_{\rm V} < -1$ is that in the VCDM model the magnitude-redshift curve for high-redshift supernovae exhibits a higher maximum and a more rapid falloff at high z than the corresponding curve for the ΛCDM model. This behavior is apparent in Fig. 2, which has the same parameter values as in Fig. 1. A value of $\Omega_{m0} \equiv \Omega_{\rm CDM} + \Omega_b$ lower than the one considered would raise the maximum in both SNe-Ia curves and fit the higher data points better.

It is clear that better supernovae data at high redshifts will be able to distinguish between the magnitude-redshift curves of classical models and those of our VCDM model. It should be remarked that magnitude-redshift curves derived from quintessence models tend to have an even lower maximum than the ΛCDM model.

The number of gravitational lensing events of quasars by galaxies may also serve to differentiate between models, but currently the uncertainties in theoretical models of sources and lenses appear too great for us to draw a meaningful conclusion [14].

Finally, we note that hybrid models are also possible, in which a nonzero vacuum expectation value (VEV) of the scalar field contributes energy and pressure similar to that of a quintessence field, in addition to the quantum vacuum energy and pressure considered here. Similarly, one could include a nonzero cosmological constant. However, we have considered here the simplest VCDM model, with zero cosmological constant and zero VEV of the scalar field.

IV. Conclusion.—We have considered the effect of the vacuum energy and pressure of a *free* quantized scalar field of very low mass on a universe containing matter and radiation, but having *zero* cosmological constant. This VCDM cosmology is based on quantum field theory and general relativity, and is logically as simple as the ΛCDM model. We showed that the VCDM model is distinct from other proposed models. Furthermore, we found that the VCDM model is in at least as good agreement as other models with the present CMBR and SNe-Ia observations, and may be distinguished through further SNe-Ia data at high redshift. If the universe is indeed acting, through its own acceleration, as a detector of this very low mass quantized field, then there would be a wealth of implications for particle physics and cosmology.

We thank Lloyd Knox for use of the RADPACK software package, and Manoj Kaplinghat for use of the Fortran version of this package as well as helpful suggestions related to its implementation. We also thank Scott Dodelson, Josh Frieman, Micheal Turner, Sean Carroll, and Laura Mersini for useful discussions. This work was partially supported by the National Science Foundation.

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