Theory for Toroidal Momentum Pinch and Flow Reversal in Tokamaks

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It is demonstrated that besides the well-known toroidal momentum diffusion flux there is a pinchlike flux in the fluctuation-induced toroidal stress. A toroidal flow profile is determined up to a constant, e.g., the value of the flow at the magnetic axis, by balancing these two fluxes. The remaining residual toroidal stress determines the value of the flow at the axis. It is illustrated that the direction of the flow at the axis can change after plasma confinement is improved. The theory is applied to explain the toroidal flow reversal in tokamak experiments.

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It is observed in recent experiments in Alcator C-MOD tokamak that toroidal flow can change direction after plasma confinement is improved from a low confinement mode (L mode) to a high confinement mode (H mode) with no apparent toroidal momentum source [1]. A similar phenomenon is observed in other tokamaks as well [2]. Judging from the importance of the toroidal rotation profile to the core plasma confinement improvement it is imperative to understand this peculiar toroidal flow reversal phenomenon.

The key to unravel the mystery of the toroidal flow reversal is to investigate the toroidal stress, which controls the flow relaxation. Because it is known that neoclassical toroidal stress is not adequate to explain the toroidal momentum confinement, we develop a theory for the fluctuation-induced toroidal stress. To this end, we adopt a neoclassical quasilinear theory [3,4].

Even though the fluctuation-induced stress is calculated for a double periodic system, such as a tokamak, the results can be easily translated to a slab geometry. Our theory is, thus, applicable in other areas of plasma physics as well.

We find that besides the well-known diffusion flux in the radial toroidal momentum transport, there is a pinchlike flux. The direction and the magnitude of the pinchlike flux depend on the mode frequency spectrum. By balancing the diffusion and the pinchlike fluxes, toroidal rotation profile is determined up to a constant, e.g., the flow speed at the magnetic axis $U_{\parallel 0}$. To determine $U_{\parallel 0}$, we need to consider the remaining small fluxes, the residual stress. It is demonstrated that the fluctuation-induced residual stress can change sign as the mode frequency decreases from the ion diamagnetic drift frequency or as the mode shifts from one side of the mode rational surface to the other. This provides a natural explanation for the toroidal flow reversal observed in tokamak experiments. (Note that the sign convention for the mode frequency ω employed here is opposite to the conventional one.)

We seek the solution to the drift kinetic equation [5]

$$\frac{\partial f}{\partial t} + (v_\parallel \hat{n} + \boldsymbol{v}_d) \cdot \nabla f + \frac{e}{M} \frac{\partial \Phi}{\partial t} \frac{\partial f}{\partial \epsilon} = C(f), \quad (1)$$

where f is the particle distribution, $v_\parallel$ is the parallel (to $\boldsymbol{B}$) particle speed, $\boldsymbol{B}$ is the magnetic field, $\hat{n} = \boldsymbol{B}/B$, $\boldsymbol{v}_d$ is the drift velocity, e is the electric charge, M is the mass, Φ is the electrostatic potential, $\epsilon = v^2/2 + e\Phi/M$ is the particle energy, v is the particle speed, and C is the Coulomb collision operator. The independent variables of Eq. (1) are $(\epsilon, \mu, \psi, \theta, \zeta)$ where μ is the magnetic moment, ψ is the poloidal flux function, θ is the poloidal angle, and ζ is the toroidal angle.

For simplicity, we are interested only in the electrostatic fluctuation-induced toroidal stress. Thus, only fluctuation-induced $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity is kept in the linearization process. The ∇B and curvature drifts that contribute to the neoclassical toroidal stress are neglected [6].

