

## Suppression of Superconductivity in Mesoscopic Superconductors

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We propose a new boundary-driven phase transition associated with vortex nucleation in mesoscopic superconductors (of size of the order of, or larger than, the penetration depth). We derive the rescaling equations and we show that boundary effects associated with vortex nucleation lower the conventional transition temperature in mesoscopic superconductors by an amount which is a function of the size of the superconductor. This result explains recent experiments in small superconductors where it was found that the transition temperature depends on the size of the system and is lower than the critical Berezinski-Kosterlitz-Thouless temperature.

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Phase transitions in two dimensions (2D) have been the subject of long-standing interest. The main reason is that the Berezinski-Kosterlitz-Thouless (BKT) transition [1–3] must arise in 2D systems like, for example, superfluids and quasi-2D superconductors. Berezinski [1] was the first to show that topological defects such as vortices may play a significant role in phase transitions. The fact that the energy required to create a vortex depends on the size of the system was the main obstacle to this fundamental discovery. Berezinski, however, recognized that the creation of a vortex-antivortex (V-A) pair is energetically more favorable, because the pair energy depends on the separation distance of the pair only. The mechanism of the BKT transition is the creation of V-A pairs decreases the superfluid density, which in turn decreases the binding energy of the V-A pair. As a result, it becomes easier for more pairs to be created and again this reduces the superfluid density further. This renormalization process continues until the screening effect is large enough for vortices and antivortices to nucleate freely and spontaneously. Since single vortices destroy the phase correlation needed for 2D superfluidity, the superfluidity is destroyed and there is a jump discontinuity in the superfluid density at the transition.

In three dimensions (3D), the creation of a V-A pair requires an enormous amount of energy depending on the size of the system and it was believed that a generalization of the BKT transition to 3D systems was not possible. However, Williams [4] and Shenoy [5] independently recognized that pairs of vortex loops may play the same role as V-A pairs in 3D. With these ideas, Williams [4] constructed a model of the superfluid transition in  $^4\text{He}$ . An analogous theory was developed by Shenoy [5] to describe the phase transition of the 3D XY model. More recently, Kusmartsev [6] proposed a mechanism for vortex nucleation in a flow of rotating superfluid  $^4\text{He}$ , based on a mechanism similar to the BKT transition, where, however, half-vortex rings (HVR's) play a role similar to V-A pairs. The HVR's penetrate the barrier with the help of critical fluctuations via the creation of an HVR "plasma." This leads to a topological phase transition where the barrier for vortex nucleation disappears and spontaneous thresh-

old vortex generation starts. Kusmartsev derived the scaling laws in the critical region and introduced and estimated the scaling relation and scaling exponent for the critical velocity.

The properties of a superconductor are expected to change radically when its size becomes comparable to that of the Cooper pairs since the creation energy of a vortex is then of the same order as that of a V-A pair. Recent progress in microfabrication techniques has made it possible to study experimentally (mesoscopic and nanoscopic) superconducting samples of micrometer and nanometer dimensions [7–9]. This has led to the discovery of new phenomena in the superconductivity of mesoscopic systems, particularly the discovery of the paramagnetic Meissner effect [9–12]. Moreover, while the value of the Ginzburg-Landau parameter  $\lambda/\xi$  (ratio of the coherence length  $\xi$  to the penetration length  $\lambda$ ) is sufficient to determine the type of bulk superconductors, both experiments [8,9] and theoretical work [13–17] on the magnetization of mesoscopic disks have shown that both the type and order of the transitions between the superconducting and the normal states depend on the size of the disk.

In this Letter we propose a boundary-driven phase transition associated with vortex nucleation in mesoscopic superconductors *not* previously reported in the literature. The nucleation of free vortices from the boundary drives the system to a topological phase transition with a lower critical temperature than the conventional or BKT critical temperature of the system. Nucleation of single vortices is prevented by their attraction to the boundary—vortices induced in the system, just like 2D electric charges, are attracted to their mirror antivortices (vortices of opposite polarity). A single "free" vortex can penetrate into the system only by overcoming the vortex-image antivortex (V-IA) interaction or, in other words, a surface or Bean-Livingston barrier. However, creation of other V-IA pairs close to the boundary may renormalize the "Coulomb" attraction to the boundary: the V-IA plasma screens the attraction of the vortex to the boundary just as in the BKT transition where the creation of V-A pairs screens an

effective interaction between a vortex and an antivortex. Eventually this leads to the creation of free vortices. These penetrate into the system and the order parameter associated with superconductivity is destroyed.