Assuming $(\omega, \omega_d) < (\omega_t, \nu)$, where ω_d is the drift frequency, ω_t is the transit frequency, and ν is the collision frequency, we obtain the lowest order equation

$$v_\parallel \hat{n} \cdot \nabla f_0 = C(f_0). \quad (2)$$

The solution to Eq. (2) is

$$f_0 = f_M = \frac{N}{\pi^{3/2} v_t^3} \exp\left(-\frac{v^2}{v_t^2} - \frac{2e\tilde{\Phi}}{M v_t^2}\right), \quad (3)$$

where N is the equilibrium plasma density, $\tilde{\Phi} = \Phi - \langle \Phi \rangle$ is the fluctuating potential, $\langle \Phi \rangle$ is the equilibrium potential, $v_t = (2T/M)^{1/2}$ is the thermal speed, T is the temperature, and f_M is a Maxwellian distribution. The next order equation is then

$$\frac{\partial f_1}{\partial t} + (v_\parallel \hat{n} + \boldsymbol{v}_d) \cdot \nabla f_1 + \frac{e}{M} \frac{\partial \tilde{\Phi}}{\partial t} \frac{\partial f_1}{\partial \epsilon} + \boldsymbol{v}_d \cdot \nabla f_0 + \frac{e}{M} \frac{\partial \tilde{\Phi}}{\partial t} \frac{\partial f_0}{\partial \epsilon} = C(f_1). \quad (4)$$

To solve Eq. (4), we expand f_1 as [6]

$$f_1 = g + \frac{2 v_\parallel U_\parallel}{v_t^2} f_M + \frac{2 v_\parallel^2 U_\parallel^2}{v_t^4} f_M + \dots, \quad (5)$$

where g is a localized (in phase space) distribution, and $U_\parallel$ is the parallel mass flow speed. Note that the second and

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the third terms on the right side of Eq. (5) are the expansion of a shifted Maxwellian distribution. Substituting Eq. (5) into Eq. (4), we obtain an equation for g

$$\frac{\partial g}{\partial t} + (v_\parallel \hat{n} + \boldsymbol{V}_E) \cdot \nabla g - C^T(g) = -(\tilde{D}_0 + \tilde{D}_1 + \tilde{D}_2 + \tilde{D}_3), \quad (6)$$

where $\boldsymbol{V}_E$ is the equilibrium $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity, C^T is the test particle collision operator, $\tilde{D}_0 = -(2v_\parallel/v_t^2) f_M (c\boldsymbol{B} \times \nabla\psi/B^2) \cdot (\nabla\theta \partial\tilde{\Phi}/\partial\theta + \nabla\zeta \partial\tilde{\Phi}/\partial\zeta) \partial U_\parallel/\partial\psi$, $\tilde{D}_1 = -(e/T) f_M [\partial\tilde{\Phi}/\partial\theta (U_p - 2/5 L_1^{(3/2)} q_p/p) + \partial\tilde{\Phi}/\partial\zeta (U_t - 2/5 L_1^{(3/2)} q_t/p)]$, $\tilde{D}_2 = -(v_\parallel U_\parallel e/T)(2 f_M/v_t^2)[\partial\tilde{\Phi}/\partial\theta (U_p - 2/5 L_1^{(5/2)} q_p/p) + \partial\tilde{\Phi}/\partial\zeta (U_t - 2/5 L_1^{(5/2)} q_t/p)]$, $\tilde{D}_3 = -(e/T)(\partial\tilde{\Phi}/\partial t) f_M (1 + 2v_\parallel U_\parallel/v_t^2 + 2v_\parallel^2 U_\parallel^2/v_t^4)$, $L_1^{(3/2)} = (5/2) - x^2$, $L_1^{(5/2)} = (7/2 - x^2)$, and $x^2 = v^2/v_t^2$. Note that $\tilde{D}_1 \sim \tilde{D}_3$ contributes mainly to the particle and the heat fluxes [4], and $\tilde{D}_0 \sim \tilde{D}_2$ contributes mainly to the momentum flux. The explicit definitions for U_p, U_t, q_p, and q_t are $U_p = U_\parallel \hat{n} \cdot \nabla\theta + (T/M)(\boldsymbol{B} \times \nabla\psi \cdot \nabla\theta/B\Omega)(p'/p + e\Phi'/T)$, $U_t = U_\parallel \hat{n} \cdot \nabla\zeta + (T/M)(\boldsymbol{B} \times \nabla\psi \cdot \nabla\zeta/B\Omega)(p'/p + e\Phi'/T)$, $q_p/p = (5/2)(T/M)(\boldsymbol{B} \times \nabla\psi \cdot \nabla\theta/B\Omega) T'/T$, and $q_t/p = (5/2)(T/M)(\boldsymbol{B} \times \nabla\psi \cdot \nabla\zeta/B\Omega) T'/T$, where p is plasma pressure.

In the quasilinear limit, Eq. (6) can be solved by approximating $C^T(q)$ with a Krook model, i.e., $C^T(g) = -\nu g$, and expanding $\tilde{\Phi} = \sum_{m,n,\omega} \tilde{\Phi}_{mn\omega} e^{i[\omega t + (m\theta - n\zeta + \eta_{mn})]}$ and $g = \sum_{m,n,\omega} g_{mn\omega} e^{i[\omega t + (m\theta - n\zeta + \eta_{mn})]}$, where m and n are mode numbers, and η_{mn} is the phase, to obtain

$$g_{mn\omega} = \frac{D_{mn\omega}}{\omega_{mn}^E + v_\parallel \tilde{n} \cdot \nabla\theta (m - nq) - i\nu}, \quad (7)$$

where

$$D_{mn\omega} = \omega \frac{e\tilde{\Phi}_{mn\omega}}{T} f_M \left(1 + \frac{2v_\parallel U_\parallel}{v_t^2} + \frac{2v_\parallel^2 U_\parallel^2}{v_t^4}\right) + \frac{e}{T} f_M \left[m\tilde{\Phi}_{mn\omega}\left(U_p - \frac{2}{5} L_1^{(3/2)} \frac{q_p}{p}\right) - n\tilde{\Phi}_{mn\omega}\left(U_t - \frac{2}{5} L_1^{(3/2)} \frac{q_t}{p}\right)\right]$$
$$+ \frac{e}{T} \frac{2f_M}{v_t^2} v_\parallel U_\parallel \left[m\tilde{\Phi}_{mn\omega}\left(U_p - \frac{2}{5} L_1^{(5/2)} \frac{q_p}{p}\right) - n\tilde{\Phi}_{mn\omega}\left(U_t - \frac{2}{5} L_1^{(5/2)} \frac{q_t}{p}\right)\right]$$
$$+ \frac{2v_\parallel}{v_t^2} f_M \frac{c\boldsymbol{B} \times \nabla\psi}{B^2} \cdot (m\tilde{\Phi}_{mn\omega} \nabla\theta - n\tilde{\Phi}_{mn\omega} \nabla\zeta) \frac{\partial U_\parallel}{\partial\psi}.$$

The Doppler-shifted frequency $\omega_{mn}^E = \omega + \omega_E$, and $\omega_E = (c\Phi'/B^2)(m\boldsymbol{B} \times \nabla\psi \cdot \nabla\theta - n\boldsymbol{B} \times \nabla\zeta \cdot \nabla\theta)$. In obtaining Eq. (7), we neglected the $(U_\parallel^3/v_t^3)$ and higher order terms. In the quasilinear limit $\nu \to 0$, we obtain the resonant part of the solution

$$g_{mn\omega}^R - i\pi\delta[\omega_{mn}^E + v_\parallel (m - nq)\hat{n} \cdot \nabla\theta] D_{mn\omega}, \quad (8)$$

where δ is the delta function.