To illustrate this effect, let us consider a circular flat superconducting disk of radius  $R = O(\lambda)$  and thickness  $d \ll R$  and a vortex of vorticity  $\kappa = h/m^*$  ( $m^*$  is the mass of the Cooper pairs) at a distance  $r < R$  from the center of the disk. Then it is straightforward to show that the image vortex (of vorticity  $-\kappa$ ) is a distance  $r' = R^2/r$  from the center of the disk on a straight line joining the center to the vortex at  $r$  (see, for example, Ref. [18]). The interaction energy  $U_0$  of the V-IA pair depends logarithmically on the separation  $r' - r$ ,

$$U_0 = 2q^2 \ln \frac{R^2 - r^2}{rr_c} + E_c,$$

where  $E_c$  is the potential energy associated with the core of the vortex and  $r_c$  is the effective core radius. By analogy with the 2D Coulomb gas,

$$q = \left(\frac{\rho_s}{4\pi}\right)^{1/2} \kappa = (\pi\rho_s)^{1/2} \left(\frac{\hbar}{m^*}\right)$$

is the effective vortex charge.  $\rho_s \equiv \rho_s^{2D} = \rho_s^{3D}d$  is the 2D superfluid density.  $U_0(r)$  is the energy with which the vortex charge at  $r$  is attracted to the boundary (surface) of the superconducting material.

At low temperatures, near  $T = 0$  K, it is unlikely that more than only a few vortices will be present. However, at higher temperatures, there are likely to be many more vortex excitations, including some located in the space between  $r$  and  $R$ . These have an attenuating effect on, and screen, the interaction  $U_0(r)$ . Following Kosterlitz and Thouless [2,3] and Williams [4] and Shenoy [5], we take into account this screening effect by introducing a scale-dependent dielectric constant

$$\varepsilon(r) = 1 + 4\pi\chi(r). \quad (1)$$

The effective susceptibility  $\chi(r) = \int_{r_c}^R \alpha(r') dn(r')$ , where  $\alpha(r) = q^2(R - r)^2/2k_B T$  is the polarizability and

$n(r)$  is the number density of vortices. It is straightforward to show that  $dn(r) = 2\pi r dr \exp[-U(r)/k_B T]/r_c^4$ .  $U(r)$  is the screened interaction,

$$U(r) = 2q^2 \int_{r_c}^{(R^2-r^2)/r} \frac{dr'}{\varepsilon(r')r'} + E_c.$$

Introducing the dimensionless superfluid density  $K = q^2/(\pi k_B T)$  and the renormalized density  $K_r = K/\varepsilon(r)$ , Eq. (1) takes the form

$$K_r^{-1} = K^{-1} + \frac{4\pi^3 y_0}{r_c^4} \int_{r_c}^R dr (R - r)^2 r \times \exp\left[-2\pi K \int_{r_c}^{(R^2-r^2)/r} \frac{dr'}{\varepsilon(r')r'}\right],$$

where  $y_0 = \exp(-E_c/k_B T)$ . This derivation implicitly assumes a rather low density of vortices. This is evident, for example, in the fact that we have used the unrenormalized charge  $q$  instead of  $q_r = q/\varepsilon(r)$  to determine the polarizability. Next we also neglect the correction term in the V-IA interaction energy. Although the principal result is not changed, these approximations are necessary to prevent the equations from being intractable and lead to

$$K_r^{-1} = K^{-1} + \frac{4\pi^3 y_0}{r_c^4} \int_{r_c}^R dr (R - r)^2 r \times \exp\left[-2\pi K \ln \frac{R^2 - r^2}{rr_c}\right], \quad (2)$$

To make Eq. (2) self-consistent, one needs to replace  $K$  in the exponential by the renormalized density  $K_r$ . However, at low temperatures, the integral is small, and Eq. (2) is the first two terms in the expansion of  $K_r^{-1}$ .

At temperatures near the phase transition where the superfluid density tends to zero, the perturbation series is not valid. In this regime, we use the vortex-core rescaling technique proposed by José *et al.* [19] (see also the review by Wallace [20]). The procedure is to split the range of integration  $[r_c, R]$  into two parts,  $[r_c, br_c]$  and  $[br_c, R]$ , with  $b - 1 \cong \ln b \ll 1$ , and evaluate only the nonsingular contribution of small  $r$ . Rescaling  $r \rightarrow rb$  in the second integral to restore the original cutoff  $r_c$ , we find a perturbative expansion for  $K_r^{-1}$ :

$$\begin{aligned} \frac{1}{b} K_r^{-1} = & \frac{1}{b} \left[ K^{-1} + \frac{4\pi^3}{r_c^4} r_c^2 (R - r_c)^2 y_0 \exp\left(-2\pi K \ln \frac{R^2 - r_c^2}{r_c^2}\right) \ln b \right] \\ & + \frac{4\pi^3 y_0}{r_c^4} \int_{r_c}^{R/b} dr \left(\frac{R}{b} - r\right)^2 r b^3 \exp(-2\pi K \ln b) \exp\left[-2\pi K \ln \frac{R^2/b^2 - r^2}{rr_c}\right]. \end{aligned} \quad (3)$$