The toroidal momentum equation is

$$\frac{\partial}{\partial t}\langle R^2 \nabla\zeta \cdot NM\boldsymbol{U}\rangle = -\langle R^2 \nabla\zeta \cdot \nabla \cdot \boldsymbol{P}\rangle + \frac{1}{c}\langle \boldsymbol{J} \cdot \nabla\psi\rangle, \quad (9)$$

where R is the major radius, $\boldsymbol{U}$ is the mass flow velocity, $\boldsymbol{J}$ is the plasma current density, c is the speed of light, and $\boldsymbol{P}$ is that total stress tensor. The mass in Eq. (9) is approximately the ion mass, and $\boldsymbol{U}$ is approximately the ion flow velocity. Because $\langle \boldsymbol{J} \cdot \nabla\psi\rangle$ is related to $\partial\langle \boldsymbol{E} \cdot \nabla\psi\rangle/\partial t$, at the steady state $\langle R^2 \nabla\zeta \cdot \nabla \cdot \boldsymbol{P}\rangle = 0$, or [7]

$$\frac{1}{V'} \frac{d}{d\psi} V' \langle R^2 \nabla\zeta \cdot \boldsymbol{P} \cdot \nabla\psi\rangle = 0, \quad (10)$$

where $V' = dV/d\psi$, and V is the volume enclosed by a flux surface. The kinetic expression for the toroidal angular momentum flux $\mathcal{P} = \langle R^2 \nabla\zeta \cdot \boldsymbol{P} \cdot \nabla\psi\rangle$ is $\langle R^2 \nabla\zeta \cdot \boldsymbol{P} \cdot \nabla\psi\rangle = \langle \int d^3v\, M(R^2 \nabla\zeta \cdot \hat{n}) v_\parallel (\boldsymbol{v}_d \cdot \nabla\psi) f\rangle$ [7].

To obtain the fluctuation-induced toroidal stress, we take the $(v_\parallel \boldsymbol{v}_d \cdot \nabla\psi)$ moment of $g_{mn\omega}^R$ and perform the random phase average over η_{mn}. The result is

$$\mathcal{P} = \langle \mathcal{P}_1\rangle + \langle \mathcal{P}_2\rangle + \langle \mathcal{P}_3\rangle, \quad (11)$$

where

$$\langle \mathcal{P}_1\rangle = -\sqrt{\pi} \frac{N v_t M I}{B} \sum_{m,n,\omega} \frac{\omega_{mn}^E \mathrm{sgn}(m - nq)}{[v_t (m - nq)\hat{n} \cdot \nabla\theta]^2} \left|\frac{e\Phi_{mn\omega}}{T}\right|^2 e^{-x_0^2} \cdot \left(m \frac{T}{M} \frac{c\boldsymbol{B} \times \nabla\psi \cdot \nabla\theta}{B^2}\right)^2$$
$$\times \left\{\left[\omega + (mU_p - nU_t) - \frac{2}{5}\left(m\frac{q_p}{p} - n\frac{q_t}{p}\right)\left(\frac{3}{2} - x_0^2\right)\right]\left(m \frac{T}{M} \frac{\boldsymbol{B} \times \nabla\psi \cdot \nabla\theta}{B\Omega}\right)^{-1}\right\},$$
$$\langle \mathcal{P}_2\rangle = -2\sqrt{\pi} \frac{N U_\parallel M I}{B} \sum_{m,n,\omega} \left(m \frac{T}{M} \frac{\boldsymbol{B} \times \nabla\psi \cdot \nabla\theta}{\Omega B}\right)^2 e^{-x_0^2} \frac{(\omega_{mn}^E)^2 |e\Phi_{mn\omega}/T|^2}{(v_t |m - nq| \hat{n} \cdot \nabla\theta)^3} \quad (12)$$
$$\cdot \left\{\left[\omega + (mU_p - nU_t) - \frac{2}{5}\left(\frac{5}{2} - x_0^2\right)\left(m\frac{q_p}{p} - n\frac{q_t}{p}\right)\right]\left(m \frac{T}{M} \frac{\boldsymbol{B} \times \nabla\psi \cdot \nabla\theta}{\Omega B}\right)^{-1}\right\},$$
$$\langle \mathcal{P}_3\rangle = -2\sqrt{\pi} \frac{NMI}{B} \frac{\partial U_\parallel}{\partial\psi} \sum_{m,n,\omega} \frac{e^{-x_0^2} (\omega_{mn}^E)^2 |e\Phi_{mn\omega}/T|^2}{(v_t |m - nq| \hat{n} \cdot \nabla\theta)^3} \left(m \frac{T}{M} \frac{\boldsymbol{B} \times \nabla\psi \cdot \nabla\theta}{\Omega B}\right)^2.$$

The x_0 here is $x_0 = \omega_{mn}^E/(v_t|m - nq|\hat{n}\cdot\nabla\theta)$, the symbol sgn$(m - nq)$ denotes the sign of $(m - nq)$, $I = RB_t$, and B_t is the toroidal magnetic field strength. We adopt $|\boldsymbol{B}\times\nabla\psi\cdot\nabla\zeta/\boldsymbol{B}\times\nabla\psi\cdot\nabla\theta| \ll 1$ in obtaining Eq. (12). Note that the parallel flow dependence in $(mU_p - nU_t)$, which appears in $\langle\mathcal{P}_1\rangle$ and $\langle\mathcal{P}_2\rangle$, is neglected for depending on $(m - nq)U_\parallel$ and because for localized mode $m - nq \approx 0$.

The $\langle\mathcal{P}_3\rangle$ is the well-known toroidal momentum diffusion flux [8–12]. The $\langle\mathcal{P}_2\rangle$ term is a new fluctuation-induced pinchlike flux. Because the pinchlike flux $\langle\mathcal{P}_2\rangle$ is a consequence of the $(\boldsymbol{v}_d\cdot\nabla\psi v_\parallel)$ moment of the distribution function, it is not the convective momentum flux associated with the particle flux Γ_p [11] which is the $(\boldsymbol{v}_d\cdot\nabla\psi)$ moment of the distribution function. Thus, even when the divergence of Γ_p vanishes at the steady state without particle sources, the $\langle\mathcal{P}_2\rangle$ term most likely will persist because the mode number dependence and energy-dependent coefficients for $\langle\mathcal{P}_2\rangle$ and Γ_p are not the same. The pinchlike term has been neglected in other quasilinear treatments [8,9,11] either because they are fluid treatments [9,10] or because they have different orderings in the kinetic treatments [12]. There is also a pinchlike flux in neoclassical toroidal stress [6]. The pinch velocity here is proportional to the gradients of plasma pressure and temperature. Note that it can also be an outward pinch. The direction of the pinch velocity depends on the details of the fluctuation spectrum. The $\langle\mathcal{P}_1\rangle$ flux originates from $\tilde{D}_0$ and $\tilde{D}_3$. It depends on the sign of $(m - nq)$. Thus for the symmetric modes, $\langle\mathcal{P}_1\rangle = 0$. However, because the spectrum $\Phi_{mn\omega}$ usually has a radial gradient and is increasing toward the outer edge, $\langle\mathcal{P}_1\rangle$ is, in general, finite.

Without taking the residual term $\langle\mathcal{P}_1\rangle$ into account, the toroidal angular momentum flux Γ_ϕ can be written as

$$\Gamma_\phi = -\chi_\phi\frac{\partial}{\partial\psi}\left(\frac{NMIU_\parallel}{B}\right) - \chi_\phi L_\psi\left(\frac{NMIU_\parallel}{B}\right), \quad (13)$$