We now require that Eq. (3) has the same functional form as Eq. (2). This is achieved by introducing new variables at the increased scale,

$$K'^{-1} = \frac{1}{b} \left[ K^{-1} + 4\pi^3 \frac{(R - r_c)^2}{r_c^2} y \ln b \right], \quad (4a)$$

$$y' = b^3 y \exp(-2\pi K \ln b), \quad (4b)$$

together with  $R' = b^{-1}R$  and  $K_r = bK_r$ , where we have introduced the fugacity  $y = y_0 \exp\{-2\pi K \ln[(R^2 - r^2)/rr_c]\}$ . In deriving the above transformation, we have retained only terms of  $O(y)$ . It is convenient to build up a large increase in the core radius  $r_c$  by successive repetition of this transformation. In this way, one arrives

at differential renormalization group equations for the effective couplings  $K_l$  and  $y_l$ :

$$\frac{dK_l}{dl} = K_l - 4\pi^3 \frac{(R_l - r_c)^2}{r_c^2} K_l^2 y_l, \quad (5a)$$

$$\frac{dy_l}{dl} = (3 - 2\pi K_l)y_l, \quad (5b)$$

with the definition  $dl = \ln b$ . The scaled radius  $R_l$  of the disk satisfies  $dR_l/dl = -R_l$ .

The fixed point of the rescaling equations is defined by

$$(3 - 2\pi K_l)y_l = 0 \quad \text{and} \\ K_l - 4\pi^3 \frac{(R - r_c)^2}{r_c^2} K_l^2 y_l = 0,$$

which have the nontrivial solution

$$K^* = \frac{3}{2\pi} \quad \text{and} \quad y^* = \frac{r_c^2}{6\pi^2(R - r_c)^2}. \quad (6)$$

The critical point  $(K^*, y^*)$  separates the two phases of the system: the first is the low temperature phase, characterized by growing superfluid (renormalized) density  $K_l \approx K_0 e^l$  (with  $K_0$  being the initial value of  $K_l$  at scale size  $r_c$ ) and vanishing fugacity  $y_l \approx e^{-l/\xi_0}$ ,  $\xi_0 = (\pi K_0)^{-1}$ . The second one is the high temperature phase, characterized by exponentially growing fugacity: for infinite temperatures,  $K_l = 0$  gives  $y_l = y_0 e^{3l}$ , i.e., vortices proliferate.

The phase transition between these two regimes is easy to understand: At low temperatures,  $T = 0 + K$ , there are only a few vortices in the system (small fugacity). These

are attracted to the boundary and cannot nucleate. The coherence length  $\xi \sim O(r_c)$ . As the temperature increases, there is a growing number of vortex excitations and these screen the attraction to the boundary. The superfluid density decreases, the scaled radius  $R_l$  decreases, and the coherence length increases (as  $\sim e^l$ ). As the temperature increases further, there comes a point at which the screening is large enough for vortices to nucleate freely.  $R_l \sim \xi$ , the scaling stops, and a phase transition occurs.

To find the behavior of the scaling near the critical point, we rewrite the rescaling equations (5) in terms of scaled deviations from the fixed point: we expand  $K_l$  and  $y_l$  around  $K^*$  and  $y^*$  as  $K_l = K^*(1 + k')$  and  $y_l = y^*(1 + y')$ . The scaling equations (5) then become, to first order in  $k'$  and  $y'$ ,

$$\frac{d}{dl} \begin{pmatrix} k \\ y \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -3 & 0 \end{pmatrix} \begin{pmatrix} k \\ y \end{pmatrix}, \quad (7)$$

where we have dropped the primes. Expanding  $k$ ,  $y$ , and  $R$  in eigenstates  $A_{\pm}(l) = A_{\pm} e^{\lambda_{\pm} l}$  of the fixed-point stability matrix above, the eigenvalues are  $\lambda_+ = (\sqrt{13} - 1)/2$  and  $\lambda_- = -(\sqrt{13} + 1)/2$ . These define the relevant and irrelevant axes in the  $K_l$ - $y_l$  plane. We assume, following the existing procedure, that the relevant scaling field  $A_+$  is the temperature axis,  $A_+ \approx A|\epsilon|$ , where  $\epsilon = (1 - T/T_c)$  is the deviation of the temperature  $T$  from the transition (superconducting critical temperature)  $T_c$ , and  $A$  is a constant.