where $\chi_\phi = \langle\mathcal{P}_3\rangle/[\partial(NMIU_\parallel/B)/\partial\psi]$, and $L_\psi = \{[\omega + (mU_p - nU_t) - (2/5)(5/2 - x_0^2)(mq_p/p - nq_t/p)] \times (mT\boldsymbol{B}\times\nabla\psi\cdot\nabla\theta/M\Omega B)^{-1}\}$. The second term on the right side of Eq. (13) is a pinch term if L_ψ is positive. For $\omega_{mn}^E \approx \omega_{*i}$, and assuming $x_0^2 < 1$, $\omega + (mU_p - nU_t) \approx 0$, and $L_\psi \approx -(5/2)(T'/T) \geq 0$ for usual temperature profiles. Here, $\omega_{*i} = -(mT\boldsymbol{B}\times\nabla\theta/MB\Omega)\times(p'/p)$. Note that $|q_t/p| \ll |q_p/p|$. In this case, the pinch velocity is $v_\phi \cong -(5/2)(T'/T)\chi_\phi$. As ω_{mn}^E decreases from ω_{*i}, the pinch velocity reduces because $[\omega + (mU_p - nU_t)] < 0$. If ω_{mn}^E reverses the sign or the density gradient is much steeper than the temperature gradient, pinch velocity can reverse the sign and becomes an outward convective velocity.

There does not seem to be a consensus on whether a momentum pinch flux is required to explain experimental observations on the toroidal rotation [13,14]. Based on our theory, the magnitude of the pinch velocity depends on the frequency spectrum. It is possible that the characteristics of the fluctuations in those two experiments are different. These differences could lead to qualitatively different pinch velocities.

In Ohmically heated tokamak plasmas, there is no obvious toroidal momentum source. The steady-state toroidal rotation velocity profile is determined approximately by balancing the diffusion term $\langle\mathcal{P}_3\rangle$ and the pinchlike term $\langle\mathcal{P}_2\rangle$. Note that those terms inside the curly brackets in Eq. (12) are not sensitive to the mode numbers m and n if we assume ω_{mn}^E is of the order of ω_* and $x_0^2 \lesssim 1$. With these assumptions, and $\omega_{mn}^E \approx \omega_{*i}$, the steady-state toroidal rotation profile is governed by

$$\frac{\partial U_\parallel}{\partial\psi} - \frac{5}{2}\frac{T'}{T}U_\parallel = 0. \quad (14)$$

To obtain Eq. (14), we have assumed that $B_p \ll B$ so that toroidal flow speed is approximated by $U_\parallel$. Note that Eq. (14) does not depend on $|\Phi_{mn\omega}|^2$ explicitly. It only depends on the fluctuations through frequency spectrum ω. A solution to Eq. (14) is

$$U_\parallel = U_{\parallel 0}\left(\frac{T}{T_0}\right)^{5/2}, \quad (15)$$

where the subscript 0 denotes the values at the magnetic axis. To determine $U_{\parallel 0}$, we need to consider the residual term $\langle\mathcal{P}_1\rangle$. The toroidal momentum equation is then

$$\frac{\partial U_\parallel}{\partial\psi} - \frac{5}{2}\frac{T'}{T}U_\parallel = R, \quad (16)$$

where R is the residual stress.

A general solution to Eq. (16) is

$$U_\parallel = U_{\parallel 0}\left(\frac{T}{T_0}\right)^{5/2} + \int_0^\psi d\psi' R\left[\frac{T(\psi')}{T(\psi)}\right]^{-5/2}. \quad (17)$$

The magnitude and the sign of $U_{\parallel 0}$ are now determined by imposing the boundary value of $U_\parallel$ at the plasma edge where $\psi = \psi_a$. If $U_\parallel(\psi_a) = 0$, $U_{\parallel 0}$ is

$$U_{\parallel 0} = -\int_0^{\psi_a} d\psi' R\left[\frac{T(\psi')}{T_0}\right]^{-5/2}. \quad (18)$$

The toroidal rotation profile is, thus, completely determined. It is interesting to note that toroidal rotation profile is related to temperature profiles if $\omega_{mn}^E \approx \omega_{*i}$.

If we assume that the fluctuation-induced toroidal stress dominates, the sign of $U_{\parallel 0}$ is the same as that of $-R$ which is $-R = (-\langle\mathcal{P}_1\rangle)/[(-\langle\mathcal{P}_3\rangle)/(\partial U_\parallel/\partial\psi)]$. Because $-\langle\mathcal{P}_3\rangle/(\partial U_\parallel/\partial\psi)$ is always positive, the sign of $U_{\parallel 0}$ is the same as that of $-\langle\mathcal{P}_1\rangle$. The sign of $-\langle\mathcal{P}_1\rangle$ is determined by the factor $S = -\Sigma_{m,n,\omega}\omega_{mn}^E \times \mathrm{sgn}(m - nq)/[v_t(m - nq)\hat{n}\cdot\nabla\theta]^2|\Phi_{mn\omega}|^2 e^{-x_0^2} \times [\omega + (mU_p - nU_t) - 2/5(mq_p/p - nq_t/p)(3/2 - x_0^2)]$. The sign of S is determined by three factors, ω_{mn}^E, $[\omega + (mU_p - nU_t) - (2/5)(mq_p/p - nq_t/p)(3/2 - x_0^2)]$, and $\mathrm{sgn}(m - nq)|\Phi_{mn\omega}|^2$. Because there is a m^2 factor in $|v_t(m - nq)\hat{n}\cdot\nabla\theta|^2$, the mode number dependences in ω_{mn}^E and $[\omega + (mU_p - nU_t) - (2/5)\times(mq_p/p - nq_t/p)(3/2 - x_0^2)]$ cancel roughly with that in $|v_t(m - nq)\hat{n}\cdot\nabla\theta|^2$ if ω_{mn}^E has the same mode number dependence as ω_* and if the mode width is not sensitive to the mode numbers. With this approximation the summation is mainly operated on $\mathrm{sgn}(m - nq)\,|\Phi_{mn\omega}|^2$. To determine the sign of $\mathrm{sgn}(m - nq)\,|\Phi_{mn\omega}|^2$, we assume the $\Phi_{mn\omega}$ is symmetrical relative to the mode rational surface where $m = nq$. Because experimentally $\Sigma_{m,n\omega}|\Phi_{mn\omega}|^2$ increases towards the outer radius we assume that $|\Phi_{mn\omega}|$ increases when it is centered at a larger radius. With these two assumptions, we conclude $\mathrm{sgn}(m - nq)\,|\Phi_{mn\omega}|^2 \propto \Delta\psi d|\Phi_{mn\omega}|^2/d\psi > 0$ for normal $\Sigma_{m,n\omega}|\Phi_{mn\omega}|^2$ profiles, and monotonic increasing q profiles. Here $\Delta\psi$ is the typical mode width measured in terms of ψ. Because $\mathrm{sgn}(m - nq)\,|\Phi_{mn\omega}|^2$ is positive, the sign of S is determined by ω_{mn}^E. If $\omega_{mn}^E \approx \omega_{*i}$, $\omega + (mU_p - nU_t) \approx 0$, $S < 0$ if $x_0^2 < 1$. This implies that $U_{\|0}$ is negative if $\omega_{mn}^E \approx \omega_{*i}$. When $\omega_{nn}^E < \omega_{*i}$, S can become positive because $\omega + (mU_p - nU_t) < 0$. In that case, $U_{\|0} > 0$. We see that as the mode frequency changes, the direction of the toroidal flow can be reversed.

We can also assume that the mode is not symmetric relative to the mode rational surface, when the radial electric field has a gradient in ψ. There is a theoretical evidence that the modes can shift away from the mode rational surface [15]. If, on the one hand, the mode is shifted to the larger radius, $\mathrm{sgn}(m - nq)\,|\Phi_{mn\omega}|^2$ is negative for monotonic increasing q profiles. If $\omega_{mn}^E \approx \omega_{*i}$ in this case, $S > 0$ and $U_{\|0}$ is positive. If, on the other hand, the mode is shifted to the smaller radius, $\mathrm{sgn}(m - nq)\,|\Phi_{mn\omega}|^2$ is positive, $S < 0$ and $U_{\|0} < 0$ if $\omega_{mn}^E \approx \omega_{*i}$.