Then the rescaling law for the free energy  $F$  per unit area implies

$$Z(K_0, y_0, R_0) = e^{-(F_l - F_0)L^2} Z(K_l, y_l, R_l) \\ = e^{-(F_l - F_0)L^2} Z(A|\epsilon|e^{\lambda_+ l}, A_- e^{\lambda_- l}, R_0 e^{-l}), \quad (8)$$

where  $Z$  is the partition function and  $R_0$  is the (unscaled) radius of the superconducting disk. The scaling stops when  $R_l = R_0 e^{-l}$  reaches a critical radius  $R_c = \xi = r_c e^l$ . This happens when

$$l = l_c = \frac{1}{2} \ln \frac{R_0}{r_c}. \quad (9)$$

Setting  $l = l_c$  in Eq. (8), the partition function is well defined only if

$$\xi (= r_c e^{l_c}) = r_c |\epsilon|^{-1/\lambda_+}. \quad (10)$$

Combining Eqs. (9) and (10) gives  $|\epsilon|^2 = (r_c/R_0)^{\lambda_+}$ , whence

$$T = T_c \left[ 1 - \left( \frac{r_c}{R_0} \right)^{\lambda_+/2} \right]. \quad (11)$$

This result implies that the superconductivity breaks down at a temperature which is lower than the conventional superconducting or BKT critical temperature  $T_c$ . The mechanism of vortex nucleation into the disk we have described leads to a depression of  $T_c$  by an amount which varies inversely with the radius of the superconducting disk

$$\Delta T_c \propto T_c \left( \frac{r_c}{R_0} \right)^{\lambda_+/2}. \quad (12)$$

This depression in the critical superconducting/BKT temperature has been observed in the recent experiments of Geim *et al.* [21] on the magnetization of mesoscopic superconducting disks of various radii, typically with  $d \sim 0.1 \mu\text{m}$  and  $R_0 \sim 1-10 \mu\text{m}$ . For these (aluminum) disks,  $r_c$  (equal to the coherence length  $\xi_0$  at zero temperature) can be estimated as  $r_c \sim \xi_0 \sim 0.18 \mu\text{m}$ .

The phenomenon described above is very general. Obviously it may arise in any small system, such as superfluid droplets, quantum dots in a superconducting state, and so on. It may arise not only in superfluid or superconducting systems, but also in other condensed states, such as the magnetic state. In any case, topological defects originating from the surface/boundary (like vortices in superfluids and superconductors) have the potential to destroy a condensed state provided that the system has a small size and, in general, the critical temperature must decrease with the size of the system.

We have described a mechanism of vortex nucleation in superconducting systems of size comparable to the characteristic size (coherence length) of the quasiparticles of the systems, the Cooper pairs. A vortex created in a small superconductor is strongly attracted to the boundary of the system and hence cannot nucleate. The attraction is due to the Coulomb attraction of the “vortex charge” to the vortex charge of the image antivortex. However, a fluctuating creation of several of these topological defects in the bulk of the superconductor, in the space between the vortex and the boundary, screens the V-IA attraction by renormalizing the superfluid density. This further improves the condition for the creation of more such fluctuations. The screening effect of the plasma of vortex fluctuations continues until vortices nucleate freely. This leads to a phase transition at which the order parameter associated with the superconductivity of the system is destroyed.

We have used a real-space renormalization group method to derive the scaling laws in the vicinity of this phase transition and shown that the phase transition occurs at a temperature which is lower than the superconducting or BKT temperature  $T_c$ . The amount by which  $T_c$  is lowered is equal to  $T_c(r_c/R_0)^a$  (with  $a > 0$ ), showing that the depression in the transition temperature increases with decreasing disk radius  $R_0$ , in agreement with recent experiments [21] on mesoscopic superconductors.

The properties of a mesoscopic superconductor are remarkably different from those of macroscopic or even microscopic superconductors, being dependent on the size of the system. In this Letter, we have predicted a new phase transition that occurs in such confined systems and driven by the boundary of the systems. More precisely, this phase transition, although similar to the BKT transition, is driven by the creation of a plasma between the vortex and the boundary which screens the Coulomb attraction of the vortex to the boundary. In this respect, the driving mechanism for the phase transition is analogous to the mechanism proposed by Kusmartsev [6] for the nucleation of vortices in a flow of rotating superfluid  $^4\text{He}$ .

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