It is observed in C-MOD that toroidal flow reverses when plasmas make an L-H transition [1]. Based on the theory developed here, this phenomenon can be explained in a few scenarios.

The first scenario: Assuming, in L-mode plasmas, ion temperature gradient driven mode is unstable and the mode is symmetrical, we have the situation where $\omega_{mn}^E \sim \omega_{*i}$ and toroidal rotation is negative. When plasmas make L-H transition, ion temperature gradient driven turbulence is more stable, because the ion temperature profile in the core region is flatter, ω_{mn}^E becomes less than ω_{*i} [16], and toroidal rotation becomes positive. The second scenario: Assuming the mode is not symmetrical and is shifted to the smaller radius in L mode, and assuming $\omega_{mn}^E \approx \omega_{*i}$, $U_{\|0} < 0$. When plasmas make a transition to H mode, because of the change of the radial electric profile, the mode may shift to the larger radius side and $U_{\|0}$ becomes positive assuming $\omega_{mn}^E \approx \omega_{*i}$, or any combinations of the first scenario and the second scenario.

The magnitude of the toroidal flow speed is of the order R, which is $R \sim (k_\| v_{ti}/\omega)(\Delta r/L_\phi)v_{ti}$, where L_ϕ is the radial scale length of the fluctuation spectrum $\Sigma_{mn\omega}|\Phi_{mn\omega}|^2$ and Δr is the typical mode width. For $\Delta r \sim 3\rho_i$, $L_\phi \sim a/2$, $\rho_{pi} \sim 10\rho_i$, and $k_\| v_{ti}/\omega \sim 2$, $R \simeq (\rho_{pi}/a)v_{ti}$ which is typically a tenth of the ion thermal speed. Here, a is the minor radius, ρ_i is ion gyroradius, and ρ_{pi} is the ion poloidal gyroradius. The observed magnitude of the toroidal rotation speed is of the order of $0.1 v_{ti}$.

In summary, we have developed a theory for toroidal momentum confinement in tokamaks. We find, besides the known diffusion flux, there is a pinchlike flux. The direction of the toroidal flow can reverse when plasmas make transitions from L mode to H mode. This provides an explanation for the observations made in C-MOD.

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[1] I. H. Hutchinson *et al.,* Phys. Rev. Lett. **84**, 3330 (2000).
[2] J. H. Coffey, in *Proceedings of the 11th Colloquium on UV and X-ray Spectroscopy of Astrophysical and Laboratory Plasmas, Nagoya, Japan, 1995,* edited by K. Yamashita and T. Watanabe, Frontiers Science Series No. 15 (Universal Academy Press, Tokyo, Japan, 1996).
[3] K. C. Shaing, Phys. Fluids **31**, 8 (1988).
[4] K. C. Shaing, Phys. Fluids **31**, 2249 (1988).
[5] F. L. Hinton and R. D. Hazeltine, Rev. Mod. Phys. **42**, 239 (1976).
[6] K. T. Tsang and E. A. Frieman, Phys. Fluids **19**, 767 (1976).
[7] R. D. Hazeltine, Phys. Fluids **17**, 961 (1974).
[8] T. Tange *et al.,* J. Phys. Soc. Jpn. **46**, 266 (1979).
[9] N. Mattor and P. H. Diamond, Phys. Fluids **31**, 1180 (1988).
[10] S-I. Itoh, Phys. Fluids **4**, 796 (1992).
[11] R. E. Waltz *et al.,* Phys. Plasmas **4**, 2482 (1997).
[12] H. Sugama and W. Horton, Phys. Plasmas **6**, 250 (1999).
[13] S. Scott *et al.,* Phys. Fluids B **2**, 1300 (1990).
[14] K. Nagashima, Y. Koida, and H. Shirai, Nucl. Fusion **34**, 449 (1994).
[15] W. Horton *et al.,* Phys. Plasmas **7**, 1494 (2000).
[16] M. Kotschenreuther (private communication).